ON-COMPACTNESS AND - CONNECTEDNESS IN INTUITIONISTIC TOPOLOGICAL SPACES

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Abstract. The aim of this paper is to introduce -compactness and -connectedness in intuitionistic topological spaces. Also some of their fundamental properties are investigated.

1. Introduction

Coker[2] after the introduction of the concept of intuitionistic sets and intuition-istic topological spaces studied some properties of intuitionistic continuity and in-tuitionistic compactness. Dogan Coker and Selma Ozcag initiated connectedness in intuitionistic topological spaces. Further, several researchers [6,8,10] studied some weak forms of intuitionistic topological spaces. In this paper some properties of in-tuitionistic -compactness and -connectedness in intuitionistic topological spaces are studied.

2. Preliminaries

De nition 2.1. [4] An intuitionistic set A is an object having the form hX; A_1; A_2i where A_1 and A_2 are subsets of X satisfying A_1 \ A_2 = . The set A_1 is called the set of members of A, while A_2 is called the set of non-members of A. Furthermore, let fA_i : i 2 Ig be an arbitrary family of intuitionistic sets in X,

where A_i = X; A^1_i; A^2_i then

= hX; ; Xi, X = hX; X; i A B if A_1 B_1 and A_2 B_2

A = hX; A_2 ; A_1i

\ A B = A B

[ ] A = hX; A_1; A^c_1i hi A = hX; A^c_2; A_2i

\$A_i$ = X; \$A^1_i$; [$A^2_i$ and [A_i = X; [A^1_i; \$A^2_i$.

De nition 2.2. [4] An intuitionistic topological space on a nonempty set X is a family of intuitionistic sets in X satisfying the following axioms:

, X 2

G_1 \ G_2 2 for G_1; G_2 2

[G_i 2 for any arbitrary family fG_i : i 2 Jg

In this case the pair (X;) is called intuitionistic topological space and any intu-itionistic set in is known as an intuitionistic open set in X, and the complement of an intuitionistic open set in X is intuitionistic closed.

De nition 2.3. [7] Let (X;) be an intuitionistic topological space. An intuition-istic set A of X is said to be

Intuitionistic semiopen if A Icl(Iint(A)) Intuitionistic preopen if A Iint(Icl(A))

Intuitionistic regular open if A = Iint(Icl(A))

The family of all intuitionistic preopen and intuitionistic regular open sets of (X;) are denoted by IP OS(X) and IROS(X) respectively.

De nition 2.4. [4] Let (X;) be an intuitionistic topological space and

A = hX; A1; A2i be an intuitionistic set in X. Then the several topologies generated by (X;) are

$_{0;1}$ = f[ ] A : A 2 g
$_{0;2}$ = fh i A : A 2 g
$_1$ = fA1 : hX; A1; A2i 2 g
$_2$ = f(A2)^c : hX; A1; A2i 2 g

De nition 2.5. [4]

If B = hY; B1; B2i is an intuitionistic set in Y , then the preimage of B under f, denoted by f ^{1}(B), is the intuitionistic set in X de ned by

f ^{1}(B) = X; f ^{1}(B1); f ^{1}(B2)

If A = hX; f(A1); f(A2)i is an intuitionistic set in X, then the image of A under f, denoted by f(A) is the intuitionistic set in Y de ned by

f(A) = hY; f(A1); f (A2))i where f (A2) = Y (f(X A2)).

De nition 2.6. [8] Let (X;) be an intuitionistic topological space. Then X is called intuitionistic disconnected if there exists intuitionistic open sets A 6= and B 6= such that A[B = X and A\B = . X is called intuitionistic connected if X is not intuitionistic disconnected.

De nition 2.7. [9] An intuitionistic topological space X is called intuitionistic C5-disconnected if there exists an intuitionistic set A which is both intuitionistic open and intuitionistic closed such that 6= A 6= X . X is called intuitionistic C5-connected, if X is not C5- disconnected.

3. I -Compactness in Intuitionistic Topological Spaces

De nition 3.1. Let (X;) be an ITS. If a family X; Gi^1; Gi^2 : i 2 K of intu-itionistic -open sets in X satis es the condition [X; Gi ; Gi: i 1 2 2 K = X then it is called an intuitionistic -open cover(brie y I 12

A nite subfamily of an intuitionistic -open cover X; Gi ; Gi : i 2 K of X, is also an intuitionistic -open cover of X is called a nite subcover of

X; Gi ; Gi : i 2 K .

De nition 3.2. An ITS (X;) is said to be intuitionistic -compact (I -compact) i each intuitionistic -open cover has a nite subcover.

De nition 3.3. Let (X;) be an ITS and A be an intuitionistic set in X. The fam-ily X; Gi^1; Gi^2 : i 2 K of intuitionistic -open sets in X is called an intuitionistic open cover of A if A X; G^1; G^2 : i K

De nition 3.4. An intuitionistic set A = hX; A_1; A_2i in an intuitionistic topolog-ical space (X;) is called intuitionistic -compact if and only if every intuitionistic -open cover of A has a nite subcover. Also an intuitionistic set A = hX; A_1; A_2i in (X;) is intuitionistic -compact i for each family G = f G : i 2 K g where G_i = X; G_i^1; G_i^2 : i 2 K of intuitionistic -open sets in X, A [i2K G_i and there exists a nite subfamily G : i = 1; 2:::n of such that

A $[_{i=1}G_i$ and A $\backslash_{i=1} G_i$

Theorem 3.5. Let (X;) be an intuitionistic topological space. Then (X;) is in-tuitionistic -compact i the intuitionistic topological space (X; $_{0;1}$) is intuitionistic -compact.

Proof. Necessity:Let (X;) be intuitionistic -compact and consider an intuitionis-tic -open cover f[ ] G_j : j 2 Kg of X in (X; $_{0;1}$). Since [([ ] G_j) = X , we obtain $[G^1_j$ = X and hence G^2_j $G^1_j{}^c$) $\backslash G^2_j$ $[G^1_j{}^c$ =) $[G_j$ = X . Since (X;) is intuitionistic -compact, there exists G_1,G_2...G_n such that $[^n_{i=1}G_i$ = X which implies $[^n_{i=1}G^1_i$ = X and $\backslash^n_{i=1}G^2_i$ = . So (X; $_{0;1}$) is intuitionistic - compact.

Su ciency: Suppose (X; $_{0;1}$) is intuitionistic -compact. Consider an intuitionis-tic -open cover f G_j^c : j 2 K g of X in (X;). Since $[G_j$ = X , $[G_j^1$ = X and hence $\backslash G_j^1$ = which implies $[G_j$ = X . Since (X; $_{0;1}$) is intuitionistic -compact, there exists G_1,G_2 ...G_n such that $[^n_{i=1} G^1_i$ = X and $\backslash^n_{i=1} G^2_i$ = . Hence (X;) is) X = $[_{i=1}G_i$ $\backslash_{i=1}$ G_i) $\backslash_{i=1}G_i$ = . Thus $[_{i=1}G_i$ = X . So intuitionistic -compact.

Theorem 3.6. The intuitionistic topological space (X;) is intuitionistic -compact i the intuitionistic topological space (X; $_1$) is intuitionistic -compact.

Proof. Similar to the Theorem 3.5.

Theorem 3.7. Intuitionistic -continuous image of an intuitionistic -compact space is intuitionistic compact.

Proof. Let f : (X;) ! (Y;) be intuitionistic -continuous from an intuitionistic -compact space X onto intuitionistic topological space Y . Let X; G_i^1; G_i^2 : i 2 K be an intuitionistic open cover of Y . Then f (G_i) : i 2 K is an intuitionistic -open cover of X. Since X is intuitionistic

f $^{-1}$ (G_1) ; f $^{-1}$ (G_2) ; ::::f $^{-1}$ (G_n) . Since f is onto, fG_1; G_2:::G_ng is an intuition-istic open cover of Y , which is nite. Therefore Y is intuitionistic compact.

Theorem 3.8. Every intuitionistic -compact space is intuitionistic compact.

Proof. Let X be intuitionistic -compact. Let X; G_i^1; G_i^2 : i 2 K be an intu-itionistic open cover of X. Then X; G_i ; G_i : i K is an intuitionistic -open cover of X as every intuitionistic itionistic -compact, intuitionistic -open cover X; G_i^1 ; G_i^2 : i 2 K of X has a nite subcover of X. Hence X is intuitionistic compact.

De nition 3.9. An intuitionistic topological space (X;) is said to be IT -space if every intuitionistic -closed set in (X;) is intuitionistic closed.

Theorem 3.10. If (X;) is intuitionistic compact and IT -space then (X;) is intuitionistic -compact.

i i 2
Proof. Let X; Gi^1 ; Gi^2 : i 2 K be an intuitionistic -open cover of X. Since
X is intuitionistic i i 2
(X;) is IT -space, X; G^1; G^2 : i K is an intuitionistic open cover of X. As
compact, intuitionistic open cover X; G^1; G^2 : i K of X
compact.

has a nite subcover. Hence (X;) is intuitionistic -

De nition 3.11. The function f is said to be intuitionistic -continuous if f ^{1}(V) is intuitionistic -open in (X;) for every intuitionistic open set V of (Y;).

Theorem 3.12. Let f : (X;) ! (Y;) be an intuitionistic -continuous map from an intuitionistic -compact space (X;) onto an intuitionistic topological space (Y;). If (Y;) is IT -space then Y is intuitionistic -compact.

is IT - i i 2
Proof. Let X; Gi^1; Gi^2 : i 2 K be an intuitionistic -open cover of Y . Since Y
tuitionistic - i 2
space, X; G^1; G^2 : i K is an intuitionistic open cover of Y . As f is in-
continuous, f ^{1} (G) : i K is an intuitionistic -open cover of X.
of X has a nite subcover f (G$_1$) ; f (G$_2$) ; ::::f (G$_n$) i=1 i
Since X is intuitionistic -compact, i i 2
= X and \i=1f Gi i
f 1 2 g 1 1 i 1i such that n f ^{1} G^1
n 1 2 = . This implies n G^1 = f(X) and n G^2 = f( ).
Hence G ; G ::::G$_n$ is a nite subcover of X; G^1; G^2 : i K of Y . Therefore

Y is intuitionistic -compact.

Theorem 3.13. Let f : (X;) ! (Y;) be surjective intuitionistic -continuous map. If (X;) is intuitionistic -compact then (Y;) is intuitionistic compact.

tuitionistic i 2
Proof. Let X; Gi^1; Gi^2 : i 2 K be an intuitionistic open cover of Y . As f is in-
such that -continuous, f ^{1} (G) : i K is an intuitionistic -open cover of X.
i=1f Gi = X and i=1f subcover Gi = . This implies i=1Gi = f(X
Since X is I -compact, it has a nite f 2 1 2 n
and \i=1Gi = f( ).
[n 1 1 \ n 1 1 2 ng n 1 [ )
n 2 Since f is surjective G ; G ::::G is an open cover of Y and

hence (Y;) is intuitionistic compact.

De nition 3.14. A mapping f : (X;) ! (Y;) is said to be intuitionistic -irresolute if the inverse image of every intuitionistic -open set of Y is intuition-istic -open in X.

Theorem 3.15. If f : (X;) ! (Y;) is an intuitionistic -irresolute mapping and A is intuitionistic -compact relative to X, then f(A) is intuitionistic -compact relative to Y .

Proof. Let fGi : i 2 Kg be an intuitionistic -open set of Y such that
f(A) [fGi : i 2 Kg. Then A [f ^{1} (Gi) : i 2 K where f ^{1} (Gi) is intuition-
istic -open in X for each i. Since A

exists a nite subcollection fG$_1$; G$_2$::::G$_n$g such that A [f ^{1} (Gi) : i = 1; 2:::n .

Hence f(A) is intuitionistic -compact relative to Y .

Theorem 3.16. Let f : (X;) ! (Y;) be an intuitionistic -irresolute mapping. If X is intuitionistic -compact, then Y is also an intuitionistic -compact space.

Proof. Let f : (X;) ! (Y;) be an intuitionistic -irresolute mapping from an intuitionistic -compact space X onto an intuitionistic topological space Y . Let f(Gi) : i 2 Kg be an intuitionistic -open cover of Y . Then f ^{1}(Gi) : i 2 K is an intuitionistic -open cover of X. Since X is intuitionistic -compact, there is a nite subfamily f ^{1}(A$_{i1}$); f ^{1}(A$_{i2}$):::f ^{1}(A$_{in}$) of f ^{1}(Ai) : i 2 K such that

$[^n_{j=1}G_{ij}$ = X: Since f is onto, f(X) = X and f $[^n_{j=1}f^{1}(G_{ij})$ = $[^n_{j=1}f\ f^{1}(G_{ij})$

= $[^n_{j=1}G_{ij}$. It follows that $[^n_{j=1}G_{ij}$ = X and the family fG_{i1}; G_{i2}:::G_{in}g is an intu-itionistic nite subcover of fG_i : i 2 Kg. Hence Y is intuitionistic -compact.

De nition 3.17. A function f : (X;) ! (Y;) is said to be strongly intuitionistic -continuous (brie y strongly I -continuous) if the preimage of every intuitionistic -open set of Y is intuitionistic open in X.

Theorem 3.18. If f : (X;) ! (Y;) is strongly intuitionistic -continuous from an intuitionistic compact space X onto an intuitionistic topological space Y , then
Y is intuitionistic -compact.

Proof. Let X; G_i^1 ; G_i^2 : i 2 K be an intuitionistic -open cover of Y . Since f is strongly intuitionistic -continuous, f ^{1}(G) : i K is an intuitionistic open cover of X. Since X is intuitionistic of X has a nite subcover f (G_1); f (G_2):::f (G_n) such that $[_n$ f ^{1}(G^1) = X and $\backslash^n_{i=1}f^{1}(G^2_i)$ = which implies $[^n_{i=1}G^1_i$ = f(X) and $\backslash^n_{i=1}G^2_i$ = f( ). Hence Y = $[^n_{i=1}G^1_i$ i.e., fG_1; G_2:::G_ng is a nite subcover of Y . Hence Y is intuitionistic
-compact.

De nition 3.19. An intuitionistic function f : (X;) ! (Y;) is said to be per-fectly intuitionistic -continuous if the inverse image of every intuitionistic -open set in Y is both intuitionistic open and intuitionistic closed in (X;).

Theorem 3.20. If f : (X;) ! (Y;) is perfectly intuitionistic -continuous from an intuitionistic compact space X onto an intuitionistic topological space Y , then
Y is intuitionistic -compact.

Proof. Since every perfectly intuitionistic -continuous is strongly intuitionistic -continuous, by previous Theorem, Y is intuitionistic -compact.

Theorem 3.21. The image of an intuitionistic -compact space under a strongly intuitionistic -continuous function is intuitionistic -compact.

Proof. Let f : (X;) ! (Y;) be strongly intuitionistic -continuous function from an intuitionistic -compact space onto an intuitionistic topological space Y . Let
X; G^1 ; G^2 : i K be an intuitionistic -open cover of Y . Then f ^{1} (G) : i K is an intuitionistic open cover of X as f is strongly intuitionistic -continuous and so f ^{1}(G_i) : i 2 K is an intuitionistic -open cover of X. Since X is intu-itionistic -compact, the intuitionistic -open cover f ^{1}(G) : i K of X has a nite subcover f ^{1}(G_1); f ^{1}(G_2):::f ^{1}(G_n) such that $[^n_{i=1}f^{1}(G^1_i)$ = X and $\backslash^n_{i=1}f^{1}(G^2_i)$ = ie., fG_1; G_2:::G_ng is a nite subcover of X; G^1_i; G^2_i : i 2 K
of Y . Hence Y is intuitionistic -compact.

Theorem 3.22. The image of an intuitionistic -compact space under an intu-itionistic -irresolute map is intuitionistic -compact.

Proof. Let f : (X;) ! (Y;) be intuitionistic -irresolute map from an in-tuitionistic -compact space X onto an intuitionistic topological space Y . Let
X; G^1 ; G^2 : i K be an intuitionistic -open cover of Y . Then f ^{1} (G) : i K
is an intuitionistic -open cover of X (since f is intuitionistic -irresolute). As X is intuitionistic -compact, the intuitionistic -open cover f ^{1}(G_i) : i 2 K of X has a nite subcover f ^{1}(G_1); f ^{1}(G_2):::f ^{1}(G_n) . Therefore $[^n_{i=1}(G^1_i)$ = f(X) and

\i n =1(Gi 2) = f( ). Thus fG1; G2:::Gng is a nite subcover of X; Gi 1 ; Gi 2 : i 2 K of Y . Hence Y is intuitionistic -compact.

De nition 3.23. An intuitionistic topological space (X;) is called intuitionistic -Lindelof if every intuitionistic -open cover of X contains a countable subcover.

Theorem 3.24. Every intuitionistic -Lindelof space is intuitionistic Lindelof.

Proof. Let (X;) be intuitionistic -Lindelof space. Let X; Gi 1; Gi 2 : i 2 K be -open cover of X. 1 2 an intuitionistic open cover of X. Then X; Gi ; Gi : i 2 K is an intuitionistic Lindelof. Since X is intuitionistic -Lindelof space, the intuitionistic -open cover X; G ; G : i 2 K of X has a countable subcover. Hence X is

Theorem 3.25. If X is intuitionistic Lindelof and IT -space then X is intuition-istic -Lindelof space.

X; Gi; Gi : i 2 K
Proof. Let X; Gi 1; Gi 2 : i 2 K be an intuitionistic -open cover of X. Then 1 2 is an intuitionistic open cover of X. Since X is IT -space and X is an intuitionistic Lindelof space, the open cover X; G 1; G 2 : i K of Lindelof.

X has a countable subcover. Hence X is intuitionistic -

Theorem 3.26. Every intuitionistic -compact space is intuitionistic -Lindelof space.

Proof. Let X be an intuitionistic -compact space. Let X; Gi 1; Gi 2 : i 2 K be cover fG1; G2::Gng. Since every nite an intuitionistic -open cover of X. Then X; Gi ; Gi : i 2 K has a nite sub- subcover is always a countable subcover,

fG1; G2:::Gng is a countable subcover of fG1; G2:::Gng of X. Therefore X is an intuitionistic -Lindelof space.

Theorem 3.27. Let f : (X;) ! (Y;) be an intuitionistic -continuous surjection and X be intuitionistic -Lindelof space then Y is a Lindelof space.

Proof. Let f : (X;) ! (Y;) be an intuitionistic -continuous surjection and X be intuitionistic -Lindelof. Let X; Gi ; Gi : i 2 K be an intuitionistic open cover of Y. Then f 1 g open cover of X. Since X is intuitionistic -Lindelof, f 1(G) : i 2K contains a countable subcover, f 1(G in) : i 2 K .

Then Gin is a countable subcover of Y.Thus Y is intuitionistic Lindelof.

Theorem 3.28. Let f : (X;) ! (Y;) be an intuitionistic -irresolute surjection and X be intuitionistic -Lindelof then Y is intuitionistic -Lindelof space.

Proof. Let f : (X;) ! (Y;) be an intuitionistic -irresolute surjection and X be intuitionistic -Lindelof. Let X; Gi ; Gi : i 2 K be an intuitionistic -open cover of Y. Then 1 2 K is an intuitionistic -open cover of X. Since X is intuitionistic -Lindelof, f 1(Gi) : i 2 K contains a countable subcover, f 1(Gin) . Then fGing is a countable subcover of Y . Thus Y is intuitionistic -Lindelof.

De nition 3.29. An intuitionistic function f : (X;) ! (Y;) is said to be intu-itionistic -open if the image f(A) is intuitionistic -open in Y for every intuition-istic open set A in (X;).

Theorem 3.30. If f : (X;) ! (Y;) is intuitionistic -open function and Y is intuitionistic -Lindelof space, then X is intuitionistic Lindelof.

ff(Gi) : i 2 g
Proof. Let X; G_i^1; G_i^2 : i 2 K be an intuitionistic open cover of X. Then
K be intuitionistic -open cover of Y . Since Y is intuitionistic -

Lindelof, ff(G_i) : i 2 Kg contains a countable subcover, ff(G_{in})g. Then fG_{in}g is a countable subcover of X. Thus X is intuitionistic Lindelof.

4. I -Connectedness In Intuitionistic Topological Spaces

De nition 4.1. An intuitionistic topological space X is called intuitionistic -disconnected (brie y I -disconnected) if there exists an intuitionistic -open sets A 6= and B 6= such that A [B = X and A \ B = . X is called intuitionistic -connected, if X is not intuitionistic -disconnected.

De nition 4.2. An intuitionistic topological space X is called intuitionistic C_5-disconnected (brie y I C_5-disconnected) if there exists an intuitionistic set A which is both intuitionistic -open and intuitionistic -closed such that 6= A 6= X . X is called intuitionistic C_5-connected, if X is not intuitionistic C_5-disconnected.

Theorem 4.3. Every intuitionistic -connected space is intuitionistic connected.

Proof. Since every intuitionistic open set is intuitionistic -open, the proof follows.

Remark 4.4. Every intuitionistic connected space need not be intuitionistic -connected which is shown in the following example.

Example 4.5. Let X be the set of all positive integers. Consider the intuitionistic sets given by A_1=hX; f2; 3:::g ; i, A_2=hX; f3; 4:::g ; f1gi, A_3=hX; f4; 5::g ; f1; 2gi,

etc A_n=hX; fn + 1; n + 2::g ; f1; 2; 3::n 1g,n 2. Then =fX; fA_n : n = 1; 2; 3:::g is an intuitionistic topological space which is connected but not intuitionistic -connected.

Theorem 4.6. Every intuitionistic C_5-connected space is intuitionistic C_5-connected

Proof. Let (X;) be intuitionistic C_5-connected space and suppose that (X;) is intuitionistic C_5-disconnected. Then there exists a proper intuitionistic set A such that A is both intuitionistic open and intuitionistic closed. Since every intuitionistic open set is intuitionistic -open and every intuitionistic closed set is intuitionistic -closed, X is intuitionistic C_5-disconnected which is a contradiction. Hence every intuitionistic C_5-connected space is intuitionistic C_5-connected.

Remark 4.7. Every intuitionistic C_5-connected space need not be intuitionistic C_5-connected which is shown in the following example.

Example 4.8. Let X =fa; b; cg , = f ; X ; hX; ; fa; bgi ; hX; ; fbgi ;

hX; fa; cg ; fbgi ; hX; fcg ; fbgi ; hX; ; fb; cgi ; hX; fcg ; fa; bgi which is intuitionis-tic C_5-connected but not intuitionistic C_5-connected.

Theorem 4.9. Every intuitionistic C_5-connected space is intuitionistic -connected

Proof. Let X be an intuitionistic -disconnected. Then there exists nonempty disjoint intuitionistic -open sets A and B in X such that A B_1 = X and A_2 \ B_2 = , A_1 \ B_1 = and A_2 [B_2 = X which implies A = B . Hence A is both intuitionistic -open and intuitionistic -closed. Hence A is intuitionistic C_5-connected.

Theorem 4.10. An intuitionistic topological space (X;) is intuitionistic C_5-connected i there exists no nonempty intuitionistic -open set A and B in X such that A = B^c

Proof. Necessity:

Suppose A and B are intuitionistic -open sets such that A 6= 6= B and A = B^c. Since A = B^c and B is an intuitionistic -open set implies B^c = A is intuitionistic -closed and B 6= . Also B^c 6= X implies A 6= X . Hence there exist a proper intuitionistic -open set A in X such that A is both intuitionistic -open and intuitionistic -closed. But this is a contradiction to the fact that X is intuitionistic C_5-connected.

Su ciency:

Let (X;) be an intuitionistic topological space and intuitionistic C_5-disconnected then there exists an intuitionistic set A which is both intuitionistic -open and intuitionistic -closed in X such that 6= A 6= X . Let B = A^c, in this case

B is an intuitionistic -open set and since A 6= X which implies B = A^c 6= . Hence A 6= which is a contradiction that there exists no nonempty intuitionistic proper subset of X which is both intuitionistic -open and intuitionistic -closed.

Therefore, X is intuitionistic C_5-connected.

Theorem 4.11. An intuitionistic topological space (X,) is intuitionistic C_5-connected i there exists no nonempty intuitionistic sets A and B in X such that

B = (I cl(A))^c, B = A^c and A = (I cl(B))^c

Proof. Necessity:

Let A and B be any two intuitionistic sets such that A 6= 6= B,B = A^c, B = (I cl(A))^c, A = (I cl(B))^c. Since (I cl(A))^c and (I cl(B))^c are intuitionistic - open sets in X, A and B are intuitionistic -open sets in X which is a contradiction. Su ciency:

Let (X;) be an intuitionistic C_5-disconnected. Then there exists intuitionistic set A which is both intuitionistic -open and intuitionistic -closed in X such that

6= A 6= X . Taking B = A^c we obtain a contradiction.

Theorem 4.12. Let f : (X;) ! (Y;) be an intuitionistic -continuous surjective function. If X is intuitionistic -connected, then Y is intuitionistic connected.

Proof. Let Y be intuitionistic disconnected. Then there exist intuitionistic open sets E 6= , F 6= in Y such that E[F = Y , E\F = . Since f is intuitionistic - continuous, there exists intuitionistic -open sets A = f ^{1}(E) and B = f ^{1}(F) in X. But E 6= implies A = f ^{1}(E) 6= and F 6= implies B = f ^{1}(F) 6= . Now E [F = Y implies f ^{1}(E) [f ^{1}(F) = f ^{1}(Y) = X = A [B = X and E \ F = implies f ^{1}(E) \ f ^{1}(F) = f ^{1}( ) = which implies A \ B = .

Hence X is intuitionistic -disconnected, which is a contradiction to our hypothesis.

Therefore Y is intuitionistic connected.

Theorem 4.13. Let f : (X;) ! (Y;) be an intuitionistic -irresolute and onto. If X is intuitionistic -connected, then Y is intuitionistic -connected.

Proof. Suppose Y is intuitionistic -connected. Then Y = A [B and A \ B = where A; B 6= and intuitionistic -open in Y . Since f is intuitionistic - irresolute and onto, X = f 1(A) [f 1(B) is disjoint nonempty open subset of X. This contradicts the fact that X is intuitionistic -connected. Hence Y is intuitionistic -connected.

Theorem 4.14. Let f : (X;) ! (Y;) be an intuitionistic -irresolute surjection and X be intuitionistic -C_5connected, then Y is also intuitionistic -C_5connected.

Proof. Let Y be intuitionistic -C_5 disconnected. Then there exist an intuitionistic set A of Y which is both intuitionistic -open and intuitionistic -closed. Since f is intuitionistic -irresolute and surjective, f 1(A) is also an intuitionistic set of X which is both intuitionistic -open and intuitionistic -closed. Hence X is not intuitionistic -C_5connected which is a contradiction. Hence Y intuitionistic -C_5connected.

De nition 4.15. An intuitionistic topological space (X;) is said to be strongly intuitionistic -connected, if there exists no non-empty intuitionistic -closed sets A and B in X such that A \ B = .

Theorem 4.16. X is strongly intuitionistic -connected i there exists no intu-itionistic -open sets A and B in X such that A 6= X 6= B and A [B = X .

Proof. Let A and B be intuitionistic -open sets in X such that A 6= X 6= B and A[B = X . If C = A^c and D = B^c then C and D are intuitionistic -closed sets in
X and as C 6= 6= D and C \D = , X is strongly intuitionistic -disconnected.

Theorem 4.17. Let f : (X;) ! (Y;) be an intuitionistic -irresolute surjective function. If X is strongly intuitionistic -connected, then so is Y .

Proof. Assume Y is not strongly intuitionistic -connected. Then there exists in-tuitionistic -closed sets C and D in Y such that C 6= 6= D, C \ D = .
Since f is intuitionistic -irresolute, f 1(C) and f 1(D) are intuitionistic -closed sets in X and f 1(C) \ f 1(D) = , f 1(C) = and f 1(D) = . So X is strongly intuitionistic -disconnected, which is a contradiction. Hence Y is strongly intuitionistic -connected.

De nition 4.18. Let N be an intuitionistic set in an intuitionistic topological space (X,). If there exists intuitionistic -open sets A and B in X satisfying the following properties then N is called intuitionistic C_i-disconnected.

I C_1: N A [B, A \ B N^c, N \ A 6= , N \ B 6=

I C_2: N A [B, A \ B \ N = , N \ A 6= , N \ B 6= .

N is said to be I C_i-connected if N is not I C_i-disconnected.

Example 4.19. Let X = fc; dg, = f ; X ; hX; fcg ; i ; hX; fdg ; i ; hX; fdg ; fcgi ; hX; ; fcgi ; hX; ; i. Then the intuitionistic set hX; fcg ; fdgi is both intuition-istic C_i-connected and intuitionistic C_i-connected.

Example 4.20. Let X = fa; b; c; dg, = f ; X ; hX; fag ; fc; dgi ; hX; fdg ; fa; cgi ; hX; fa; dg ; fcgi ; hX; ; fa; c; dgi. Let N = hX; fag ; fc; dgi then N is intuitionis-tic C_2-connected but not intuitionistic C_1-connected.

De nition 4.21. The intuitionistic sets A and B in (X;) is said to be intuition-istic -weakly separated (brie y I -weakly separated) if I cl(A) B^c and I cl(B) A^c.

De nition 4.22. The intuitionistic topological space (X;) is said to be intuition-istic C_S-disconnected if there exists an intuitionistic -weakly separated non-empty sets A and B in (X;) such that X = A [B. (X;) is called intuitionistic
C_S-connected if it is not intuitionistic C_S-disconnected

Theorem 4.23. If the intuitionistic closure of the subsets of (X;) are intu-itionistic -closed, then the nonempty sets A and B are intuitionistic intuition-istic -weakly separated i there exists M; W 2 I OS(X) such that A M, B W ,A W ^{c} and B M^c

Proof. Necessity:
Let I cl(A) B^c and I cl(B) A^c and let W = (I cl(A))^c, M = (I cl(B))^c which are intuitionistic -open sets in X. Then W ^{c} B^c and M^c A^c this implies A M and B W . Since W = (I cl(A))^c A^c, A W ^{c} and M = (I cl(B))^c
B^c this implies B M^c. Su ciency:
Let M and W be intuitionistic -open sets in X such that A M, B W , A W ^{c}, B M^c. Since intuitionistic closure of intuitionistic sets of (X;) are
intuitionistic -closed, I cl_C(A) c I cl(W) = W c c c
This implies I cl_C(A) WB and I cl(B) M A . Hence I cl(A) B
and I cl(B) A . Hence A and B are intuitionistic -weakly separated.

De nition 4.24. The intuitionistic set N in the intuitionistic topological space (X;) is said to be I C_S-disconnected i there are two non-empty I -weakly sep-arated sets A and B in (X;) such that N = A [B. N is called I C_S-connected i N is not I C_S-disconnected

Theorem 4.25. If I cl(A) is intuitionistic -closed, for every intuitionistic set A in (X;), then N is intuitionistic C_S-connected if N is intuitionistic C_1-connected

Proof. Let N be intuitionistic C_S-disconnected. Then there exists intuitionistic nonempty sets A and B such that N = A [B where A and B are intuitionistic -weakly separated. So, I cl(A) B^c, I cl(B) A^c and I cl(A) \ I cl(B) B^c \ A^c = (A [B)^c = N^c. Let P = (I cl(A))^c and Q = (I cl(B))^c. Then P and
Q are intuitionistic -open sets. Now N (I cl(A) \ I cl(B))^c = (I cl(A))^c [(I cl(B))^c = P [Q this implies N P [Q. Now, P \Q = (I cl(A))^c \ (I cl(B))^c =(I cl(A)[I cl(B))^c (A[B)^c = N^c. If P \N = then P N^c) N P ^{c}) N I cl(A) B^c. i.e.,A [B B^c which is a contradiction. Hence P \ N 6= .
Similarly Q \ N 6= . Hence P and Q are intuitionistic C_1-disconnected.

References

[1] Andrijevic D, Some properties of the topology of -sets, Mat.Vesnik, 36 (1984), 1-10.
[2] Coker D, A note on intuitionistic sets and intuitionistic points, Turkish J.Math(1996), 343-351.
[3] Coker D, An introduction to intuitionistic fuzzy topological spaces, Fuzzy Sets and Systems, (1997), 81-89.
[4] Coker D, An introduction to intuitionistic topological spaces, Busefal,(2000), 51-56.
[5] Olav Njastad, On some classes of nearly open sets, Paci c Journal of Mathematics, 15, (1965), 961-970.
[6] Sadik Bayhan and Coker, On separation axioms in intuitionistic topological spaces, IJMMS 27:10(2001), 621-630.

[7] Selvanayaki and Gnanambal Ilango, IGPR continuity and compactness in intuitionistic topo-logical spaces, BJMCS, 11(2):1-8, 2015.

[8] Selvanayaki and Gnanambal Ilango, On IGPR connectedness on intuitionistic topological spaces, Journal Of Advanced Studies In Topology 6:3(2015), 90-98.

[9] Selma Ozcag and D.Coker, On connectedness in intuitionistic fuzzy special topological spaces. Int.J.Math.Sci., 21(1)(1998), 33-40.

[10] Younis.J.Yaseen and Asmaa G. Raouf, On generalization of closed set and generalized conti-nuity on intuitionistic topological spaces, J.of Al-Anbar University for Pure Science, (2009), 3(1).

[11] Zadeh L.A ,Fuzzy sets, Inform. and Control, (1965), 338-353.