Solution of Abel's Integral Equation by Aboodh Transform Method

Sudhanshu Aggarwal[1*], Swarg Deep Sharma[2]

[1*]*Assistant Professor, Department of Mathematics, National P.G. College, Barhalganj, Gorakhpur-273402, U.P., India*

[2]*Assistant Professor, Department of Mathematics, Nand Lal Singh College Jaitpur Daudpur Constituent of Jai Prakash University Chhapra-841205, Bihar, India*

ABSTRACT: Abel's integral equation is an important singular integral equation and generally appears in many branches of sciences such as atomic scattering, mechanics, radio astronomy, physics, electron emission, X-ray radiography and seismology. In this paper, we use Aboodh transform method to solve Abel's integral equation and some numerical applications in application section are given to demonstrate the effectiveness of Aboodh transform method to solve Abel's integral equation.

KEYWORDS: Abel's integral equation, Aboodh transforms method, Inverse Aboodh transform, Convolution theorem.

AMS SUBJECT CLASSIFICATION 2010: 44A05, 44A35, 45E10.

I. INTRODUCTION: In 1823, Niels Henrik Abel discussed the motion of particle on smooth curve lying on a vertical plane using Abel's integral equation in mathematical form as [1-2]

$$f(x) = \int_0^x \frac{1}{\sqrt{x-t}} u(t)\, dt \quad (1)$$

Here the kernel of integral equation, $K(x,t) = \frac{1}{\sqrt{x-t}}$ becomes ∞ at $t = x$, the function $f(x)$ is known function and the function $u(t)$ is unknown function.

Integral transforms are widely used mathematical techniques for solving advanced problems of science and engineering which mathematically express in terms of differential equations, partial differential equations, integral equations, partial integro-differential equations, integro-differential equations etc. Many researchers used different integral transforms (Laplace transform [3-4], Fourier transform [3], Hankel transform [3], Kamal transfom [5, 16-19, 38], Mahgoub transform [8-12, 24], Elzaki transform [6-7, 30-31], Mohand transform [20-22, 36-37, 39-40], Aboodh transform [13-15, 23, 32-35], Sumudu transform [41-42], Wavelet transform [3]) for solving many problems of science and engineering. Aggarwal and others [25-29] discussed the comparative study between these transforms.

The Aboodh transform of the function $F(t)$ for all $t \geq 0$ is defined as [43]:

$$A\{F(t)\} = \frac{1}{v}\int_0^\infty F(t)e^{-vt}dt = K(v), 0 < k_1 \leq v \leq k_2, \quad (2)$$

where the operator A is called the Aboodh transform operator.

The Aboodh transform of the function $F(t)$ for $t \geq 0$ exist if $F(t)$ is piecewise continuous and of exponential order. These conditions are only sufficient conditions for the existence of Aboodh transforms of the function $F(t)$. Aggarwal et al. [44] defined Aboodh transform of Bessel's functions.

In this paper, we are giving the solution of Abel's integral equation using Aboodh transform method and explain all procedure by giving some numerical applications in application section.

II. SOME USEFUL PROPERTIES OF ABOODH TRANSFORM:

2.1 Linearity property of Aboodh transform [13-15, 28]:

If Aboodh transform of functions $F_1(t)$ and $F_2(t)$ are $K_1(v)$ and $K_2(v)$ respectively then Aboodh transform of $[aF_1(t) + bF_2(t)]$ is given by $[aK_1(v) + bK_2(v)]$, where a, b are arbitrary constants.

2.2 Change of scale property of Aboodh transform [28]:

If Aboodh transform of function $F(t)$ is $K(v)$ then Aboodh transform of function $F(at)$ is given by $\frac{1}{a^2}K\left(\frac{v}{a}\right)$.

2.3 Shifting property of Aboodh transform [28]:

If Aboodh transform of function $F(t)$ is $K(v)$ then Aboodh transform of function $e^{at}F(t)$is given by $\frac{(v-a)}{v}K(v-a)$.

2.4 Aboodh transform of the derivatives of the function $F(t)$[13-15, 28]:

If $A\{F(t)\} = K(v)$then

a) $A\{F^{'}(t)\} = vK(v) - \frac{F(0)}{v}$

b) $A\{F^{''}(t)\} = v^2K(v) - F(0) - \frac{F'(0)}{v}$

c) $A\{F^{(n)}(t)\} = v^nK(v) - \frac{F(0)}{v^{2-n}} - \frac{F'(0)}{v^{3-n}} - \cdots \ldots - \frac{F^{(n-1)}(0)}{v}$

2.5 Convolution theorem for Aboodh transforms [13-14, 28]:

If Aboodh transform of functions $F_1(t)$ and $F_2(t)$are $K_1(v)$and $K_2(v)$ respectively then Aboodh transform of their convolution $F_1(t) * F_2(t)$ is given by $A\{F_1(t) * F_2(t)\} = vA\{F_1(t)\}A\{F_2(t)\}$

$\Rightarrow A\{F_1(t) * F_2(t)\} = v\, K_1(v)K_2(v)$, where $F_1(t) * F_2(t)$ is defined by

$$F_1(t) * F_2(t) = \int_0^t F_1(t-x)\, F_2(x)dx = \int_0^t F_1(x)\, F_2(t-x)dx$$

III. ABOODH TRANSFORM OF FREQUENTLY ENCOUNTERED FUNCTIONS [13-15, 28, 44]:

Table: 1

S.N.	$F(t)$	$A\{F(t)\} = K(v)$
1.	1	$\frac{1}{v^2}$
2.	t	$\frac{1}{v^3}$
3.	t^2	$\frac{2!}{v^4}$
4.	$t^n, n \in N$	$\frac{n!}{v^{n+2}}$
5.	$t^n, n > -1$	$\frac{\Gamma(n+1)}{v^{n+2}}$
6.	e^{at}	$\frac{1}{v^2 - av}$
7.	$sinat$	$\frac{a}{v(v^2 + a^2)}$
8.	$cosat$	$\frac{1}{v^2 + a^2}$
9.	$sinhat$	$\frac{a}{v(v^2 - a^2)}$
10.	$coshat$	$\frac{1}{v^2 - a^2}$

11	$J_0(t)$	$\frac{1}{v\sqrt{(1+v^2)}}$
12	$J_1(t)$	$\frac{1}{v}-\frac{1}{\sqrt{(1+v^2)}}$

IV. INVERSE ABOODH TRANSFORM [13-15, 28]:

If $K(v)$ is the Aboodh transforms of $F(t)$ then $F(t)$ is called the inverse Aboodh transform of $K(v)$and in mathematical terms, it can be expressed as$F(t) = A^{-1}\{K(v)\}$, where A^{-1} is an operator and it is called as inverse Aboodh transform operator.

V. LINEARITY PROPERTY OF INVERSE ABOODH TRANSFORMS:

If $A^{-1}\{H(v)\} = F(t)$ and $A^{-1}\{I(v)\} = G(t)$ then $A^{-1}\{aH(v) + bI(v)\} = aA^{-1}\{H(v)\} + bA^{-1}\{I(v)\}$

$\Rightarrow A^{-1}\{aH(v) + bI(v)\} = aF(t) + bG(t)$, where a, b are arbitrary constants.

VI. INVERSE ABOODH TRANSFORM OF FREQUENTLY ENCOUNTERED FUNCTIONS [13-15, 28]:

Table: 2

S.N.	$T(v)$	$F(t) = A^{-1}\{K(v)\}$
1.	$\frac{1}{v^2}$	1
2.	$\frac{1}{v^3}$	t
3.	$\frac{1}{v^4}$	$\frac{t^2}{2!}$
4.	$\frac{1}{v^{n+2}}, n\epsilon N$	$\frac{t^n}{n!}$
5.	$\frac{1}{v^{n+2}}, n > -1$	$\frac{t^n}{\Gamma(n+1)}$
6.	$\frac{1}{v^2 - av}$	e^{at}
7.	$\frac{1}{v(v^2+a^2)}$	$\frac{sinat}{a}$
8.	$\frac{1}{v^2+a^2}$	$cosat$
9.	$\frac{1}{v(v^2-a^2)}$	$\frac{sinhat}{a}$
10.	$\frac{1}{v^2-a^2}$	$coshat$
11.	$\frac{1}{v\sqrt{(1+v^2)}}$	$J_0(t)$

12.	$\frac{1}{v}-\frac{1}{\sqrt{(1+v^2)}}$	$J_1(t)$

VII. ABOODH TRANSFORM METHOD FOR SOLVING ABEL'S INTEGRAL EQUATION: In this section, we present Aboodh transform method for the solution of Abel's integral equation.

Taking Aboodh transform of both sides of (1), we have

$$A\{f(x)\} = A\left\{\int_0^x \frac{1}{\sqrt{x-t}}u(t)\,dt\right\}$$

$$\Rightarrow A\{f(x)\} = A\{x^{-1/2} * u(x)\} \quad (3)$$

Applying convolution theorem of Aboodh transform in (3), we have

$$A\{f(x)\} = vA\{x^{-1/2}\}A\{u(x)\}$$

$$\Rightarrow A\{f(x)\} = v(\sqrt{\pi}v^{-3/2})A\{u(x)\}$$

$$\Rightarrow A\{u(x)\} = \frac{v^{1/2}}{\sqrt{\pi}}A\{f(x)\}$$

$$\Rightarrow A\{u(x)\} = \frac{v}{\pi}\left[v(\sqrt{\pi}v^{-3/2})A\{f(x)\}\right]$$

$$\Rightarrow A\{u(x)\} = \frac{v}{\pi}\left[vA\{x^{-1/2}\}A\{f(x)\}\right]$$

$$\Rightarrow A\{u(x)\} = \frac{v}{\pi}A\{x^{-1/2} * f(x)\}$$

$$\Rightarrow A\{u(x)\} = \frac{v}{\pi}\left[A\left\{\int_0^x \frac{1}{\sqrt{x-t}}f(t)\,dt\right\}\right]$$

$$\Rightarrow A\{u(x)\} = \frac{v}{\pi}A\{F(x)\} \quad (4)$$

where $F(x) = \int_0^x \frac{1}{\sqrt{x-t}}f(t)\,dt$ (5)

Now applying the property, Aboodh transform of derivative of the function, on (5), we have

$$A\{F'(x)\} = vA\{F(x)\} - \frac{F(0)}{v}$$

$$\Rightarrow A\{F'(x)\} = vA\{F(x)\}$$

$$\Rightarrow A\{F(x)\} = \frac{1}{v}A\{F'(x)\} \quad (6)$$

Now from (4) and (6), we have

$$A\{u(x)\} = \frac{1}{\pi}A\{F'(x)\} \quad (7)$$

Applying inverse Aboodh transform on both sides of (7), we get

$$u(x) = \frac{1}{\pi}F'(x) = \frac{1}{\pi}\frac{d}{dx}F(x) \quad (8)$$

Using (5) in (8), we have

$$u(x) = \frac{1}{\pi}\left[\frac{d}{dx}\int_0^x \frac{1}{\sqrt{x-t}}f(t)\,dt\right] \quad (9)$$

which is the required solution of (1).

VIII. APPLICATIONS: In this section, we present some numerical applications to demonstrate the effectiveness of Aboodh transform method to solve Abel's integral equation**.**

8.1 Consider the Abel's integral equation:

$$x = \int_0^x \frac{1}{\sqrt{x-t}} u(t)\, dt \quad (10)$$

Taking Aboodh transform of both sides of (10), we have

$$A\{x\} = A\left\{\int_0^x \frac{1}{\sqrt{x-t}} u(t)\, dt\right\}$$

$$\Rightarrow \frac{1}{v^3} = A\{x^{-1/2} * u(x)\} \quad (11)$$

Applying convolution theorem of Aboodh transform in (11), we have

$$\frac{1}{v^3} = vA\{x^{-1/2}\}A\{u(x)\}$$

$$\Rightarrow \frac{1}{v^3} = v(\sqrt{\pi} v^{-3/2})A\{u(x)\}$$

$$\Rightarrow A\{u(x)\} = \frac{v^{-5/2}}{\sqrt{\pi}} \quad (12)$$

Applying inverse Aboodh transform on both sides of (12), we get

$$u(x) = \frac{1}{\sqrt{\pi}} A^{-1}\{v^{-5/2}\}$$

$$\Rightarrow u(x) = \frac{2}{\pi} x^{1/2} \quad (13)$$

which is the required solution of (10).

8.2 Consider the Abel's integral equation:

$$1 + x + x^2 = \int_0^x \frac{1}{\sqrt{x-t}} u(t)\, dt \quad (14)$$

Taking Aboodh transform of both sides of (14), we have

$$A\{1\} + A\{x\} + A\{x^2\} = A\left\{\int_0^x \frac{1}{\sqrt{x-t}} u(t)\, dt\right\}$$

$$\Rightarrow \frac{1}{v^2} + \frac{1}{v^3} + \frac{2}{v^4} = A\{x^{-1/2} * u(x)\} \quad (15)$$

Applying convolution theorem of Aboodh transform in (15), we have

$$\frac{1}{v^2} + \frac{1}{v^3} + \frac{2}{v^4} = vA\{x^{-1/2}\}A\{u(x)\}$$

$$\Rightarrow \frac{1}{v^2} + \frac{1}{v^3} + \frac{2}{v^4} = v(\sqrt{\pi} v^{-3/2})A\{u(x)\}$$

$$\Rightarrow A\{u(x)\} = \frac{1}{\sqrt{\pi}}[v^{-3/2} + v^{-5/2} + 2v^{-7/2}] \quad (16)$$

Applying inverse Aboodh transform on both sides of (16), we get

$$u(x) = \frac{1}{\sqrt{\pi}} A^{-1}\{v^{-3/2} + v^{-5/2} + 2v^{-7/2}\}$$

$$\Rightarrow u(x) = \frac{1}{\sqrt{\pi}}\left[A^{-1}\{v^{-3/2}\} + A^{-1}\{v^{-5/2}\} + 2A^{-1}\{v^{-7/2}\}\right]$$

$$\Rightarrow u(x) = \frac{1}{\pi}\left[x^{-1/2} + 2x^{1/2} + \frac{8}{3}x^{3/2}\right] \quad (17)$$

which is the required solution of (14).

8.3 Consider the Abel's integral equation:

$$3x^2 = \int_0^x \frac{1}{\sqrt{x-t}} u(t)\, dt \quad (18)$$

Taking Aboodh transform of both sides of (18), we have

$$3A\{x^2\} = A\left\{\int_0^x \frac{1}{\sqrt{x-t}} u(t)\, dt\right\}$$

$$\Rightarrow \frac{6}{v^4} = A\{x^{-1/2} * u(x)\} \quad (19)$$

Applying convolution theorem of Aboodh transform in (19), we have

$$\frac{6}{v^4} = vA\{x^{-1/2}\}A\{u(x)\}$$

$$\Rightarrow \frac{6}{v^4} = v(\sqrt{\pi}v^{-3/2})A\{u(x)\}$$

$$\Rightarrow A\{u(x)\} = \frac{6}{\sqrt{\pi}} v^{-7/2} \quad (20)$$

Applying inverse Aboodh transform on both sides of (20), we get

$$u(x) = \frac{6}{\sqrt{\pi}} A^{-1}\{v^{-7/2}\}$$

$$\Rightarrow u(x) = \frac{8}{\pi} x^{3/2} \quad (21)$$

which is the required solution of (18).

8.4 Consider the Abel's integral equation:

$$\frac{4}{3}x^{3/2} = \int_0^x \frac{1}{\sqrt{x-t}} u(t)\, dt \quad (22)$$

Taking Aboodh transform of both sides of (22), we have

$$\frac{4}{3}A\{x^{3/2}\} = A\left\{\int_0^x \frac{1}{\sqrt{x-t}} u(t)\, dt\right\}$$

$$\Rightarrow \sqrt{\pi}v^{-7/2} = A\{x^{-1/2} * u(x)\} \quad (23)$$

Applying convolution theorem of Aboodh transform in (23), we have

$$\sqrt{\pi}v^{-7/2} = vA\{x^{-1/2}\}A\{u(x)\}$$

$$\Rightarrow \sqrt{\pi}v^{-7/2} = v(\sqrt{\pi}v^{-3/2})A\{u(x)\}$$

$$\Rightarrow A\{u(x)\} = \frac{1}{v^3} \quad (24)$$

Applying inverse Aboodh transform on both sides of (24), we get

$$u(x) = A^{-1}\left\{\frac{1}{v^3}\right\}$$

$$\Rightarrow u(x) = x \tag{25}$$

which is the required solution of (22).

8.5 Consider the Abel's integral equation:

$$2\sqrt{x} + \frac{8}{3}x^{3/2} = \int_0^x \frac{1}{\sqrt{x-t}}u(t)\,dt \tag{26}$$

Taking Aboodh transform of both sides of (26), we have

$$2A\{x^{1/2}\} + \frac{8}{3}A\{x^{3/2}\} = A\left\{\int_0^x \frac{1}{\sqrt{x-t}}u(t)\,dt\right\}$$

$$\Rightarrow \sqrt{\pi}v^{-5/2} + 2\sqrt{\pi}v^{-7/2} = A\{x^{-1/2} * u(x)\} \tag{27}$$

Applying convolution theorem of Aboodh transform in (27), we have

$$\sqrt{\pi}v^{-5/2} + 2\sqrt{\pi}v^{-7/2} = vA\{x^{-1/2}\}A\{u(x)\}$$

$$\Rightarrow \sqrt{\pi}v^{-5/2} + 2\sqrt{\pi}v^{-7/2} = v\left(\sqrt{\pi}v^{-3/2}\right)A\{u(x)\}$$

$$\Rightarrow A\{u(x)\} = \frac{1}{v^2} + \frac{2}{v^3} \tag{28}$$

Applying inverse Aboodh transform on both sides of (28), we get

$$u(x) = A^{-1}\left\{\frac{1}{v^2}\right\} + 2A^{-1}\left\{\frac{1}{v^3}\right\}$$

$$\Rightarrow u(x) = 1 + 2x \tag{29}$$

which is the required solution of (26).

8.6 Consider the Abel's integral equation:

$$\frac{3}{8}\pi x^2 = \int_0^x \frac{1}{\sqrt{x-t}}u(t)\,dt \tag{30}$$

Taking Aboodh transform of both sides of (30), we have

$$\frac{3}{8}\pi A\{x^2\} = A\left\{\int_0^x \frac{1}{\sqrt{x-t}}u(t)\,dt\right\}$$

$$\Rightarrow \frac{3}{4}\left(\frac{\pi}{v^4}\right) = A\{x^{-1/2} * u(x)\} \tag{31}$$

Applying convolution theorem of Aboodh transform in (31), we have

$$\frac{3}{4}\left(\frac{\pi}{v^4}\right) = vA\{x^{-1/2}\}A\{u(x)\}$$

$$\Rightarrow \frac{3}{4}\left(\frac{\pi}{v^4}\right) = v\left(\sqrt{\pi}v^{-3/2}\right)A\{u(x)\}$$

$$\Rightarrow A\{u(x)\} = \frac{3}{4}\sqrt{\pi}v^{-7/2} \tag{32}$$

Applying inverse Aboodh transform on both sides of (32), we get

$$u(x) = \frac{3}{4}\sqrt{\pi}A^{-1}\{v^{-7/2}\}$$

$$\Rightarrow u(x) = x^{3/2} \qquad (33)$$

which is the required solution of (30).

IX. CONCLUSION: In this paper, we have successfully discussed Aboodh transform method for the solution of Abel's integral equation. The given numerical applications in the application section explain the complete procedure for the solution of Abel's integral equation using Aboodh transform method. The results show that Aboodh transform method is a powerful integral transform method for the solution of Abel's integral equation. In the future, Aboodh transform method can be used for solving other singular integral equations.

REFERENCES

[1] Wazwaz, A.M., Linear and nonlinear integral equations: Methods and applications, Higher Education Press, Beijing, 2011.
[2] Rahman, M., Integral equation and their applications, Wit Press Southampton, Boston, 2007.
[3] Lokenath Debnath and Bhatta, D., Integral transforms and their applications, Second edition, Chapman & Hall/CRC, 2006.
[4] Aggarwal, S., Gupta, A.R., Singh, D.P., Asthana, N. and Kumar, N., Application of Laplace transform for solving population growth and decay problems, International Journal of Latest Technology in Engineering, Management & Applied Science, 7(9), 141-145, 2018.
[5] Aggarwal, S. and Gupta, A.R., Solution of linear Volterra integro-differential equations of second kind using Kamal transform, Journal of Emerging Technologies and Innovative Research, 6(1), 741-747, 2019.
[6] Aggarwal, S., Chauhan, R. and Sharma, N., Application of Elzaki transform for solving linear Volterra integral equations of first kind, International Journal of Research in Advent Technology, 6(12), 3687-3692, 2018.
[7] Aggarwal, S., Singh, D.P., Asthana, N. and Gupta, A.R., Application of Elzaki transform for solving population growth and decay problems, Journal of Emerging Technologies and Innovative Research, 5(9), 281-284, 2018.
[8] Aggarwal, S., Sharma, N., Chauhan, R., Gupta, A.R. and Khandelwal, A., A new application of Mahgoub transform for solving linear ordinary differential equations with variable coefficients, Journal of Computer and Mathematical Sciences, 9(6), 520-525, 2018.
[9] Aggarwal, S., Chauhan, R. and Sharma, N., A new application of Mahgoub transform for solving linear Volterra integral equations, Asian Resonance, 7(2), 46-48, 2018.
[10] Aggarwal, S., Sharma, N. and Chauhan, R., Solution of linear Volterra integro-differential equations of second kind using Mahgoub transform, International Journal of Latest Technology in Engineering, Management & Applied Science, 7(5), 173-176, 2018.
[11] Aggarwal, S., Sharma, N. and Chauhan, R., Application of Mahgoub transform for solving linear Volterra integral equations of first kind, Global Journal of Engineering Science and Researches, 5(9), 154-161, 2018.
[12] Aggarwal, S., Pandey, M., Asthana, N., Singh, D.P. and Kumar, A., Application of Mahgoub transform for solving population growth and decay problems, Journal of Computer and Mathematical Sciences, 9(10), 1490-1496, 2018.
[13] Aggarwal, S., Sharma, N. and Chauhan, R., Application of Aboodh transform for solving linear Volterra integro-differential equations of second kind, International Journal of Research in Advent Technology, 6(6), 1186-1190, 2018.
[14] Aggarwal, S., Sharma, N. and Chauhan, R., A new application of Aboodh transform for solving linear Volterra integral equations, Asian Resonance, 7(3), 156-158, 2018.
[15] Aggarwal, S., Asthana, N. and Singh, D.P., Solution of population growth and decay problems by using Aboodh transform method, International Journal of Research in Advent Technology, 6(10), 2706-1190, 2710.
[16] Aggarwal, S., Chauhan, R. and Sharma, N., A new application of Kamal transform for solving linear Volterra integral equations, International Journal of Latest Technology in Engineering, Management & Applied Science, 7(4), 138-140, 2018.
[17] Gupta, A.R., Aggarwal, S. and Agrawal, D., Solution of linear partial integro-differential equations using Kamal transform, International Journal of Latest Technology in Engineering, Management & Applied Science, 7(7), 88-91, 2018.
[18] Aggarwal, S., Sharma, N. and Chauhan, R., Application of Kamal transform for solving linear Volterra integral equations of first kind, International Journal of Research in Advent Technology, 6(8), 2081-2088, 2018.
[19] Aggarwal, S., Gupta, A.R., Asthana, N. and Singh, D.P., Application of Kamal transform for solving population growth and decay problems, Global Journal of Engineering Science and Researches, 5(9), 254-260, 2018.
[20] Aggarwal, S., Sharma, N. and Chauhan, R., Solution of population growth and decay problems by using Mohand transform, International Journal of Research in Advent Technology, 6(11), 3277-3282, 2018.
[21] Aggarwal, S., Sharma, N. and Chauhan, R., Solution of linear Volterra integral equations of second kind using Mohand transform, International Journal of Research in Advent Technology, 6(11), 3098-3102, 2018.
[22] Aggarwal, S., Chauhan, R. and Sharma, N., Mohand transform of Bessel's functions, International Journal of Research in Advent Technology, 6(11), 3034-3038, 2018.

[23] Aggarwal, S., Sharma, N. and Chauhan, R., Application of Aboodh transform for solving linear Volterra integral equations of first kind, International Journal of Research in Advent Technology, 6(12), 3745-3753, 2018.

[24] Chauhan, R. and Aggarwal, S., Solution of linear partial integro-differential equations using Mahgoub transform, Periodic Research, 7(1), 28-31, 2018.

[25] Aggarwal, S. and Chaudhary, R., A comparative study of Mohand and Laplace transforms, Journal of Emerging Technologies and Innovative Research, 6(2), 230-240, 2019.

[26] Aggarwal, S., Sharma, N., Chaudhary, R. and Gupta, A.R., A comparative study of Mohand and Kamal transforms, Global Journal of Engineering Science and Researches, 6(2), 113-123, 2019.

[27] Aggarwal, S., Mishra, R. and Chaudhary, A., A comparative study of Mohand and Elzaki transforms, Global Journal of Engineering Science and Researches, 6(2), 203-213, 2019.

[28] Aggarwal, S. and Chauhan, R., A comparative study of Mohand and Aboodh transforms, International Journal of Research in Advent Technology, 7(1), 520-529, 2019.

[29] Aggarwal, S. and Sharma, S.D., A comparative study of Mohand and Sumudu transforms, Journal of Emerging Technologies and Innovative Research, 6(3), 145-153, 2019.

[30] Elzaki, T.M. and Ezaki, S.M., On the Elzaki transform and ordinary differential equation with variable coefficients, Advances in Theoretical and Applied Mathematics, 6(1), 41-46, 2011.

[31] Elzaki, T.M. and Ezaki, S.M., Applications of new transform ‘‘Elzaki transform’’ to partial differential equations, Global Journal of Pure and Applied Mathematics, 7(1), 65-70, 2011.

[32] Aboodh, K.S., Application of new transform “Aboodh Transform” to partial differential equations, Global Journal of Pure and Applied Mathematics, 10(2), 249-254, 2014.

[33] Aboodh, K.S., Farah, R.A., Almardy, I.A. and Osman, A.K., Solving delay differential equations by Aboodh transformation method, International Journal of Applied Mathematics & Statistical Sciences, 7(2), 55-64, 2018.

[34] Aboodh, K.S., Farah, R.A., Almardy, I.A. and Almostafa, F.A., Solution of partial integro-differential equations by using Aboodh and double Aboodh transforms methods, Global Journal of Pure and Applied Mathematics, 13(8), 4347-4360, 2016.

[35] Mohand, D., Aboodh, K.S. and Abdelbagy, A., On the solution of ordinary differential equation with variable coefficients using Aboodh transform, Advances in Theoretical and Applied Mathematics, 11(4), 383-389, 2016.

[36] Kumar, P.S., Saranya, C., Gnanavel, M.G. and Viswanathan, A., Applications of Mohand transform for solving linear Volterra integral equations of first kind, International Journal of Research in Advent Technology, 6(10), 2786-2789, 2018.

[37] Kumar, P.S., Gomathi, P., Gowri, S. and Viswanathan, A., Applications of Mohand transform to mechanics and electrical circuit problems, International Journal of Research in Advent Technology, 6(10), 2838-2840, 2018.

[38] Abdelilah, K. and Hassan, S., The use of Kamal transform for solving partial differential equations, Advances in Theoretical and Applied Mathematics, 12(1), 7-13, 2017 .

[39] Sathya, S. and Rajeswari, I., Applications of Mohand transform for solving linear partial integro-differential equations, International Journal of Research in Advent Technology, 6(10), 2841-2843, 2018.

[40] Kumar, P.S., Gnanavel, M.G. and Viswanathan, A., Application of Mohand transform for solving linear Volterra integro-differential equations, International Journal of Research in Advent Technology, 6(10), 2554-2556, 2018.

[41] Watugula, G.K., Sumudu transform: A new integral transform to solve differential equations and control engineering problems, International Journal of Mathematical Education in Science and Technology, 24(1), 35-43, 1993.

[42] Belgacem, F.B.M. and Karaballi, A.A., Sumudu transform fundamental properties investigations and applications, Journal of Applied Mathematics and Stochastic Analysis, 1-23, 2006.

[43] Aboodh, K.S., The new integral transform “Aboodh Transform”, Global Journal of Pure and Applied Mathematics, 9(1), 35-43, 2013.

[44] Aggarwal, S., Gupta, A.R. and Agrawal, D., Aboodh transform of Bessel’s functions, Journal of Advanced Research in Applied Mathematics and Statistics, 3(3), 1-5, 2018.