GENERALIZED PRODUCTS OF DIRECTABLE AUTOMATA

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Abstract: In this paper, we introduce generalized direct product, generalized restricted direct product of automata. Using the above concept, we proved that generalized direct product, and generalized restricted direct product of directable and trap-directable automata is directable and trap-directable. Also, we proved that generalized direct product of strongly connected directable automata is strongly connected directable automata.

***IndexTerms* - Generalized direct product, Directable automata, Trap-directable automata.**

I. INTRODUCTION

The directable automata were first introduced by J. Cerny [4] and later studied by many authors [2, 5, 6]. In [3] authors were studied directable automata in another point of view using the notions of necks. Trapped and trap-directable automata were introduced in [7]. Directle automata is also known as synchronizable, cofinal automata. The main purpose of this paper is to discuss properties of directable automata using the concept of generalized direct product, generalized restricted direct product. We proved that generalized direct product, and generalized restricted direct product of directable and trap-directable automata is directable and trap-directable automata. Also, we proved that generalized direct product of strongly connected directable automata is strongly connected directable automata.

II PRELIMINARIES

$M = (Q, X, \delta)$ be an automaton, where Q and X are non-empty finite sets called a state set and alphabet. δ is a transition function, defined as $\delta(q_i, a) = q_j$, $q_i, q_j \in Q$ and $a \in X$. Extended transition function is denoted by δ^* and is defined as $\delta(q_i, \epsilon) = q_i$, $\delta^*(q_i, aw) = \delta^*(\delta(q_i, a), w)$, $q_i \in Q$, $a \in X$, $w \in X^*$.

$M = (Q, X, \delta)$ be an automaton. M is said to deterministic if $\delta : Q \times X \to Q$. M is said to be non deterministic if $\delta : Q \times X \to 2^Q$. 2^Q is the power set of Q, the set of all subsets of Q [1].

M is said to be strongly connected if for every $q_i, q_j \in Q$, there exists $u, v \in X^*$ such that $\delta^*(q_i, u) = q_j$ and $\delta^*(q_j, v) = q_i$.

M is said to be directable automaton if for every $q_i, q_j \in Q$ there exists a word $w \in X^*$ such that $\delta^*(q_i, w) = \delta^*(q_j, w)$. In this case w is called directing word of M [2].

A state $q_i \in Q$ is said to be trap of M if $\delta^*(q_i, w) = q_i \ \forall \ w \in X^*$. The set of all traps of M is denoted by $Tr(M)$. M is said to be trapped automaton if there exists $w \in X^*$ such that $\delta^*(q_i, w) = q_j, \forall \ q_i \in Q$, $q_j \in Tr(M)$. M is said to be trap-directable automaton if it is directable automaton and has a trap [7].

III GENRALIZED PRODUCTS OF DIRECTABLE AUTOMATA

3.1 Generalized Direct Product of Automata:

Let $M_i = (Q_i, X_i, \delta_i)$, $i = 1, 2, \ldots, n$ be arbitrary automata. The generalized direct product of automata of M_i is given by $M = \prod_{i=1}^{i=n} M_i = (\prod_{i=1}^{i=n} Q_i, \prod_{i=1}^{i=n} X_i, \prod_{i=1}^{i=n} Q_i)$.

Define $\prod_{i=1}^{i=n} \delta_i : \prod_{i=1}^{i=n} Q_i \times \prod_{i=1}^{i=n} X_i \to \prod_{i=1}^{i=n} Q_i$ is as follows:

$\prod_{i=1}^{i=n} \delta_i ((q_1, q_2, \ldots, q_n), a) = (\delta_1(q_1, a_1), \delta_2(q_2, a_2), \ldots, \delta_n(q_n, a_n))$,

Where $q_1, q_2, \ldots q_n \in \prod_{i=1}^{i=n} Q_i$, and $a_1, a_2, \ldots, a_n \in \prod_{i=1}^{i=n} X_i$.

3.2 Generalized Restricted Direct Product of AutoMata:

$M_i = (Q_i, X_i, \delta_i)$, $i = 1, 2, \ldots, n$ be arbitrary automata. The generalized restricted direct product of M_i is given by $\bigcap_{i=1}^{i=n} M_i = (\prod_{i=1}^{i=n} Q_i, X, \bigcap_{i=1}^{i=n} \delta_i)$.

Define $\bigcap_{i=1}^{i=n} \delta_i : \prod_{i=1}^{i=n} Q_i \times X \to \prod_{i=1}^{i=n} Q_i$ is as follows,

$\bigcap_{i=1}^{i=n} \delta_i ((q_i, q_j, \ldots, q_n), a) = (\delta_1(q_i, a), \delta_2(q_j, a), \ldots, \delta_n(q_n, a))$, $q_i, q_j, \ldots q_n \in \prod_{i=1}^{i=n} Q_i$ and $a \in X$.

Theorem 3.1:

Generalized direct product of directable automata is directable.

Proof:

Let $M_i = (Q_i, X_i, \delta_i)$, $i = 1, 2, \ldots, n$ be arbitrary directable automata. The generalized direct product of M_i is given by

$M = \prod_{i=1}^{i=n} M_i = (\prod_{i=1}^{i=n} Q_i, \prod_{i=1}^{i=n} X_i, \prod_{i=1}^{i=n} \delta_i)$.

Define $\prod_{i=1}^{i=n} \delta_i : \prod_{i=1}^{i=n} Q_i \times \prod_{i=1}^{i=n} X_i \to \prod_{i=1}^{i=n} Q_i$.

Since M_i are directable automata, then there exist directing words $u_i \in X_i^*$, $i = 1, 2, \ldots, n$ and states $q_k \in Q_1$, $q_l \in Q_2$, ..., $q_y \in Q_n$ such that
$\delta_1^*(q_i, u_1) = \delta_1^*(q_i', u_1) = q_k$, $\forall q_i, q_i' \in Q_1$, $\delta_2^*(q_j, u_2) = \delta_2^*(q_j', u_2) = q_l$, $\forall q_j, q_j' \in Q_2$,........, and
$\delta_n^*(q_n, u_n) = \delta_n^*(q_n', u_n) = q_y$, $\forall q_n, q_n' \in Q_n$.
$w \in \prod_{i=1}^{i=n} X_i^*$ such that $w = u_1 u_2 \ldots \ldots \ldots u_n$.
Now, $\prod_{i=1}^{i=n} \delta_i^*((q_i, q_j, \ldots\ldots, q_n), w) = \prod_{i=1}^{i=n} \delta_i^*((q_i, q_j, \ldots\ldots, q_n), u_1 u_2 \ldots\ldots u_n)$

$$= (\delta_1^*(q_i, u_1), \delta_2^*(q_j, u_2), \ldots\ldots \delta_n^*(q_n, u_n))$$
$$= (\delta_1^*(q_i', u_1), \delta_2^*(q_j', u_2), \ldots\ldots, \delta_n^*(q_n', u_n))$$
$$= (q_k, q_l, \ldots\ldots, q_y)$$

Therefore, $\prod_{i=1}^{i=n} \delta_i^*((q_i, q_j, \ldots\ldots, q_n), w) = (q_k, q_l, \ldots\ldots, q_y)$.
Hence, $\prod_{i=1}^{i=n} M_i$ is directable automata.

Theorem 3.2:

Generalized direct product of trap-directable automata is trap-directable.

Proof:

Let $M_i = (Q_i, X_i, \delta_i)$, $i = 1, 2, \ldots\ldots, n$ be arbitrary trap-directable automata. The generalized direct product of M_i is given by
$M = \prod_{i=1}^{i=n} M_i = (\prod_{i=1}^{i=n} Q_i, \prod_{i=1}^{i=n} X_i, \prod_{i=1}^{i=n} \delta_i)$.
Define $\prod_{i=1}^{i=n} \delta_i$: $\prod_{i=1}^{i=n} Q_i \times \prod_{i=1}^{i=n} X_i \to \prod_{i=1}^{i=n} Q_i$.
Since M_i are trap-directable automata, then there exist trap-directing words $u_i \in X_i^*$, $i = 1, 2, \ldots\ldots, n$ and states $q_k \in Q_1$, $q_l \in Q_2$,...., $q_y \in Q_n$ such that $\delta_1^*(q_i, u_1) = q_k$, $\forall q_i \in Q_1$, $\delta_2^*(q_j, u_2) = q_l$, $\forall q_j \in Q_2$,........,and
$\delta_n^*(q_n, u_n) = q_y$,$\forall q_n \in Q_n$.
Choose $w \in \prod_{i=1}^{i=n} X_i^*$ such that $w = u_1 u_2 \ldots\ldots u_n$, $u_i \in X_i^*$, $i = 1, 2, \ldots. n$.
Now, $\prod_{i=1}^{i=n} \delta_i^*((q_i, q_j, \ldots\ldots, q_n), w) = \prod_{i=1}^{i=n} \delta_i^*((q_i, q_j, \ldots\ldots, q_n), u_1 u_2 \ldots\ldots u_n)$

$$= (\delta_1^*(q_i, u_1), \delta_2^*(q_j, u_2), \ldots\ldots, \delta_n^*(q_n, u_n))$$
$$= (q_k, q_l, \ldots\ldots, q_y).$$

$\prod_{i=1}^{i=n} \delta_i^*((q_i, q_j, \ldots\ldots, q_n), w) = (q_k, q_l, \ldots\ldots, q_y)$ and $q_k, q_l, \ldots\ldots, q_y$ is a trap state
Therefore, $\prod_{i=1}^{i=n} M_i$ is trap-directable automata.

Theorem 3.3:

Generalized restricted direct product of directable automata is directable.

Theorem 3.4:

Generalized restricted direct product of trap-directable automata is trap-directable.

Proof:

The Proof of above Theorems is easily follows from proof of the Theorem 3.1 and Theorem 3.2.

Theorem 3.5:

The generalized direct product of strongly conneceted directable automata is strongly conneceted directable.

Proof:

Let $M_i = (Q_i, X_i, \delta_i)$, $i = 1, 2, \ldots\ldots, n$ be arbitrary strongly connected directable automata. Since M_i are strongly connected, if for any $q_j, q' \in Q_1$, $q_k, q_k' \in Q_2$,........, $q_n, q_n' \in Q_n$ then there exists $u_i, v_i \in X_i^*$, $i = 1, 2, \ldots\ldots, n$ such that
$\delta_1(q_j, u_1) = q_j'$ and $\delta_1(q_j', v_1) = q_j$,......, $\delta_n(q_n, u_n) = q_n'$ and δ_n(q_n', v_n) = q_n.
Since M_i are diectable, there existing directing words $w_i \in X_i^*$ such that $\delta_i^*(q_i, w_i) = \delta_i^*(q_i', w_i)$ $\forall q_i, q_i' \in Q_i$, $i = 1, 2, \ldots\ldots, n$.
Now, choose $w = w_1 w_2 \ldots. w_n \in \prod_{i=1}^{i=n} X_i^*$.
$\prod_{i=1}^{i=n} \delta_i^*((q_j, q_k, \ldots\ldots, q_n), w) = \prod_{i=1}^{i=n} \delta_i^*((q_j, q_k, \ldots\ldots, q_n), w_1 w_2 \ldots. w_n)$

$$= \delta_1^*(q_j, w_1), \delta_2^*(q_k, w_2), \ldots, \delta_n^*(q_n, w_n)$$
$$= \delta_1^*(q_j', w_1), \delta_2^*(q_k', w_2), \ldots, \delta_n^*(q_n', w_n) \text{-------------------}(1)$$
$$= \delta_1^*(q_j', v_1 u_1), \delta_2^*(q_k', v_2 u_2), \ldots, \delta_n^*(q_n', v_n u_n),$$
$$where,\ w_i = v_i u_i,\ i = 1, 2, \ldots, n.$$
$$= \delta_1(q_j, u_1), \delta_2(q_k, u_2), \ldots. \delta_n(q_n, u_n)$$
$$= (q_j' q_k' \ldots. q_n') \text{------------------------}(2).$$

FroM (1) and (2)
$\prod_{i=1}^{i=n} M_i$ is strongly connected directable automata.

IV Conclusion

In this paper, we discuss properties of directable automata through generalized direct products,and generalized restricted direct products. Here, we proved that the generalized direct product,and generalized restricted direct product of directable and trap-directable automata is directable and trap-directable automata. Finally, we proved that generalized direct product of strongly connected directable is strongly connected directable automata.

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