A Generalized Mixed Sensitivity Control Design Framework for Multivariable System

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Abstract—*Work presented here is a very powerful control system design for Linear Time-Invariant (LTI) Single-Input Single-Output (SISO) stable and unstable plants. The system has been constructed to show an extensive set of closed loop parameters based on various constraints. The generic issue presented –a generalized weighted mixed-sensitivity problem based on constraints – privileged the designers to directly report and tradeoff multivariable properties at different loop diverging points; e.g. at plant input and output. As such, the proposed system is particularly powerful for both the Single Input Single Output stable and unstable plants. The Youla based parameterization is enforced to characterized the set of all stabilizing LTI controllers. It is used to quantify the general problem being addressed. Quantitative based approaches are used to turn the resulting infinite-dimensional problem into a finite-dimensional problem for which there exist many efficient convex optimization methods. A basic cutting plane method is utilized inside the framework. Academic and physical example is presented to illustrate the utility of the presented work.*

Keywords: SISO, MIMO, LTI, QFT, PID.

I. Introduction and Overview

Controller configuration is a development issue since it requires numerous components identification with the consideration of performance and robustness[37]. Numerous highlights can be caught by planning the structure issue as a compelled enhancement issue [3, 6, 14, 15]. Convex programming [35] is a loop-breaking improvement strategy, which has ensured assembly and effective calculations that have been bundled in simple to-utilize devices. There is an alteration called convex-concave optimization which concedes non convex criteria and requirements [36]. In this paper we will consider h_∞ based convex and constrained controllers following the thoughts in [38].We consider here the calculation of peak frequency subject to weighted function on the sensitivities and other constraints.

The utilization of the recurrence reaction for registering SISO-PID controllers by arched improvement is exhibited by [36]. This strategy utilizes indistinguishable sort of linearization of the imperatives from [37] yet displayed it as a convex–concave approximation method. An expansion of [38] for the structure of MIMO PID controllers by linearization of quadratic network imbalances is exhibited by[37] for stable plants. A comparable methodology, with a similar kind of linearization, is utilized for planning of LP-MIMO controllers. This methodology isn't just viewed as the stable plants, yet in addition incorporates the conditions for the stability of the closed-loop framework.

Traditional loop-shaping techniques and Quantitative Feedback Theory (QFT) likewise utilizes Frequency-domain information where previous frequency domain technique utilized for processing stable SISO plants [4]. In these methodologies, the controller with different working parameters utilizes graphical strategies. Most recent optimization based method has additionally been proposed[15]. The arrangement of all set of PID controllers with H∞ execution is acquired by just frequency domain information[18, 22]. The proposed technique is stretched out to construct fixed order controllers by [26, 27, 28]. The frequency domain information is utilized in [39] to register the frequency response of a controller that accomplishes a closed loop pole location. An information driven frequency based procedure for settled structure controller plan issues with H∞ execution is proposed by [2] Another frequency domain approach is exhibited by [24] configured reduced order controllers with ensured limited error on the contrast between the ideal and accomplished size of sensitivity function.

Standard H1 Mixed-Sensitivity Minimization Problem: Pros and Cons: The Standard Weighted H1/H2 mixed sensitivity optimization problem that addresses closed loop maps at plant output is mentioned in below equation:

$$K = \arg\left\{ \min_{K\ stabilizing} \gamma \;\middle|\; \left\| \begin{bmatrix} W_1 S_o \\ W_2 S_o \\ W_3 S_o \end{bmatrix} \right\|_{H_\infty} < \gamma \right\} \quad \text{-----------------------------(1)}$$

where W1, W2, W3 are frequency-dependent weighted matrices that are used to tradeoff the properties of So, KSo, and To.

One of the main disadvantage of having only the transfer function matrices from reference r to output y is that, it might result in worst feedback properties at the plant input. In other words, if we get good feedback properties at plant output it does not mean that at the plant input also will the good properties [11].

II. Proposed Design Framework

Here in this part, we demonstrate the proposed generalized H1 mixed-sensitivity control design framework. The main purpose of the environment is to be able to address specifications at distinct loop shaping points (e.g. output and input). More specifically, we want to make an environment that permits a designer to shape multiple sensitivity transfer function matrices; e.g. the sensitivity $S_o = [1 + PK]^{-1}$ associated with breaking the loop at the plant output as well as that the sensitivity $S_i = [1 + PK]^{-1}$ associated with breaking the loop at the plant input also the sensitivity $PS_i = \text{P}\,[1 + PK]^{-1}$input disturbance to output transfer function. Toward this end, we propose the following generalized mixed-sensitivity optimization.

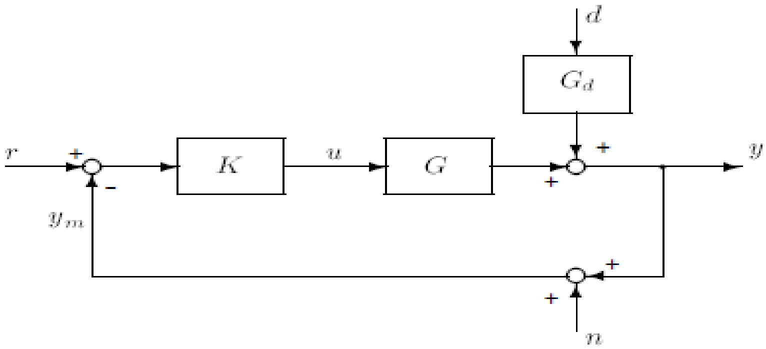

Figure 1: One degree-of-freedom control configuration

Proposed Generalized H^1 Mixed-Sensitivity Optimization:

To address the above, we propose the following generalized weighted H^1 mixed-sensitivity problem:

$$K = \arg\left\{ \min_{K\ stabilizing} \gamma \;\middle|\; \max\left(\left\| \begin{bmatrix} W_1 S_o \\ W_2 K S_o \\ W_3 T_o \end{bmatrix} \right\|_{H\infty}, \rho \left\| \begin{bmatrix} W_4 S_i \\ W_5 P S_i \\ W_6 T_i \end{bmatrix} \right\|_{H\infty} \right) < \gamma \right\} \text{---(2)}$$

Here, W_1-W_6 are RHfrequency-dependent weighting matrices that are used to shape the closed properties of S_o, KS_o, T_o, S_i, $P\,S_i$ and T_i. The above framework captures the traditional mixed-sensitivity of the output problem that has been widely addressed within the control literature [17–19] ($W_{4;\,5;6}$ = 0 with no constraints) as well the not so broadly addressed mixed-sensitivity of the input problem ($W_{1;\,2;3}$ = 0 with no constraints). The former can be used to systematically achieve desirable properties at the output, while the latter can be used to achieve desirable properties at the input. By combining the two as above, a designer can use the weightings to systematically shape and tradeoff properties at both loop breaking points.

Solution Method:

Achieving Convexity Via Youla Parameterization. The approach relies on using the Youla Q-Parameterization [25] to transform the transfer matrices (So, KSo To, Si, PSi and Ti) that depend nonlinearly on K into transfer matrices that depend affinely on the stable Youla parameter on Q (stable transfer matrix). This results in a transfer function matrix that are convex in Q. Since the H1 norm is also a convex functional [2], the Youla parameterization results in a convex problem in Q. Obtaining a Finite-Dimensional Convex Problem. Because Q can be an arbitrary stable transfer function matrix, the resulting problem is infinite-dimensional. This permits us to transform the infinite-dimensional convex problem in Q to a finite-dimensional convex optimization problem in the coefficients defining the above linear combination. The proposed optimization problem may be posed as follows:

$$K = \arg\left\{ \min_{K\ stabilizing} \gamma \;\middle|\; \max\left(\left\| \begin{bmatrix} W_1 S_o \\ W_2 K S_o \\ W_3 T_o \end{bmatrix} \right\|_{H\infty}, \rho \left\| \begin{bmatrix} \rho W_4 S_i \\ \rho W_5 P S_i \\ \rho W_6 T_i \end{bmatrix} \right\|_{H\infty} \right) < \gamma \right\} \text{---(3)}$$

$$C_i \begin{pmatrix} W_{1c}^i S_0 \\ W_{2c}^i K S_0 \\ W_{3c}^i T_0 \\ W_{4c}^i S_i \\ W_{5c}^i P S_i \\ W_{6c}^i S_i \end{pmatrix} \le C_i \quad i = 1,2, \ldots \ldots$$

Where$C_K(.)$ denotes the k^th constraint functional and $C_K \epsilon R$.

- The above optimization problem for K is nonlinear and infinite-dimensional.
- It is difficult to generate a closed loop solution or direct approach exists for the above mentioned problem.

It should be clearly mentioned here is that the demonstrated plant contains all subsystems necessary to apply optimization. After the optimization process is applied successfully, the resulting controller K can then be inserted into the unity feedback system.
The general guidelines for the selection of weights [6][9]:

1) For systems with $PM \le 90°$, it is well known that $W_B \le W_C \le W_{BT}$where W_B, W_{BT} *and* $W_C$are the closed loop bandwidth measured on the basis of S, the closed loop frequency calculated on the basis of T, and the gain crossover frequency, respectively. Therefore, it is required that $W_B^* \le W_{BT}^*$.
Since $|L(jW_c)| = 1$
Therefore $|S(jW_c)| = |T(jW_c)|$
Thus when $PM = 90°$, we get

$|S(jW_c)| = |T(jW_c)| = 0.707$

And we have $W_B = W_C = W_{BT}$

when $PM < 90°$, we get

$|S(jW_c)| = |T(jW_c)| > 0.707$

Since W_B is the frequency where $|S(jW_c)|$ crosses 0.707 from below we must have $W_B < W_C$.

Similarly, since W_{BT} is the frequency where $|T(jW_c)|$ crosses 0.707 from above we must have $W_{BT} > W_C$.

Here one thing should be keep in mind that the presence of non-minimum phase zeros places restriction on the achievable bandwidth. Also, for high performance measurement applications with distinct measurement noise it usually becomes mandatory to make a compromise and instead choose $W_{BT}^* < W_B^*$

2) When disturbance attenuation is the control objective ,the general rule is to increase W_B^* as much as possible.However, increasing W_B^*more than necessary causesthe appearance of a peak in the sensitivity curve. Thisimplies that the system will have less stability marginswhich manifests itself in an increased overshoot in thestep response.
3) When the control objective is to reduce the effect of the measurement noise, the general rule is to decreaseW_{BT}^* as much as possible. However, decreasing W_{BT}^*more than as per the requirement may cause a decrement in the system frequency and this signifies itself in a poor tracking performance.
4) Increasing m and n can enhance the disturbance elimination and observed noise attenuation, respectively. However, m and n should be kept as low as possible since large values of these parameters adversely affect the stability margins, and the controller order becomes unnecessarily high. (Controller order is N + n + m)

III. Implementation of Design Framework

The way by which we can easily convert a basic nonlinear infinite-dimensional optimization problem into a finite-dimensional convex optimization problem is demonstrated here in several steps.

1. *Achieving Convexity.*

The closed loop transfer matrices at plant output and plant input are augmented separately to form two distinct transfer function matrices.

$$T_{rz} = \begin{bmatrix} S_o \\ KS_o \\ T_o \end{bmatrix} ; \; T_{diz} = \begin{bmatrix} S_i \\ KS_i \\ T_i \end{bmatrix} \qquad \text{------------------------------------(4)}$$

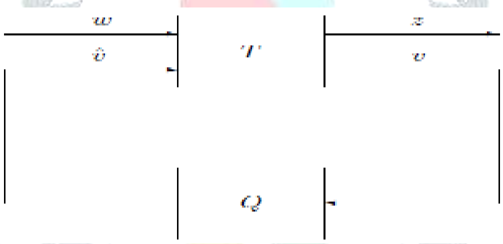

Figure 2: Representation of Closed Loop System T_{rz} and T_{diz}in terms of T and Q

The closed loop system can be represented as shown in Figure 3 where Q is Youla's parameter and the system T is to be determined below. The general transfer function matrices can be visualized as both T_{rz} and T_{diz}. Hence it is sufficient to show the affine relation between T and Q. With x denoting the states of F and x_k, the states of K_{mbc} (model based compensator)

$$\dot{x} = Ax + BF_x - BF_{(x-x_k)} + Bw + B\hat{v}$$

$$\frac{d}{dt}(x - x_k) = (A+LC)\ (x - x_k) + (B+LD)w$$

$$z = (C + DF)x - DF(x - x_k) + Dw + D\hat{v}$$

$$v = C\ (x - x_k) + Dw \qquad \text{--------------------------------------(5)}$$

$$T = \begin{bmatrix} T_1 & T_2 \\ T_3 & 0 \end{bmatrix} = \left[\begin{array}{cc|cc} A+BF & -BF & B & B \\ 0 & A+LC & B+LD & 0 \\ \hline C+DF & -DF & D & D \\ 0 & C & D & 0 \end{array}\right] \qquad \text{-----------------------(6)}$$

Given the above, it follows that the closed loop transfer function matrix T is given by

$$T(Q) = F_l(T, Q)$$

$$T(Q) = T_1 + T_2 Q T_3 \qquad \text{--(7)}$$

Equation (7) represents that the closed loop transfer function matrix depends affinely on Q and therefore our general control problem is convex in Q.

Given the above, we no longer have to search for a stabilizing controller K. Instead, we search over the convex set consisting of all stable transfer function matrices Q. As such, we still have an infinite-dimensional problem. This problem can be transformed to a finite-dimensional problem if Q is appropriately approximated.

2. Achieving Finite-Dimensionality

To obtain a finite-dimensional problem, we express the Q-parameter as a finite linear combination of a priori selected stable transfer functions qk; i.e.

$$Q_N = \sum_{k=1}^{N} X_k q_k \qquad \text{---(8)}$$

Where

$$X_k = \begin{bmatrix} x_k^{11} & \cdots & x_k^{1n_e} \\ \vdots & \ddots & \vdots \\ x_k^{n_u 1} & \cdots & x_k^{n_u n_e} \end{bmatrix} \in R^{n_u \times n_e} \qquad \text{---(9)}$$

$$q_k = \left(\frac{s-\alpha_a+\alpha_b}{s+\alpha_a+\alpha_b}\right)^{k-1} k = 1,2, \ldots N \qquad \text{---(10)}$$

Where both α_a and α_b are positive real numbers. Therefore Q_N can be written as,

$$Q_N = X_1 + X_2 \left(\frac{s-\alpha_a+\alpha_b}{s+\alpha_a+\alpha_b}\right) + X_3 \left(\frac{s-\alpha_a+\alpha_b}{s+\alpha_a+\alpha_b}\right)^2 + \ldots \ldots + X_N \left(\frac{s-\alpha_a+\alpha_b}{s+\alpha_a+\alpha_b}\right)^{N-1} \text{---(11)}$$

Uniform Real-Rational Approximation in H_∞:

A function $Q_N \in H_\infty$ can be uniformly approximated by real-rational H_∞ functions if and only if Q is continuous on the extended imaginary axis.

Substituting Q_N into (5),

$$T_{wz} = T_1 + T_2 \left(\sum_{k=1}^{N} X_k q_k\right) T_3 \qquad \text{---(12)}$$

$$= T_1 + \sum_{k=1}^{N} T_2 X_k T_3 q_k$$

$$T = T_1 + \sum_{k=1}^{N} T_2 \left(\sum_{j=1}^{n_e} \sum_{i=1}^{n_u} B^{ij} x_k^{ij}\right) T_3 q_k$$

$$T = T_1 + \sum_{k=1}^{N} \sum_{j=1}^{n_e} \sum_{i=1}^{n_u} T_2 B^{ij} x_k^{ij} T_3 q_k$$

$$T = T_1 + \sum_{k=1}^{N} \sum_{j=1}^{n_e} \sum_{i=1}^{n_u} T_2 B^{ij} T_3 q_k x_k^{ij}$$

$$T = M_0 + \sum_{k=1}^{N} \sum_{j=1}^{n_e} \sum_{i=1}^{n_u} M_k^{ij} x_k^{ij}$$

$$T = M_0 + \sum_{l=1}^{n_u \times n_e \times N} M_l x_l \qquad \text{---(13)}$$

From this expression, it follows that T depends affinely on the elements $x_l = x_k^{ij}$. Our general control system design problem has thus been transformed to a finite dimensional convex optimization in the scalar elements $x_l = x_k^{ij}$.

IV. Results

we consider the following finite-dimensional ill-conditioned plant from Freudenberg [2]:

$$P(s) = \begin{bmatrix} \frac{1}{s+1} & 0 \\ 0 & \frac{1}{s+2} \end{bmatrix} \begin{bmatrix} 9 & -10 \\ -8 & 9 \end{bmatrix} \qquad \text{---(14)}$$

This plant possesses a large condition number over all frequencies as well as large relative gain array entries [1–7].

Table I concluded the H_1 norms for various closed loop transfer function matrices showing fundamental closed loop trade-offs that were made. Specifically, Table I shows how the framework can be used to trade-off closed loop properties at distinct loop breaking points. Design 1 (MSO) has a good peak S_0 but a very bad peak S_i. Design 2 (MSI) has reversed properties. Design 3 replicates the Freudenberg subspace -based design within [2]. It exhibits a very good peak Si and a very acceptable peak S_0. Design 4 has reversed properties. Design 5 yields S_0 and S_i with similar peaks. We refer to this as an equilibrated design. Design 6 represents an equilibrated design with a lower (perhaps more realistic) bandwidth. As claimed, our generalized mixed-sensitivity framework permits designers to systematically address input-output equilibration tradeoffs. Finally, design 7 includes a peal controls constraint. Being able to address such constraints is very important when saturating actuators are a key controls concern.

Table-I: Hinf norms for Close loop maps

Sl. No	Design	S_o	S_i	T_o	T_i	KS	PS_i
1	MSO	1.62	38.62	0.00	38.62	32.48	14.37
2	MSI	4.82	1.46	3.54	0.00	38.42	2.14
3	Freq Replica	1.78	0.00	5.28	2.56	56.22	-5.23
4	Freq Inverse	0.00	3.67	1.88	5.78	56.34	0.00
5	FreqEqui	0.00	0.00	3.02	3.02	56.52	-1.62
6	Equilibrated	4.38	1.78	1.12	1.36	38.24	8.54
7	Equi-Constr	4.68	1.92	1.65	0.00	37.25	6.34

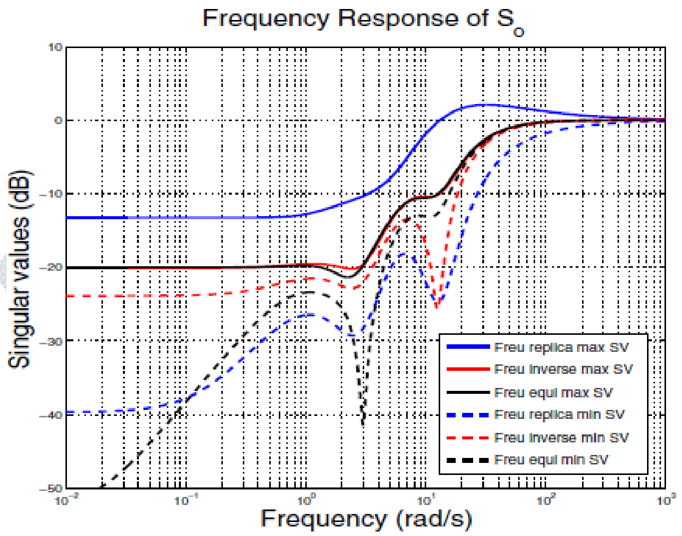

Figure 3: Maximum Singular Value So for Designs 3-5

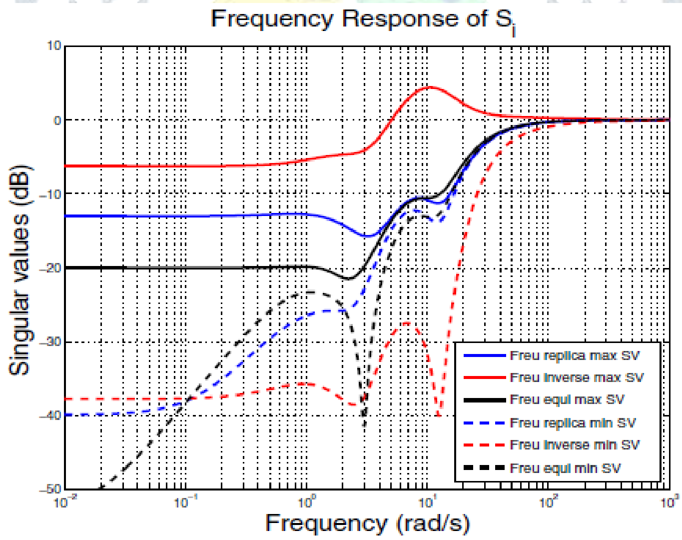

Figure 4: Maximum Singular Value Si for Designs 3-5

V. Conclusion and Future Scope

In this paper, an extensive control system design framework was developed. The utility of the design framework provides an acceptable approach forgetting performance goals at different loop-diverging points. An acceptable mixed sensitivity problem was formulated to fulfill trade-off between loop-diverging points. Design frame work of linear time invariant model smoothly illustrated the trade-offs and achieved our design objective. In future we may focus to develop generalized mixed sensitivity problems for non-linear and infinite dimensional plants. During the present work we found that for some applications, high controller order might be undesirable. So researcher may also work to reduce the order of the controller.

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