Designing of Quick Switching Bayesian Conditional RGS System-1 under Gamma – Zero Inflated Poisson Distribution

Dr. V. Kaviyarasu [1]and A. Parimala[2]

Assistant Professor[1] and Research Scholar[2], Department of Statistics, Bharathiar University,

Coimbatore – 641 046, Tamil Nadu, India

[1]Corresponding author: kaviyarasu123@gmail.com

[2]parimalaakrishnan@gmail.com

Abstract

Acceptance sampling method is an important part of Statistical quality control. In this paper Quick Switching System is studied with Conditional Repetitive group sampling plan as reference plan under Bayesian perspective. Zero-Inflated Poisson Distribution is considered for this proposed sampling plan due to modern technological advancements with more inflation of zero. The occurrence of defects in the processing units are studied with the prior probability distribution as Gamma Poisson as a prior distribution. Operating Characteristics curve and their probability values are given for the proposed methodology. Producer quality level and Consumer quality levels are determined and given in the tables. Suitable illustrations are provided for the readymade selection of plan parameters.

Introduction

Quality is playing an important role in every walk of our life, every people will accept the good quality and reject the bad quality. Acceptance sampling plans also serves the purpose either to accept or not accept the lot quality in manufacturing industries. It is a middle path in between hundred percent inspections and no inspection at all. A lot quality cannot be decided to accept without inspection at the same time if we inspect all the items the time and cost involved in such studies are high. Some times when we deal with the costly or destructive items the sampling inspection is the only tool to give a solution for such studies. Acceptance sampling techniques can be applied for finished products, semi-finished products or even for raw materials. If a product wants to be in good quality then the four M's are to be good i.e., Men, Machine, Material and Methodology are to be good. Acceptance sampling methods are involved in industries to decide whether the product meets the required standards specified by the producer and consumer. Hence these methodologies should protect both the producer and consumer simultaneously.

A company has many consumer for their day to day routine products and they decided to give certain relaxation to the their regular buyer under normal inspection and if it any rejection comes under normal inspection they want to tightened their inspection quality due to instantaneously switching between the normal and tightened inspection this system is called as Quick Switching System proposed by Romboski (1969). According to him this system is a combination of two plan system with any sampling plan as Reference plan. In this paper, Conditional Repetitive Group Sampling Plan (CRGS) is used as a Reference plan. Repetitive Group sampling plan is widely used in industries for cheaper products like nails and its types, nuts, bolts, rubber etc., Since the items are manufactured in lot by lots and they may form as a group which was repeatedly involved in inspection procedure hence it is called as Repetitive Group Sampling plan.

Quick Switching System

Dodge (1967) has proposed a two plan system with Normal and Tightened rules which are essentials for the sampling inspections. This was extensively studied by Romboski (1969) with the switching procedures involved in this system which has two types namely (n, C_N, C_T) and (n, kn, C_0). The first one suggested with two acceptance numbers with one sample size and second one has two sample sizes with one acceptance number. It shows that when one can use this plan they have two choice of having Normal and Tightened in acceptance number as usual as the sample size. These systems are widely applicable in real world environment to study the performance measure of the sampling system which illustrates the application of this system.

Bayesian Sampling Plan

The design of acceptance sampling plans are widely used quality control research especially in the development of Bayesian sampling plan which is a risk based approach for both the producer and consumer. The Bayesian sampling plan plays a predominant role in the development of acceptance sampling plans in quality control environments. Many authors developed various sampling plans for their practical utility of the plans in which the most popular methods are such as single sampling, double sampling, special type double sampling etc.,. In this paper the CRGS plan is considered with the decision can be made based on the previous lots only.

Zero-Inflated Poisson distribution

In acceptance sampling plan, the major application is to take a decision about the lot or sometimes referred as lot sentencing. Lambert D. (1992) has studied Zero-inflated Poisson regression with an application to defects in manufacturing units. Let us consider an example in a large super market receiving manufacturing products from various vendors and they involved in inspection activities with more zero inflations. This super market receives products from various suppliers to its various departments each involved with inspection methods. A supermarket manager wishes to test the products to make sure that the received products are good in conditions and there are fresh and in good quality. A sample is taken from the lot, some quality characteristics are tested and decision is made either to accept or not to accept the lot. In general, the production processes are not continuously stable in all such circumstances and the incoming items found from such processes may lead to random fluctuations that will affect the quality variations in each lot. The two major types of quality variations are studied in sampling inspection such as within-lot variation and between-lot variation. In between-lot variation the occurrences of defects are varying from the other, the proportion of nonconforming units in the lots will vary frequently.

Review of Literature

Hald (1981), Case and Keats (1982), Calvin (1990). Pandey (1972) discussed a Bayesian Single Sampling plan by attributes with three decision criteria for discrete prior distribution**. Suresh and Latha (2001)** investigated the Bayesian Single Sampling Plans for a Gamma Prior distribution.

Romboski (1969) has presented tables for the selection of QSS-1 (n, C_N, C_T) system for given p_1, p_2, α, and β. Devaraj Arumainayagam (1991) has studied the construction to the study of Quick Switching system (QSS) and its applications. Kaviyarasu and Suresh (2008) has studied the QSS-1 with Conditional RGS plan as reference plan using acceptable and limiting quality levels through incentive and filter effects are carried out to design the plans for specified AQL (or LQL) with the ratio of relative slopes h_2/h_1 have also been done. Romboski (1969) has also made certain modification and studied the merits and demerits of switching rules of QSS when it is compared with two-plan system (m, d). The rule of QSS is retained at m=1 where as tightened rule is made when d>1.

Cameron (1952) established SSP by attributes under the conditions of Poisson distribution applying unity value approach. **Golub (1953)** developed tables for determining the acceptance number for given parameter values of n, p_1 and p_2 such that the resultant sampling plan to protect the producer's and consumer's risk. **Guenther (1969)** proposed an iterative procedure to determine the SSP by attributes for specified values of the plan using binomial, hyper geometric and Poisson distributions. A detailed discussion on the studies relating to designing of such plans could be found from **Singh and Palanki (1976)**, **Hald (1981), Duncan (1986) and Stephens (2001)**. Further

Lambert (1992) reported the application to defects in manufacturing process with Zero Inflated Poisson (ZIP) model. **Bohning et al.** (1999) discussed ZIP distribution is performed better than the Poisson distribution in dental epidemiology research to measure the dental health of individuals. Some of other applications can be found in **Sim and Lim** (2008), **Yang** et al. (2011) and **Hai-yan Xu** et al. (2014). Recently, in acceptance sampling, **Loganathan and shalini** (2013) developed extensive tables and operating procedure for a single sampling attribute plan using the Zero Inflated Poisson Distribution (ZIP**). Srinivasa Rao (2017)** has designed the resubmitted lots with attribute single sampling plan for ZIP distribution**.** Designing of sampling plans under the conditions of ZIP is desperately important when the occurrence of defects would be a rare event in sampling inspection and also when the prior knowledge available for production process then the Bayesian methodology is more appropriate. Kaviyarasu .V and Sivakumar .P (2019) has studied the Bayesian single sampling plan in production and monitoring techniques under Gamma-Zero inflated Poisson distribution.

Gauri Shankar and Mohapatra (1993) developed a new sampling plan, which is an extension of classical Repetitive Group Sampling plan and designated as Conditional Repetitive Group Sampling plan. Further Kaviyarasu (2012) has studied certain designing procedure for the Quick Switching System of type 1, 2 and 3 with Conditional Repetitive Group Sampling plan as reference plan. Further Kaviyarasu and Parimala (2020) have studied a sampling system for Bayesian single sampling plan as reference plan. In this paper a new designing procedure is developed for Quick Switching Bayesian Conditional Repetitive Group sampling system of type-1 with Gamma-ZIP distribution as a probability distribution.

The designing of CRGS

The concept of repetitive group sampling (RGS) plan was introduced by Sherman (1965) in which acceptance or rejection of a lot is based on the repeated sample results on the same lot. This plan was further extended as the Conditional Repetitive Group Sampling plan which is better in sample size efficiency than the RGS plan; they also made an attempt to model the dynamics of the Conditional Repetitive Group Sampling plans through the GERT approach which has been used by several authors in quality control plans.

Operating Procedure

Following the notations similar to those of Sherman (1965), the Conditional RGS plan is carried out through the following steps:

Step 1: Draw a random sample of size *n* and determine the number of defectives *d* found therein.

Step 2: Accept the lot, if $d < c_1$ or Reject the lot, if $d > c_2$

Step 3: If $c_1 < d \leq c_2$ repeat step 1, 2 and 3 provided previous *i* lots are accepted (i.e., in each of the previous *i* lots $d \leq c_1$); otherwise reject the lot.

Thus the conditional RGS plans are characterized by four parameters, namely n, c_1, c_2 and acceptance criteria i. Here it may be observed that when $c_1 = c_2$ the resulting plan may reduced to single sampling plan. Also when i = 0, this plan becomes RGS plan due to Sherman (1965).

Operating Characteristics function

The performance of Conditional RGS plan can be studied from its operating characteristics curve according to Sherman (1965) the OC function of a CRGS plan is given by

$$P_a(p) = \frac{P_a}{1 - P_c P_a^{\,i}} \qquad (1)$$

Where $p_a = \sum_{x=0}^{c_1} \frac{e^{-np}(np)^x}{x!}$ $\quad p_r = 1 - \sum_{x=0}^{c_2} \frac{e^{-np}(np)^x}{x!}$

$$p_c = 1 - P_a - p_r$$

Operating Procedure for QSBCRGSS:

The Quick Switching Bayesian Conditional Repetitive Group Sampling plan is carried out through the following steps:

Step 1: Draw a random sample of size n and test each unit for conformance for the specified requirements.

Step 2: Under normal inspection, inspect the plan under Bayesian Conditional Repetitive Group Sampling plan with the parameters n, u_1 and u_2. If lot is accepted, continue step 2 otherwise step 3.

Step3: Under tightened inspection, inspect the plan under Bayesian Conditional Repetitive Group Sampling plan with the parameters n, v_1 and v_2. If a lot is accepted, use step 2, otherwise continue step 3.

Thus the Quick Switching Bayesian Conditional Repetitive Group Sampling plans-1 are characterized with six parameters namely, n, u_1, u_2, v_1, v_2 and i. Here, it may be observed that when $u_1 = u_2 = v_1 = v_2$ the resulting plan is reduced to the usual Bayesian Repetitive Group Sampling plan due to Sherman (1965). It may note that Quick Switching Conditional Repetitive Group Sampling plan-2 is applicable to a stream of lots and not for isolated lots.

Determination of Plan parameters

For construction and evaluation of the proposed Systems, the unity values np has obtained from the above said equation. It requires the specifications of AQL (p_1), LTPD (p_2), Producers risk (α), Consumers risk (β) and acceptance criteria i. the steps are to be followed as follows,

(1) Specify p_1 : Producer's quality level (0.95)
p_2 : Consumer's quality level (0.10)
and acceptance criteria i.

(2) From the operating ratio $R = p_2 / p_1$,

(3) Choose a plan having c_1 and c_2 associated with an operating ratio nearest to R in the corresponding table of i.

(4) Determine the sample size n = np_2 / p_2. Round up in determining the sample size.

(5) Thus, the plan consists of n, c_1, c_2 and i is chosen.

(6) The OC curve may be drawn by dividing the values of np shown for the plan by sample size n to obtain p associated with Pa (p).

(7) The ASN curve is obtained by multiplying the values of ASN/n shown by the sample size n and plotting the resulting values of ASN against p values obtained for corresponding probability of acceptance.

Designing Systems given p_1, α, p_2 and β

Table 1 can be used to design Quick Switching Bayesian Conditional Repetitive Group Sampling system–1 (QSBCRGSS-1), when two points on the OC curve (p_1, 1- α) and (p_2, β) are given. To design a QSBCRGSS-1 for the given two points on the OC curve, calculate the operating ratio, OR = p_2 / p_1. From table 2, one can determine the value of OR which is nearer to the desired ratio. Corresponding to the selected 'OR' values of u_1, u_2, v_1, v_2 and np_1 when i = 2. The sample size is determined by dividing np_1 by p_1.

For example, let $p_1 = 0.05$, $\alpha = 0.05$, $p_2 = 0.085$ and $\beta = 0.10$, calculate the Operating Ratio (OR) = $p_2 / p_1 = 0.085 / 0.05 = 1.7$. From the table 4, the value of OR (for $\alpha = 0.05$ and $\beta = 0.10$) which is the nearest to the desired ratio is 1.7055. Corresponding to this selected OR value of the values $u_1 = 1$ $u_2 = 2$, $v_1 = 0$, $v_2 = 1$, i = 2 and $np_1 = 0.6799$. The sample size is obtained as $n = np_1 / p_1 = 0.9799 / 0.05 = 19.598 \cong 20$. The desired system is QSBCRGSS-1 (20; 1, 2; 0, 1).

Designing systems for given AQL and AOQL

Table 3 can be used to design a QSCRGSS-1 indexed by AQL and AOQL. For example, given AQL = 2.2% (α= 0.05) and AOQL = 2.6%, one can compute AOQL / AQL = 2.6 / 2.2 = 1.1818. From the table, value of AOQL / AQL closest to the desired value is 1.1648. This value corresponds to $u_1 = 2$, $u_2 = 6$, $v_1 = 1$, $v_2 = 3$, i = 3 and $np_1 = 1.1648$.

The sample size is determined by, $n = np_1 / p_1 = 1.1648 / 0.025 = 46.592 \cong 46$. Thus the desired QSBCRGSS-1 parameters are n = 46, $u_1 = 2$, $u_2 = 6$, $v_1 = 1$, $v_2 = 3$ when i = 3.

Construction of Tables

The expression for probability of acceptance of Quick Switching Bayesian Conditional Repetitive Group Sampling System (QSBCRGSS)-1 sampling system, under the assumption of Gamma Zero inflated Poisson model, the composite OC function is given by equation (1) with

$$P_N = \frac{\sum_{x=o}^{u_1} e^{-np}(np)^x / x!}{1+\left[\sum_{x=0}^{u_1} e^{-np}(np)^x / x! - \sum_{x=0}^{u_2} e^{-np}(np)^x / x!\right]\left[\sum_{x=0}^{u_1} e^{-np}(np)^x / x!\right]^i} \quad (2)$$

$$P_T = \frac{\sum_{x=o}^{v_1} e^{-np}(np)^x / x!}{1+\left[\sum_{x=0}^{v_1} e^{-np}(np)^x / x! - \sum_{x=0}^{v_2} e^{-np}(np)^x / x!\right]\left[\sum_{x=0}^{v_1} e^{-np}(np)^x / x!\right]^i} \quad (3)$$

For various assumed values of u_1, u_2, v_1, v_2, i and P_a (p) the equation 1 is solved for np using iteration techniques. Utilizing the np values tabulated for different values of u_1, u_2, v_1, v_2 and i. From table 1 to 4 the various incoming quality levels, outgoing quality levels and OR values are calculated for different α and β values are given. Assuming nAOQ = np * Pa(p), value of np which maximizes nAOQ was obtained by the method of successive approximation and these values (np_m) together with nAOQL(= np_m*Pa(pm)) appear in table 4.

Examples

For example, many small scale industries want to apply this proposed methodology in their organization. Let us suppose to have the Bayesian Conditional RGS plan, for instant consider a brick manufacturing industries wants to implement the methodology and they have the responsibility to produce quality products to their customers. Suppose that the brick manufacturer appoints a quality engineer wants to run an experiment to make a decision on their product either to accept or reject a lot.

Quality is not a one time achievement, but it is a necessary part of the planning, preparation and execution production in all days and all shifts. Any lack of consideration of quality can result in a serious threat to producer in their sales and it affects the quality of the building. Assuming that the quality of brick products follows the gamma- ZIP distribution with the estimated value from the prior production process knowledge is s=5.

Suppose one wants to find the strength of the plan parameter of a BCRGS under QSS-1 for specified values of AQL and LQL say, $p_1 = 0.005, p_2 = 0.10$ with producer risk (α) 5% and consumers risk (β) 10% and estimated value of $\omega = 0.05$. The value of the operating ratio corresponding to these specifications is calculated as OR= 10. The corresponding acceptance number can be $u_1 = 1\ and\ v_2 = 2$ and $v_1 = 0\ and\ u_2 = 1$. The unity values corresponding to the values of $s, \omega, p_1, \alpha, p_2$ and β are obtained from Table, then $\frac{np1}{p1} = 88.8 \approx 89$. Therefore, the sampling system for the given specifications is (89; 1, 2, 0, 1).

For instance, the brick manufacturer produces a lot of 3000 bricks in a shift, in that they have to adhere to consider 89 samples for inspection, in that they will start with normal inspection at most 1 defective bricks can be accepted at normal level and 2 defective bricks can be accepted at tightened level, or else go to tightened inspection method as rejection occurs in normal inspection. In tightened inspection the quality engineer can inspect with 89 samples with 1 brick can be accepted as a defective brick at maximum level otherwise reject the entire lot or accept the whole lot. Switching between the inspections of tightened and normal is called the Quick Switching System.

Conclusion

Quick Switching System is an important technique in production units which deals with the products inspection in which decision to accept or reject submitted lots based on normal and tightened level. In this paper, designing of Quick Switching Bayesian Conditional Repetitive Group Sampling System has been developed under the Gamma-Zero Inflated Poisson distributions. The proposed plan can be applied as more efficient sampling system which contains more zero counts than the traditional Gamma Poisson model. The design parameters of the proposed method has investigated in terms of minimizing the risk associated with the plan parameters and its associated OC function. The effectiveness of the system has been illuminated and also shows the importance of the system under Gamma-Zero Inflated Poisson distribution. In the final touch, it may be concluded that the proposed plan will be effectively minimize the producers and consumers risks than the QSS-1 System.

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Table:1 Unity Values of QSS-1 under the conditions of BQSRGS- GZIP S=5 and i=1

					Probability of Acceptance Pa(p)					
	u_1	u_2	v_1	v_2	0.99	0.95	0.90	0.50	0.20	0.10
	0	2	0	1	0.1063	0.2392	0.3464	1.0515	2.3348	4.0689
	0	3	0	2	0.1086	0.2480	0.3624	1.1227	2.4017	4.1017
	0	3	0	1	0.1085	0.2450	0.3554	1.0740	2.3501	4.0921
	0	4	0	1	0.1113	0.2453	0.3567	1.0816	2.3625	4.0766
	1	2	0	1	0.3345	0.5688	0.7241	1.5391	2.7816	4.4459
	1	3	1	2	0.4681	0.7689	0.9815	2.2127	4.2712	6.8791
	1	3	0	1	0.4450	0.6891	0.8450	1.6371	2.8302	4.4757
ω=0.05	1	4	1	2	0.5068	0.8216	1.0406	2.2627	4.2791	6.8809
	1	4	0	2	0.4885	0.7483	0.9135	1.7279	2.9101	4.5118
	1	5	1	2	0.5122	0.8374	1.0619	2.2937	4.3083	6.8870
	1	5	0	2	0.4960	0.7602	0.9295	1.7571	2.9372	4.5289
	1	5	0	1	0.4867	0.7434	0.9086	1.7207	2.9015	4.5024
	2	3	1	2	0.6759	1.0640	1.3211	2.6717	4.6578	7.1689
	2	4	1	2	0.8599	1.2543	1.5108	2.7880	4.7145	7.2272
	2	6	1	3	1.0032	1.4260	1.6920	2.9736	4.8209	7.2998
	0	2	0	1	0.1106	0.2482	0.3627	1.1302	2.7511	7.3671
	0	3	0	2	0.1110	0.2585	0.3818	1.2117	2.8492	7.4930
	0	3	0	1	0.1110	0.2549	0.3739	1.1575	2.7698	7.3096
	0	4	0	1	0.1110	0.2554	0.3756	1.1686	2.7839	7.4101
	1	2	0	1	0.3330	0.5863	0.7450	1.6376	3.2517	7.9527
	1	3	1	2	0.4772	0.7890	1.0070	2.3256	4.8818	11.5930
ω=0.09	1	3	0	1	0.4440	0.7093	0.8749	1.7450	3.3106	7.9488
	1	4	1	2	0.5000	0.8456	1.0718	2.3838	4.9033	11.6042
	1	4	0	2	0.4898	0.7720	0.9497	1.8501	3.4129	8.0562
	1	5	1	2	0.5240	0.8632	1.0960	2.4203	4.9235	11.6195
	1	5	0	2	0.5000	0.7858	0.9680	1.8840	3.4471	8.0859
	1	5	0	1	0.4890	0.7690	0.9455	1.8428	3.3886	8.0032
	2	3	1	2	0.6692	1.0746	1.3508	2.8026	5.3309	12.1137
	2	4	1	2	0.8722	1.2807	1.5446	2.9282	5.3742	12.1389
	2	6	1	3	1.0152	1.4612	1.7412	3.1346	5.5135	12.2521

Table:2 Unity Values of QSS-1 under the conditions of BQSRGS- GZIP S=5 and i=2

					Probability of Acceptance Pa(p)					
	u_1	u_2	v_1	v_2	0.99	0.95	0.90	0.50	0.20	0.10
ω=0.05	0	2	0	1	0.0795	0.1789	0.2648	0.9194	2.2571	3.9936
	0	3	0	2	0.0798	0.1817	0.2707	0.9470	2.2770	4.0223
	0	3	0	1	0.0798	0.1806	0.2678	0.9272	2.2602	3.9989
	0	4	0	1	0.0798	0.1807	0.2682	0.9297	2.2623	3.9904
	1	2	0	1	0.3101	0.5190	0.6587	1.4322	2.7108	4.3683
	1	3	1	2	0.8517	0.6695	0.8560	2.0492	4.1896	6.8182
	1	3	0	1	0.3809	0.5932	0.7329	1.4874	2.7305	4.3718
	1	4	1	2	0.4219	0.6906	0.8817	2.0714	4.1931	6.8185
	1	4	0	2	0.4045	0.6204	0.7642	1.5247	2.7428	4.4182
	1	5	1	2	0.4290	0.6967	0.8900	2.0826	4.1961	6.8188
	1	5	0	2	0.3988	0.6251	0.7703	1.5362	2.7508	4.4222
	1	5	0	1	0.4036	0.6184	0.7621	1.5247	2.7434	4.4162
	2	3	1	2	0.6391	1.0122	1.2508	2.5591	4.6014	7.1476
	2	4	1	2	0.7940	1.1464	1.3774	2.6206	4.6138	7.1486
	2	6	1	3	0.8676	1.2363	1.4734	2.7007	4.6451	7.1513
ω=0.09	0	2	0	1	0.0807	0.1873	0.2780	0.9846	2.6532	7.2097
	0	3	0	2	0.0811	0.1904	0.2847	1.0155	2.6733	7.2276
	0	3	0	1	0.0810	0.1892	0.2815	0.9936	2.6567	7.2124
	0	4	0	1	0.0810	0.1894	0.2819	0.9967	2.6591	7.2118
	1	2	0	1	0.3161	0.5333	0.6799	1.5266	3.1710	7.8539
	1	3	1	2	0.4161	0.6837	0.8787	2.1564	4.7719	11.5308
	1	3	0	1	0.3960	0.6117	0.7587	1.5835	3.1883	7.8563
	1	4	1	2	0.4299	0.7103	0.9094	2.1797	4.7759	11.5319
	1	4	0	2	0.4099	0.6403	0.7924	1.6252	3.2117	7.8800
	1	5	1	2	0.4431	0.7168	0.9186	2.1926	4.7794	11.5333
	1	5	0	2	0.4130	0.6452	0.7992	1.6382	3.2212	7.8838
	1	5	0	1	0.4086	0.6384	0.7905	1.6253	3.2111	7.8747
	2	3	1	2	0.6550	1.0290	1.2797	2.6818	5.2549	12.0678
	2	4	1	2	0.7770	1.1697	1.4108	2.7464	5.2668	12.0692
	2	6	1	3	0.8851	1.2662	1.5141	2.8405	5.2993	12.0793

Table:3 Unity Values of QSS-1 under the conditions of BQSRGS- GZIP S=5 and i=3

					Probability of Acceptance Pa(p)					
	u_1	u_2	v_1	v_2	0.99	0.95	0.90	0.50	0.20	0.10
ω=0.05	0	2	0	1	0.0640	0.1513	0.2267	0.8611	2.2380	4.0004
	0	3	0	2	0.0642	0.1528	0.2299	0.8746	2.2414	4.0000
	0	3	0	1	0.0641	0.1521	0.2282	0.8639	2.2385	4.0003
	0	4	0	1	0.0641	0.1522	0.2284	0.8648	2.2389	4.0004
	1	2	0	1	0.2935	0.4897	0.6187	1.3855	2.6891	4.3757
	1	3	1	2	0.3706	0.6081	0.7836	1.9788	4.1610	6.8158
	1	3	0	1	0.3445	0.5413	0.6719	1.4172	2.6954	4.3760
	1	4	1	2	0.3826	0.6237	0.8012	1.9888	4.1611	6.8159
	1	4	0	2	0.3554	0.5579	0.6909	1.4333	2.7015	4.3742
	1	5	1	2	0.3789	0.6271	0.8058	1.9939	4.1611	6.8159
	1	5	0	2	0.3568	0.5606	0.6944	1.4384	2.7044	4.3742
	1	5	0	1	0.3546	0.5569	0.6901	1.4355	2.7028	4.3750
	2	3	1	2	0.6232	0.9753	1.2056	2.5027	4.5851	7.1453
	2	4	1	2	0.7398	1.0780	1.2978	2.5389	4.5887	7.1456
	2	6	1	3	0.8029	1.1389	1.3608	2.5816	4.5961	7.1462
ω=0.09	0	2	0	1	0.0694	0.1576	0.2374	0.9232	2.6333	7.2059
	0	3	0	2	0.0695	0.1592	0.2409	0.9377	2.6374	7.2068
	0	3	0	1	0.0695	0.1585	0.2391	0.9271	2.6340	7.2060
	0	4	0	1	0.0695	0.1585	0.2393	0.9283	2.6345	7.2062
	1	2	0	1	0.2967	0.5009	0.6389	1.4733	3.1502	7.8522
	1	3	1	2	0.3751	0.6245	0.8068	2.0796	4.7724	11.5256
	1	3	0	1	0.3583	0.5577	0.6950	1.5073	3.1558	7.8523
	1	4	1	2	0.3848	0.6408	0.8256	2.0906	4.7732	11.5257
	1	4	0	2	0.3669	0.5747	0.7149	1.5278	3.1619	7.8525
	1	5	1	2	0.3960	0.6444	0.8306	2.0965	4.7740	11.5258
	1	5	0	2	0.3679	0.5774	0.7185	1.5346	3.1541	7.8527
	1	5	0	1	0.3664	0.5738	0.7142	1.5300	3.1631	7.8523
	2	3	1	2	0.6274	0.9926	1.2312	2.6229	5.2379	12.0634
	2	4	1	2	0.7436	1.1011	1.3297	2.6604	5.2413	12.0636
	2	6	1	3	0.8104	1.1648	1.3967	2.7066	5.2492	12.0646

Table: 4 Operating Ratio values

Operating Ratio for 3.1				Operating Ratio for 3.2				Operating Ratio for 3.3			
α=0.01 β=0.10	α=0.01 β=0.20	α=0.05 β=0.10	α=0.05 β=0.20	α=0.01 β=0.10	α=0.01 β=0.20	α=0.05 β=0.10	α=0.05 β=0.20	α=0.01 β=0.10	α=0.01 β=0.20	α=0.05 β=0.10	α=0.05 β=0.20
38.2792	21.9659	17.0082	9.7598	7.2845	9.4078	4.1801	5.3985	0.1903	0.4283	0.2458	0.5531
37.7663	22.1133	16.5423	9.6860	6.8878	9.2075	4.0330	5.3912	0.1824	0.4164	0.2438	0.5566
37.7038	21.6537	16.7028	9.5926	7.1072	9.2138	4.0817	5.2916	0.1885	0.4255	0.2444	0.5516
36.6342	21.2305	16.6175	9.6303	7.0339	8.9865	4.0763	5.2079	0.1920	0.4233	0.2453	0.5408
13.2915	8.3160	7.8168	4.8907	2.8102	2.9896	1.7582	1.8705	0.2114	0.3595	0.2249	0.3825
14.6950	9.1240	8.9468	5.5550	2.0947	2.1362	1.3006	1.3263	0.1425	0.2341	0.1454	0.2388
10.0587	6.3605	6.4945	4.1067	2.2948	2.2474	1.4511	1.4211	0.2281	0.3533	0.2234	0.3460
13.5774	8.4435	8.3750	5.2082	1.9572	1.9732	1.2171	1.2271	0.1442	0.2337	0.1453	0.2356
9.2359	5.9571	6.0292	3.8888	2.0718	2.0470	1.3363	1.3203	0.2243	0.3436	0.2216	0.3395
13.4465	8.4116	8.2248	5.1451	1.9091	1.9524	1.1942	1.2214	0.1420	0.2321	0.1452	0.2374
9.1311	5.9221	5.9577	3.8639	2.0283	2.0162	1.3155	1.3076	0.2221	0.3405	0.2208	0.3384
9.2508	5.9616	6.0566	3.9032	2.0874	2.0547	1.3452	1.3241	0.2256	0.3446	0.2221	0.3392
10.6063	6.8912	6.7377	4.3777	1.4465	1.4795	0.9399	0.9613	0.1364	0.2147	0.1395	0.2196
8.4051	5.4828	5.7621	3.7587	1.2222	1.1630	0.7973	0.7586	0.1454	0.2121	0.1384	0.2018
7.2762	4.8053	5.1192	3.3808	1.0619	0.9968	0.7013	0.6583	0.1459	0.2074	0.1370	0.1947
66.5874	24.8660	29.6778	11.0827	10.7875	9.0385	4.0284	3.3753	0.1620	0.3635	0.1357	0.3046
67.5047	25.6686	28.9855	11.0217	10.1731	9.0090	3.8683	3.4257	0.1507	0.3510	0.1335	0.3108
65.8522	24.9531	28.6716	10.8644	10.3515	9.0090	3.9225	3.4137	0.1572	0.3610	0.1368	0.3142
66.7573	25.0804	29.0116	10.8995	10.4211	9.0090	3.9152	3.3846	0.1561	0.3592	0.1350	0.3105
23.8821	9.7649	13.5637	5.5459	4.1713	3.0030	1.7055	1.2279	0.1747	0.3075	0.1257	0.2214
24.2949	10.2306	14.6929	6.1872	3.0097	2.0957	1.2674	0.8825	0.1239	0.2048	0.0863	0.1426
17.9027	7.4563	11.2071	4.6677	3.3852	2.2523	1.4099	0.9380	0.1891	0.3021	0.1258	0.2010
23.2084	9.8066	13.7233	5.7987	2.7988	2.0000	1.1826	0.8451	0.1206	0.2039	0.0862	0.1457
16.4476	6.9679	10.4349	4.4207	3.0575	2.0416	1.2953	0.8649	0.1859	0.2930	0.1241	0.1956
22.1745	9.3960	13.4604	5.7036	2.7339	1.9084	1.1584	0.8086	0.1233	0.2031	0.0861	0.1418
16.1718	6.8942	10.2897	4.3866	2.9850	2.0000	1.2726	0.8526	0.1846	0.2901	0.1237	0.1944
16.3670	6.9300	10.4074	4.4066	3.0713	2.0451	1.3004	0.8659	0.1876	0.2951	0.1249	0.1965
18.1028	7.9666	11.2724	4.9607	2.1145	1.4944	0.9306	0.6576	0.1168	0.1876	0.0826	0.1326
13.9180	6.1619	9.4782	4.1963	1.7636	1.1466	0.7808	0.5076	0.1267	0.1861	0.0824	0.1210
12.0687	5.4310	8.3847	3.7732	1.5208	0.9850	0.6844	0.4433	0.1260	0.1814	0.0816	0.1175