SOME NEW NOTIONS ON $\breve{\theta}$-$\mathcal{J}$-CLOSEDSETS WITH RESPECT TO AN IDEAL TOPOLOGICAL SPACES

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Abstract: In this paper, we introduce the notion of $\breve{\theta}$-$\mathcal{J}$-closed sets and $\breve{\theta}$-$\mathcal{J}_\alpha$-closedsets in ideal topological spaces.We discuss about the class lies between the class of ⋆-closed sets and the class of $\breve{\theta}$-$\mathcal{J}$-closed sets.

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1. INTRODUCTION

A. Acikgoz and et al. [1], introduced the on α-I-continuous and α-I-open functions.In 1966,K. Kuratowski [5], introducedtopology. S. Jafari and N. Rajesh [6], introduced the generalized closed sets with respect to an ideal. N. Levine [7], introduced the generalized closed sets in topology.

In this paper, we introduce a new class of sets namely $\breve{\theta}$-$\mathcal{J}$-closed sets in ideal topological spaces. We discuss about the class lies between the class of ⋆-closed sets and the class of $\breve{\theta}$-$\mathcal{J}$-closed sets.

2.PRELIMINARIES

An ideal I on a topological space (briefly, TPS) (X, τ) is a nonempty collection of subsets of X which satisfies

(1)A∈ I and B⊆A⇒B∈ I and

(2)A∈ I and B∈ I⇒A∪B∈ I.

Given a topological space (X, τ) with an ideal $\mathcal{J}$on X if ℘(X) is the set of allsubsets of X, a set operator (•)⋆: ℘(X)→ ℘(X), called a local function [4] ofA with respect to τ and $\mathcal{J}$is defined as follows: for A ⊆X, A⋆($\mathcal{J}$, τ)={ x ∈ X:U∩A ∉ IforeveryU ∈ τ(x)} whereτ(x)={U ∈ τ:x ∈ U}.A Kuratowskiclosure operator cl⋆(•) for a topology τ⋆($\mathcal{J}$, τ), called the ⋆-topology and finer than τ, is defined by cl⋆(A) = A ∪A⋆($\mathcal{J}$, τ) [4]. We will simply writeA⋆for A⋆($\mathcal{J}$, τ) and τ⋆for τ⋆($\mathcal{J}$, τ). If I is an ideal on X, then (X, τ, $\mathcal{J}$) iscalled an ideal topological space(briefly, ITPS). A subset A of ani deal topological space(X,τ,$\mathcal{J}$)is ⋆-closed (briefly, ⋆-cld) [4]ifA⋆⊆A.The interior of a subset A in (X,τ⋆($\mathcal{J}$)) is denoted by int⋆(A).

Definition 2.1 A subset K of a TPS X is called:
(i) semi-open set [2] if $K \subseteq cl(int(K))$;
(ii) α-open set [8] if $K \subseteq int(cl(int(K)))$;
(iii) β-open set (Semi-pre-open) [2] if $K \subseteq cl(int(cl(K)))$;

The complements of the above-mentioned open sets are called their respective closed sets.

Definition 2.2 A subset K of a TPS X is called

(i) αgs -closedset (briefly, αgs -cld)[8] if $\alpha cl(K) \subseteq V$ whenever $K \subseteq V$ and V is semi-open.

(ii) semi-generalized closed (briefly, sg-cld)[2] if $scl(K) \subseteq V$ whenever $K \subseteq V$ and V is semi-open.

(iii) ψ -closed (briefly, ψ -cld) [10] if $scl(K) \subseteq V$ whenever $K \subseteq V$ and V is sg-open.

(iv) generalized semi-closed (briefly, gs-cld)[9] if $scl(K) \subseteq V$ whenever $K \subseteq V$ and V is open.

(v) α -generalized closed (briefly, α g-cld)[8] if α $cl(K) \subseteq V$ whenever $K \subseteq V$ and V is open.

(vi) generalized semi-pre-closed(briefly, gsp-cld) [9] if $spcl(K) \subseteq V$ whenever $K \subseteq V$ and V is open .

The complements of the above-mentioned closed sets are called their respective open sets.

Definition 2.3 A subset K of a ITPS X is called

(i) $\mathcal{J}_g$-closed (briefly, $\mathcal{J}_g$-cld) set [3] if $K^* \subseteq V$ whenever $K \subseteq V$ and V is open.

The complements of the above-mentioned closed set is called their respective open set.

3. $\breve{\theta}$-$\mathcal{J}$-CLOSED SETS

We introduce the following definition.

Definition 3.1

A subset K of X is called

(i) $\breve{\theta}$-$\mathcal{J}$-closed (briefly, $\breve{\theta}$-$\mathcal{J}$-cld) if $K^* \subseteq V$ whenever $K \subseteq V$ and V is sg-open.

The complement of $\breve{\theta}$-$\mathcal{J}$-cld is called $\breve{\theta}$-$\mathcal{J}$-open.

The family of all $\breve{\theta}$-$\mathcal{J}$-cld in X is denoted by $\breve{\theta}$-$\mathcal{J}C(X)$.

(ii) $\breve{\theta}$-$\mathcal{J}_\alpha$-closed (briefly, $\breve{\theta}$-$\mathcal{J}_\alpha$-cld) if α $cl(K^*) \subseteq V$ whenever $K \subseteq V$ and V is sg-open.

The complement of $\breve{\theta}$-$\mathcal{J}_\alpha$-cld is called $\breve{\theta}$-$\mathcal{J}_\alpha$-open.

(iii) $\mathcal{J}$-ĝ-closed (briefly, $\mathcal{J}$-ω-cld) if $K^*\subseteq V$ whenever $K\subseteq V$ and V is semi-open.

The complements of the above-mentioned closed set is called their respective open set.

Proposition 3.2

Every ⋆-cld is $\breve{\theta}$-$\mathcal{J}$-closed.

Proof

If K is any⋆-cld in X and H is any sg-open set containing K,then $H\supseteq K = K^*$. Hence K is $\breve{\theta}$-$\mathcal{J}$-cld.

The converse of Proposition 3.2 need not be true as seen from the following example.

Example 3.3

Let X = {p, q, r},τ = {ϕ, {p, q}, X} and$\mathcal{J}$ = {ϕ}. Then $\breve{\theta}$-$\mathcal{J}$C(X) = {ϕ, {r}, {p, r}, {q, r}, X}. Here, K = {p, r} is $\breve{\theta}$-$\mathcal{J}$-cld set but not ⋆-cld.

Proposition 3.4

Every $\breve{\theta}$-$\mathcal{J}$-cld is $\breve{\theta}$-$\mathcal{J}_\alpha$-cld.

Proof

If K is a $\breve{\theta}$-$\mathcal{J}$-cld subset of X and H is any sg-open set containing K, then $H\supseteq K^*\supseteq \alpha$ cl(K*). Hence K is $\breve{\theta}$-$\mathcal{J}_\alpha$-cld.

The converse of Proposition 3.4 need not be true as seen from the following example.

Example 3.5

Let X = {p, q, r}, τ = {ϕ, {q}, X} with $\mathcal{J}$ = {ϕ}. Then $\breve{\theta}$-$\mathcal{J}$C(X) = {ϕ, {p, r}, X} and $\breve{\theta}$-$\mathcal{J}_\alpha$C(X) = {ϕ, {p}, {r}, {p, r}, X}. Here, K = {p} is $\breve{\theta}$-$\mathcal{J}_\alpha$-cld but not $\breve{\theta}$-$\mathcal{J}$-cld.

Proposition 3.6

Every $\breve{\theta}$-$\mathcal{J}$-cld is ψ -cld.

Proof

If K is a $\breve{\theta}$-$\mathcal{J}$-cld subset of X and H is any sg-open set containing K, then $H\supseteq K^*\supseteq$scl(K). Hence K is ψ -cld.

The converse of Proposition 3.6 need not be true as seen from the following example.

Example 3.7

Let X = {p, q, r}, τ = {ϕ, {p},X} and $\mathcal{I}$ = {ϕ}. Then $\breve{\theta}$-$\mathcal{I}$C(X) = {ϕ, {q, r}, X} and ψ C(X) = {ϕ, {q}, {r}, {q, r}, X}. Here, K = {q} is ψ -cld but not $\breve{\theta}$-$\mathcal{I}$-cld.

Proposition 3.8

Every $\breve{\theta}$-$\mathcal{I}$-cld set is $\mathcal{I}$-ω -cld.

Proof

Suppose that K⊆H and H is semi-open in X. Since every semi-open set is sg-open and K is $\breve{\theta}$-$\mathcal{I}$-cld, therefore K*⊆H. Hence K is $\mathcal{I}$-ω -cld.

The converse of Proposition 3.8 need not be true as seen from the following example.

Example 3.9

Let X = {p, q, r, s},τ = {ϕ, {s}, {q, r}, {q, r, s}, X} and $\mathcal{I}$ = {ϕ}. Then $\breve{\theta}$-$\mathcal{I}$C(X) = {ϕ, {p}, {p, s}, {p, q, r}, X} and $\mathcal{I}$-ω C(X) = {ϕ, {p}, {p, q}, {p, r}, {p, s},{p, q, r}, {p, q, s}, {p, r, s}, X}. Here, K = {p, r, s} is $\mathcal{I}$-ω -cld but not $\breve{\theta}$-$\mathcal{I}$-cld.

Proposition 3.10

Every $\breve{\theta}$-$\mathcal{I}$-cld is $\mathcal{I}_g$-cld.

Proof

If K is a $\breve{\theta}$-$\mathcal{I}$-cld subset of X and H is any open set containing K, since every open set is sg-open, we have H⊇K*. Hence K is$\mathcal{I}_g$-cld.

The converse of Proposition 3.10 need not be true as seen from the following example.

Example 3.11

Let X = {p, q, r},τ = {ϕ, {p}, {q, r}, X} and $\mathcal{I}$ = {ϕ}. Then $\breve{\theta}$-$\mathcal{I}$C(X) = {ϕ, {p}, {q, r}, X} and $\mathcal{I}_g$-C(X) = P(X). Here, K = {p, q} is $\mathcal{I}_g$-cld but not $\breve{\theta}$-$\mathcal{I}$-cld.

Proposition 3.12

Every $\breve{\theta}$-$\mathcal{I}$-cld is αgs -cld.

Proof

If K is a $\breve{\theta}$-$\mathcal{J}$-cld subset of X and H is any semi-open set containing K, since every semi-open set is sg-open, we have H⊇K*⊇α cl(K*). Hence K is αgs -cld.

The converse of Proposition 3.12 need not be true as seen from the following example.

Example 3.13

Let X = {p, q, r},τ = {ϕ, {p}, {q, r}, X} and $\mathcal{J}$ = {ϕ}. Then $\breve{\theta}$-$\mathcal{J}$C(X) = {ϕ, {p}, {q, r}, X} and αGS C(X) = P(X). Here, K = {p, r} is αgs -cld but not $\breve{\theta}$-$\mathcal{J}$-cld.

Proposition 3.14

Every $\breve{\theta}$-$\mathcal{J}$-cld is α g-cld.

Proof

If K is a $\breve{\theta}$-$\mathcal{J}$-cld subset of X and H is any open set containing K, since every open set is sg-open, we have H⊇K*⊇α cl(K). Hence K is α g-cld.

The converse of Proposition 3.14 need not be true as seen from the following example.

Example 3.15

Let X = {p, q, r},τ = {ϕ, {r}, {p, q}, X} and $\mathcal{J}$ = {ϕ}. Then $\breve{\theta}$-$\mathcal{J}$C(X) = {ϕ, {r}, {p, q}, X} and αG C(X) = P(X). Here, K = {p, r} is α g-cld but not $\breve{\theta}$-$\mathcal{J}$-cld.

Proposition 1.3.16

Every α -cld is $\breve{\theta}$-$\mathcal{J}_\alpha$-cld.

Proof

If K is an α -cld subset of X and H is any sg-open set containing K, we have α cl(K*) = K⊆H. Hence K is $\breve{\theta}$-$\mathcal{J}_\alpha$-cld.

The converse of Proposition 3.16 need not be true as seen from the following example.

Example 3.17

Let X = {p, q, r}, τ = {ϕ, X, {p, q}} and $\mathcal{J}$ ={ϕ}. Then α C(X) = {ϕ, {r}, X} and $\breve{\theta}$-$\mathcal{J}_\alpha$C(X) = {ϕ, {r}, {p, r}, {q, r}, X}. Clearly, the set {p, r} is an $\breve{\theta}$-$\mathcal{J}_\alpha$ cld but not an α -cld.

Proposition 3.18

Every $\breve{\theta}$-$\mathcal{J}$-cld is gs-cld.

Proof

If K is a $\breve{\theta}$-$\mathcal{J}$-cld subset of X and H is any open set containing K, since every open set is sg-open, we have H⊇K*⊇scl(K). Hence K is gs-cld.

The converse of Proposition 3.18 need not be true as seen from the following example.

Example 3.19

Let X = {p, q, r},τ = {ϕ, {p}, X} and $\mathcal{J}$ = {ϕ}. Then $\breve{\theta}$-$\mathcal{J}$C(X) = {ϕ, {q, r}, X} and GS C(X) = {ϕ, {q}, {r}, {p, q}, {p, r}, {q, r}, X}. Here, K = {r} is gs-cld but not $\breve{\theta}$-$\mathcal{J}$-cld.

Proposition 3.20

Every $\breve{\theta}$-$\mathcal{J}$-cld is gsp-cld.

Proof

If K is a $\breve{\theta}$-$\mathcal{J}$-cld subset of X and H is any open set containing K, every open set is sg-open, we have H⊇K*⊇spcl(K). Hence K is gsp-cld.

The converse of Proposition 3.20 need not be true as seen from the following example.

Example 3.21

Let X = {p, q, r},τ = {ϕ, {q}, X} and $\mathcal{J}$ = {ϕ}. Then $\breve{\theta}$-$\mathcal{J}$C(X) = {ϕ, {p, r}, X} and GSP C(X) = {ϕ, {p}, {r}, {p, q}, {p, r}, {q, r}, X}. Here, K = {r} is gsp-cld but not $\breve{\theta}$-$\mathcal{J}$-cld.

Remark 3.22

The following example shows that $\breve{\theta}$-$\mathcal{J}$-cld sets are independent of α-cld sets and semi-cld sets.

Example 3.23

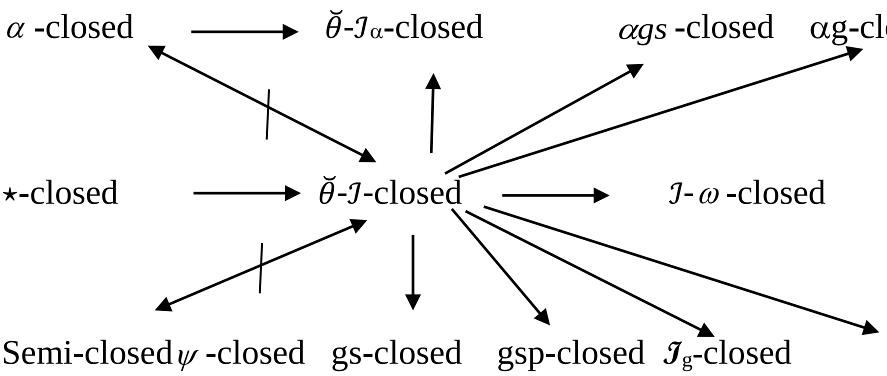
Let X = {p, q, r},τ = {ϕ, {p, q}, X} and $\mathcal{J}$ = {ϕ}. Then $\breve{\theta}$-$\mathcal{J}$C(X) = {ϕ, {r}, {p, r}, {q, r}, X} and α C(X) = S C(X) = {ϕ, {r}, X}. Here, K = {p, r} is $\breve{\theta}$-$\mathcal{J}$-cld but it is neither α-cld nor semi-cld.

Example 3.24

Let X = {p, q, r},τ = {ϕ, {p}, X} and $\mathcal{J}$ = {ϕ}. Then $\breve{\theta}$-$\mathcal{J}$C(X) = {ϕ, {q, r}, X} and α C(X) = S C(X) = {ϕ, {q}, {r}, {q, r}, X}. Here, K = {q} is α-cld as well as semi-cld but it is not $\breve{\theta}$-$\mathcal{J}$-cld.

Remark 3.25

We obtain the following diagram, where A → B (resp. A ⟷ B) represents A implies B but not conversely (resp. A and B are independent of each other).



Theorem 3.26

A set K is $\breve{\theta}$-$\mathcal{J}$-cld if and only if K^*-K contains no nonempty sg-cld set.

Proof

Necessity. Suppose that K is $\breve{\theta}$-$\mathcal{J}$-cld. Let T be a sg-cld subset of K^*-K. Then $K\subseteq T^c$. Since A is $\breve{\theta}$-$\mathcal{J}$-cld, we have $K^*\subseteq T^c$. Consequently, $T\subseteq (K^*)^c$. Hence, $T\subseteq K^*\cap (K^*)^c = \phi$. Therefore,T is empty.

Sufficiency. Suppose that K^*-K contains no nonempty sg-cld set. Let $K\subseteq H$ and H be ⋆-cld and sg-open. If $K^*\not\subset H$, then $K^*\cap H^c\neq\phi$. Since K^* is a ⋆-cld set and H^c is a sg-cld set, $K^*\cap H^c$ is a nonempty sg-cld subset of K^*-K. This is a contradiction. Therefore, $K^*\subseteq H$ and hence K is $\breve{\theta}$-$\mathcal{J}$-cld.

Theorem 3.27

A set K of X is $\breve{\theta}$-$\mathcal{J}$-open if and only if $F\subseteq int(K)$ whenever F is sg-cld and $F\subseteq K$.

Proof

Suppose that $F\subseteq int(K)$ such that F is sg-cld and $F\subseteq K$. Let $K^c\subseteq G$ where G is sg-open. Then $G^c\subseteq K$ and G^c is sg-cld. Therefore $G^c\subseteq int(K)$ by hypothesis. Since $G^c\subseteq int(K)$, we have $(int(K))^c\subseteq G$. i.e., $(K^c)^*\subseteq G$, since $(K^c)^* = (int(K))^c$. Thus,K^c is $\breve{\theta}$-$\mathcal{J}$-cld. i.e., K is $\breve{\theta}$-$\mathcal{J}$-open.

Conversely, suppose that K is $\breve{\theta}$-$\mathcal{J}$-open such that $F\subseteq K$ and F is sg-cld. Then F^c is sg-open and $K^c\subseteq F^c$. Therefore, $(K^c)^*\subseteq F^c$ by definition of $\breve{\theta}$-$\mathcal{J}$-cld and so $F\subseteq int(K)$, since $(K^c)^* = (int(K))^c$.

Lemma 3.28

For an $x\in X$, $x\in\breve{\theta}$-$\mathcal{J}$-cl(K) if and only if $V\cap K\neq\phi$ for every $\breve{\theta}$-$\mathcal{J}$-open set V containing x.

Proof

Let $x\in\breve{\theta}$-$\mathcal{J}$-cl(K) for any $x\in X$. To prove $V\cap K\neq\phi$ for every $\breve{\theta}$-$\mathcal{J}$-open set V containing x. Prove the result by contradiction. Suppose there exists a $\breve{\theta}$-$\mathcal{J}$-open set V containing x such that $V\cap K = \phi$. Then $K\subseteq V^c$ and V^c is $\breve{\theta}$-$\mathcal{J}$-cld. We have $\breve{\theta}$-$\mathcal{J}$-cl(K) $\subseteq V^c$. This shows that $x\notin\breve{\theta}$-$\mathcal{J}$-cl(K) which is a contradiction. Hence $V\cap K\neq\phi$ for every $\breve{\theta}$-$\mathcal{J}$-open set V containing x.

Conversely, let V $\cap$K$\neq\phi$ for every $\breve{\theta}$-$\mathcal{J}$-open set V containing x. To prove x $\in\breve{\theta}$-$\mathcal{J}$-cl(K). We prove the result by contradiction. Suppose x $\notin\breve{\theta}$-$\mathcal{J}$-cl(K). Then there exists a $\breve{\theta}$-$\mathcal{J}$-cld set F containing K such that x $\notin$ F. Then x $\in$ F^c and F^c is $\breve{\theta}$-$\mathcal{J}$-open. Also, $F^c\cap K = \phi$, which is a contradiction to the hypothesis. Hence x $\in\breve{\theta}$-$\mathcal{J}$-cl(K).

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