

STUDY OF JOSEPHSON VORTEX LATTICE AND EVALUATION OF VORTEX-OSCILLATIONS IN HIGH T_c -SUPERCONDUCTORS

VIKASH KUMAR AND L.K. MISHRA

PHD Research scholar MU Bodh Gaya Bihar and Professor, Department of Physics MU Bodh Gaya Bihar, India

ABSTRACT

We have studied vortex lattice oscillations of the Josephson vortex lattice at small field and frequency for high T_c - superconductors. We have obtained the simple relation for the dynamic dielectric constant which the perturbation of the superconducting, phase induces by the oscillating electric field. We have computed the loss-function as a function of frequency for c-axis and in plane dissipation parameters. These parameters are inversely proportional to the anisotropy. This case of weak and strong dissipation is realized in $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_x$ and under doped $\text{YBa}_2\text{Cu}_3\text{O}_x$ respectively. We have also explored the evaluation of the loss-function with the increasing magnetic field. We observed in additional peak in the loss function below the JPR peak which was observed experimentally in under doped YBCO. We established that this peak appears due to the frequency dependents of the in plane contribution to losses.

KEY WORDS: Vortex oscillations module, Josephson plasma resonance (JPR), Josephson vortices, vortex oscillation theory, vortex parameters, linear vortex mass, viscosity, coefficient, DC flux-below conductivity.

INTRODUCTION:

Collective oscillation in superconductors have been discussed in many place in the literature, phase oscillation with an acoustic spectrum has been founded by Bogolyubov¹ and Anderson² with neutral electrons.

In conventional superconductors Coulomb interaction transfers. Such modes into plasma oscillations with a frequency higher than the energy gap. These oscillations differ slightly from plasma oscillations in normal metals weakly damped collective oscillations in real superconductor (the carlon – Goldman modes) have been found experimentally³ and explained theoretically^{4, 5}.

The modes exist only near the transition temperature T_c . It has a line spectrum and is associated with state, oscillators in the phase of the mode parameters in an electric field.

In cuprate superconductor, the superconductivity only the c-axis is maintained by the Josephson Coupling only between the CO₂ layers the Josephson current flowing along the C-axis is coupled with the electromagnetic field, generating the Josephson plasma whose frequency appears in the range of 10 GHz and 1 THz. The frequency dispersion for the transverse and longitudinal plasma modes were calculated the Josephson plasma strongly interacts with vortices and the values of the plasma frequency depends on the vortex state.

The interaction is so strong that the plasma can be extended by the vortex flow generated by an external electric current.⁶⁻⁸ In this paper using the theoretical formalism of A.E. Koshelev⁹ and A.E. Kospelev and M.J.W. Dodgson, ¹⁰ we have theoretically evaluated vertex oscillations at small field and frequency. We have studied in role of in-plane dissipation and c-axis dissipation by evaluating loss- function as a function of frequency ω for different values of parameters h , N_2 λ_c and V_{ab}

We have also compared the result of loss-function as a function of frequency with numerical solution and vortex oscillation mode for different values of λ , V_{ab} , h and N_2 . We observed that value of the vortex oscillation mode describes the high frequency response up to half of the plasma frequency. We have also evaluated frequency dependent loss function near JPR frequency of different field for $\lambda_c=0.01$ and $V_{ab} = 0.1$ we have also evaluated the loss-function for the same $h=0.2$ and 0.4 for different N_2 keeping $v_c=0.01$ and $v_{ab}=0.1$

In this evaluation, we observed that the loss function depends upon the vortex lattice superconductor above the JPR peak. We have also evaluated the frequency dependents loss- function of different field for $v_c = 0.32$ and $v_{ab} = 6.0$ In this study we have evaluated the loss- function with increasing in plane field for under doped YBCO. Finally we evaluated the real part of conductivity σ_1 as a function of ω keeping

$$v_{ab}=6.0, v_c = 0.32, N_2 = 2 \text{ and } h = 1.$$

OBJECTIVE:

In this paper, we have studied the vortex lattice oscillation of the josephson vortex lattice at small field and frequency for high T_c superconductors. We have theoretically evaluated

vortex oscillations of Josephson vortex lattice by evaluating the frequency dependents of the loss-function.

MATHEMATICAL FORMULA USED IN THE STUDY:

One writes Maxwell's equation for layered superconductor for fields and current in terms of gauge invariant phase difference between the layers.

$$\theta_n = \phi_{n+1} - \phi_n - (2\pi s / \phi_0) A_2$$

One writes these equations in the form of phase difference and magnetic field ⁹⁻¹²

$$\frac{\sigma_c \phi_0}{2\pi s j_j} \frac{\partial \theta_n}{\partial t} + \sin \theta_n + \frac{1}{\omega_p^2} \frac{\partial^2 \theta_n}{\partial t^2} - \frac{C}{4\pi j_j} \frac{\partial B_n}{\partial x} \quad \text{-----(1)}$$

$$\frac{1}{4\pi j_j} \frac{\partial D_z}{\partial t} \left(\frac{4\pi \sigma_{ab}}{C^2} \frac{\partial}{\partial t} + \frac{1}{\lambda_{ab}^2} \right) \left(\frac{\phi_0}{2\pi s} \frac{\partial \theta_n}{\partial x} - \mathbf{B}_n \right) = \frac{\nabla_n^2 B_n}{s^2} \quad \text{----- (2)}$$

Here the magnetic field along y- axis σ_{ab} and σ_c the component of the quasiparticles conductivity λ_{ab} and λ_c are the components of the London penetration depth, J_j is Josephson current density.

$J_j = C \phi_0 / 8\pi^2 s \lambda_c^2$, ω_p is the plasma frequency.

$\omega_p = C / \sqrt{\xi_c} \lambda_c$. D_z is the external electric field and

$$\nabla_n^2 B_n = B_{n+1} + B_{n-1} - 2B_n$$

Neglecting the charging effects. ¹² The local electric field is connected with the phase difference by the Josephson relation.

$$E_z = \frac{\phi_0}{2\pi s} \frac{\partial \theta_n}{\partial t} \quad \text{-----(3)}$$

The average magnetic induction inside the superconductor By fixes the average phase gradient

$$(\partial \theta_n^{(0)} / \partial x) = 2\pi s B_y / \phi_0$$

Now, one uses a standard transformation to the reduced variables.

$$x / \lambda_j \longrightarrow x$$

$$i \omega_p \longrightarrow l$$

$$h_n = 2\pi \gamma s^2 B_n / \phi_0$$

Hence

$$\lambda_j = \gamma s$$

One introduces the dimension less parameters

$$\begin{aligned}
 l &= \frac{\lambda_{ab}}{s} \\
 \nu_c &= \frac{4\pi\sigma_c}{\xi_c\omega_p} \\
 \nu_{ab} &= \frac{4\pi\sigma_{ab}\lambda_{ab}^2\omega_p}{c^2}
 \end{aligned}
 \tag{5}$$

Both damping parameters ν_c and ν_{ab} inversely proportional to the anisotropy transfer γ . γ measures that the effecting damping is stronger in less anisotropic materials due to d-wave pairing in the high temperature superconductors both dissipation parameters ν_c and ν_{ab} do not vanish as T- O. The another important function of the high temperature superconductors is that the in-plane dissipation is much stronger than the C-axis dissipation¹² $\nu_{ab} \gg \nu_c$, this is the consequence of the rapid decreases of the in-plane scattering role with decreasing temperature. This manifest itself a large peak in the temperature dependents of the in-plane quasiparticle conductivity.^{13,14}

For an oscillating external field and using a complex parameters

$$D_z(t) = D_z \exp(-i\omega t)$$

One obtains for small oscillation

$$\left[-i\nu_c\varpi + C_n(x) - \varpi^2\right]\theta_n - l^2 \frac{\partial n}{\partial x} = \frac{i\omega}{4\pi j_l} D_z \tag{6}$$

$$\frac{\partial \theta_n}{\partial x} - h_n + \frac{l^2}{1 - i\nu_{ab}\varpi} \nabla_n^2 h_n = 0 \tag{7}$$

Where $\varpi = \frac{\omega}{\omega_p}$

$$C_n(x) = \text{Cos}[\theta_n^{(0)}(x)]$$

The static phase $\theta_n^{(0)}(x)$ are determined by the following reduced equation.

$$\frac{\partial^2 \theta_n^{(0)}}{\partial x^2} + \left(-\frac{1}{l^2} + \Delta_n\right) \text{Sin}\theta_n^{(0)} = 0 \tag{8}$$

Here $(\theta_n^{(0)} / \partial x) = h = 2\pi\gamma^2 s^2 B y / \phi_0$

Now one introduces the reduces oscillating phase

$$\theta_n = \theta_n^{(0)} + \frac{\omega_p D_z}{4\pi J_l}$$

From equation (6) and (7) are gets the following reduced equation.

$$-\frac{\partial^2 \theta_n}{\partial x^2} + \left[\frac{1}{l^2} - \frac{1}{l - i\nu_{ab}\varpi} - \nabla_n^2\right] \tag{9}$$

$$[C_n(x) - \varpi^2 - i\nu_c\varpi]\theta_n = \frac{i\varpi}{l^2}$$

From this Josephson relation (3) ones finds that

$$E_z = (-i\varpi / \xi_z) \bar{\theta} D_z \tag{10}$$

Here $\bar{\theta}$ is the oscillating phase

$$\xi_c(\omega) = -\xi_c l(-i\omega \bar{\theta}) \quad \text{-----(11)}$$

For zero magnetic field the oscillating phase is given by

$$\theta_n = \bar{\theta} \left[\frac{-i\omega}{1 - \omega^2 - i\nu_c \omega} \right] \quad \text{-----(12)}$$

In these cases equation (11) gives the well known results for the dynamic dielectric constant.

$$\xi_c^0(\omega) = D_z / E_2$$

$$\text{Then loss function } \bar{l}_0(\omega) = I_m \left(-\frac{1}{\xi_c^0(\omega)} \right)$$

In this of real units we get

$$\xi_c^0(\omega) = \xi_c - \frac{\xi_c \omega_p^2}{\omega^2} + \frac{4\pi i \sigma_c}{\omega} \quad \text{-----(13)}$$

$$L_0(\omega) = \frac{4\pi \omega^2 \sigma_c / \xi_c^2}{(\omega^2 - \omega_p^2)^2 + (4\pi \omega \sigma_c / \xi_c)^2} \quad \text{-----(14)}$$

The zero field loss function has a peak at the Josephson plasma frequency width which determined only by the C-axis quasiparticle conductivity.

The statics phase solution at high fields for transverse lattices is given by.

$$\phi_n^{(0)} = \frac{\pi n(n-1)}{2} + \frac{2}{h^2} \sin(h_x + \pi_n) \quad \text{-----(15)}$$

At high field one can neglect rapidly oscillating $C_n(x)$.

Then one obtains the solution.

$$\phi_n = \frac{C_n(x/2)}{\omega^2 - (l - i\nu_{ab} \omega)h^2 / 4 + i\nu_c \omega} \quad \text{-----(16)}$$

Finally one obtained:

$$\frac{\xi_c(\omega)}{\xi_c} = 1 + \frac{i\nu_c}{\omega} - \frac{2}{h^2 \omega^2} - \frac{[\frac{1}{i\omega^2}]}{\omega^2 + i\nu_c \omega - (l - \nu_{ab} \omega^2)h^2 / 4} \quad \text{-----(17)}$$

In this low frequency regime

$$\nu_c, \nu_{ab} \ll 1$$

This loss function has a peak at $\omega = h/2$

Corresponding to homogeneous plasma mode.¹⁵ such linear growth of plasma frequency with field has been observed in under doped ¹⁶

BSCCO ($\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$)

Now one introduces the phenomenological theory of vortex oscillations. It describes the response $\omega \ll \omega_p$ of the vortex lattice at small frequency. The Abrikosov vortex lattice theory was developed by Collay and clean.¹⁷

The dynamic dielectric constant for the Josephson vortex lattice has been derived¹⁸. Consider a superconductor in the vortex state carrying ac superconductor.

$J_j \propto \exp(-i\omega t)$ along the c-axis

The ac electric field consist of London term and the contribution from the vortex oscillation^{15,16}

$$E_z = \frac{4\pi\lambda_c^2}{c^2} i\omega j_s - \frac{By}{c} i\omega t \tag{18}$$

The vortex oscillation u can be formed from the equation¹⁷

$$(-\rho_j\omega^2 - i\eta_j\omega + k) u = \frac{\phi_0}{c} j_s \tag{19}$$

Here ρ_j is the linear mass of the Josephson vortex,¹⁹ η_j is viscosity coefficient^{19,20} and K is spring constant due to pairing. The viscosity of an isolated Josephson Vortex has been calculated considering the dissipation coset by both c-axis and in-plane quasiparticle transport.

$$\eta_j = \frac{\xi_c \omega_p \phi_0^2}{\pi(4\pi cs)^2 \gamma} (C_c \nu_c + C_{ab} \nu_{ab}) \tag{20}$$

Where the numerical constant C_c and C_{ab} are derived by the phase distribution of an isolated Josephson vortex $\phi_b^{(0)}$

$$C_c = \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} du \left[\frac{\partial(\phi_n^{(0)}) + \phi_n^{(0)}}{\partial u} \right] = 9.0 \tag{21}$$

$$C_{ab} = \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} du \left(\frac{\partial^2(\phi_n^{(0)}) + \phi_n^{(0)}}{\partial u^2} \right)^2 = 2.4 \tag{22}$$

The linear mass of the Josephson vortex is distributed by the kinetic energy E_k . This for many vortex is expressed as.

$$E_k \int \frac{d^2\bar{\gamma} \xi_c E^2}{8\pi} = s \sum_u \int d^2\bar{\gamma} \frac{\xi_c}{8\pi} \left(\frac{\phi_0}{2\pi cs} \right)^2 \theta_n^2 \tag{23}$$

By definition $E_k = L_y \rho_j u^2 / 2$

The linear vortex mass is given by

$$\rho_j = \frac{C_c \xi_c \phi_0^2}{2\pi\gamma(4\pi cs)^2} \tag{24}$$

$$\text{Here } E_z = \left(-4\pi\lambda_c^2 + \frac{By\phi_0}{-\rho_j\omega^2 - i\eta_j\omega + k} \right) \frac{i\omega j_j}{c^2} \tag{25}$$

The total conductivity $\sigma_c(B_\gamma, \omega)$ is given by

$$\sigma_c(B_\gamma, \omega) = \sigma_c n / \omega - \frac{\xi_c \omega_p^2}{4\pi L \omega} \left[1 + \frac{1}{\rho_j \omega^2 - i \eta_j \omega + k} \frac{B_\gamma \phi_0}{4\pi \lambda_c^2} \right]^{-1} \quad \text{-----(26)}$$

The real part of the conductivity,

$$\sigma_{c,s}(B_\gamma, \omega) = \frac{R_c [\sigma_{c,s}(\omega) + \eta_j \xi_c \omega_p^2 B_\gamma \phi_0 / (4\pi \lambda_c)^2]}{(k + B_\gamma \phi_0 / (4\pi \lambda_c^2) - \rho^2 \omega^2)^2 + (\eta_j \omega)^2} \quad \text{-----(27)}$$

The dynamic dielectric metal and conductivity is related with the equation.

$$\xi_c / \omega = \xi_c - 4\pi \sigma_c \omega / i \omega \quad \text{-----(28)}$$

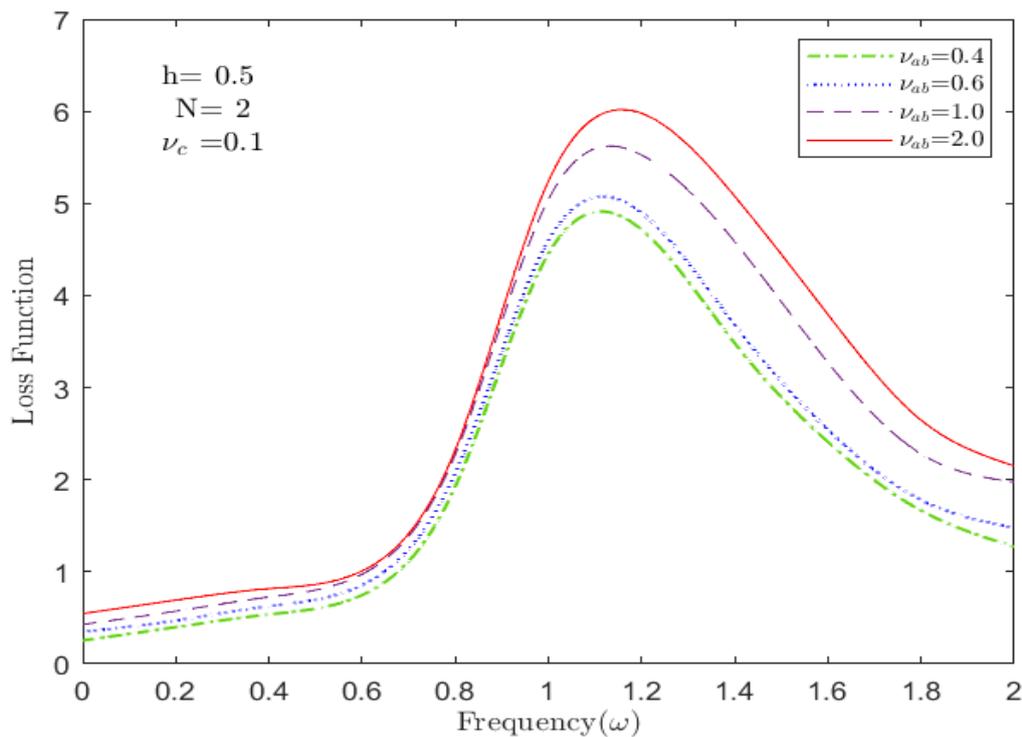
We gets

$$\frac{\xi_c(h, \varpi)}{\xi_c} = 1 + \frac{i \nu_c}{\varpi} - \frac{(1/\varpi^2)}{1 - 2\pi h / [c_e / \varpi^2 + i \nu_c \varpi] + C_{ab}(\nu_{ab} \varpi)} \quad \text{-----(29)}$$

Some recent results²¹ also reveal the same behavior.

RESULT AND DISCUSSION:

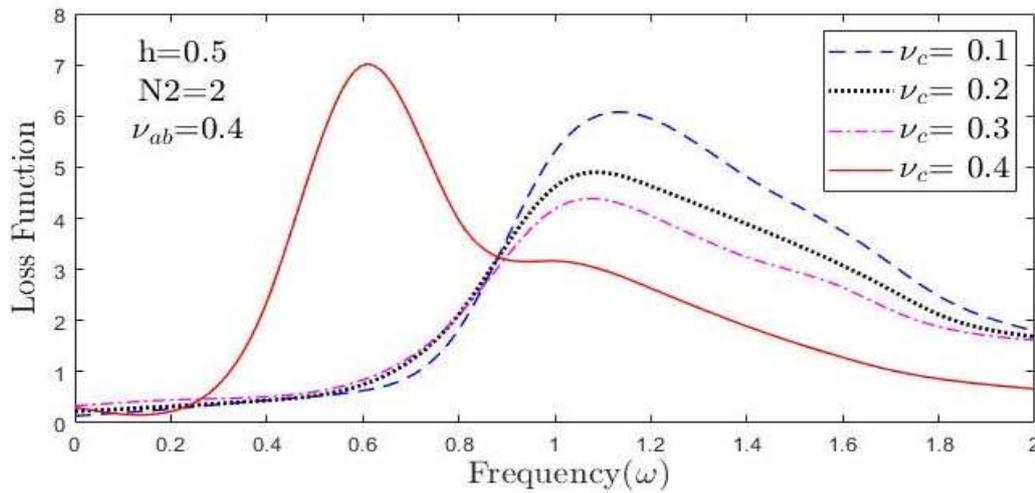
FIGURE: 1



Role of in-plane dissipation has been calculated by evaluating the loss function as a function of frequency for ω for h = 0.5, N₂ = 2 and ν_c = 0.1, N₂ is the No. of vortices.

Loss-function

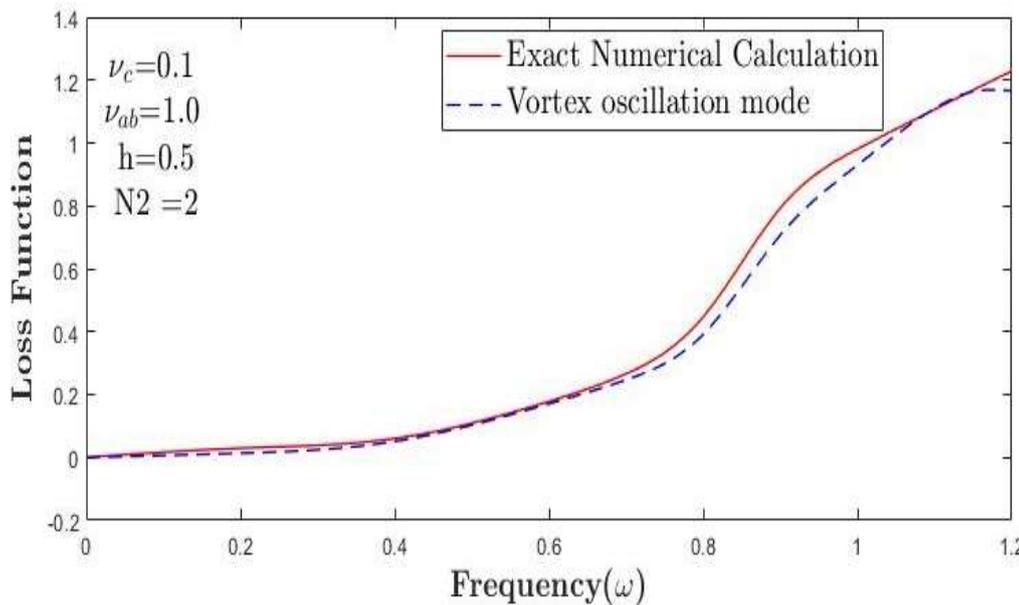
FIGURE: 2



Role of c-axis

dissipation has been calculated by evaluating the loss function as a function of frequency ω keeping $h = 0.5$, $N_2 = 2$, $\nu_{ab} = 0.4$ Loss-function

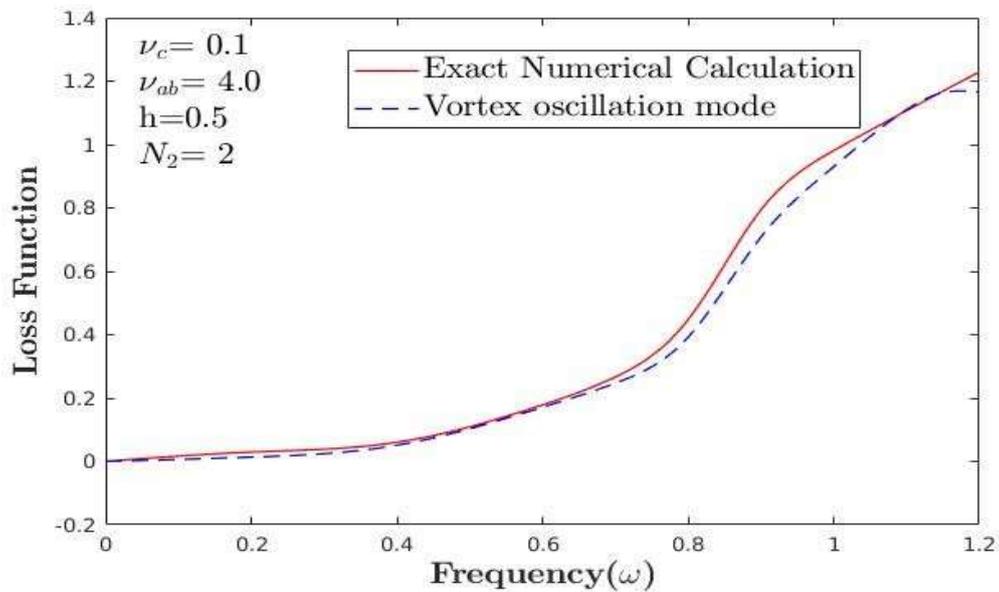
FIGURE: 3



Comparison between Vortex oscillation mode with exact numerical calculation for the loss function as a function of frequency with different values of ν_c , ν_{ab} , h and N_2 Loss-function

$$\nu_c = 0.1, \nu_{ab} = 1.0, h = 0.5 \text{ and } N_2 = 2$$

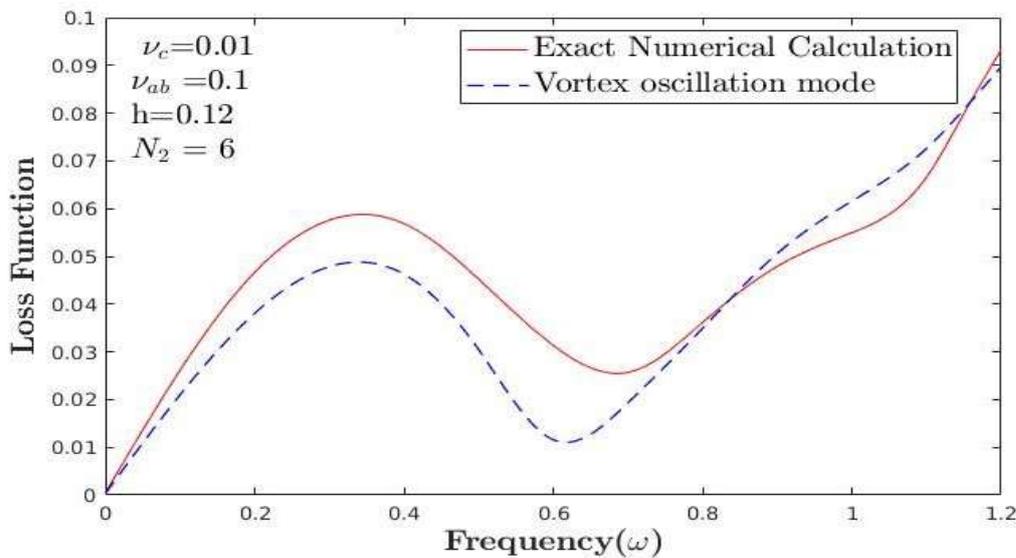
FIGURE: 4



Comparison between Vortex oscillation mode with exact numerical calculation for the loss function as a function of frequency with different values of ν_c, ν_{ab}, h and N_2 Loss-function

$$\nu_c = 0.1, \nu_{ab} = 4.0, h = 0.5 \text{ and } N_2 = 2$$

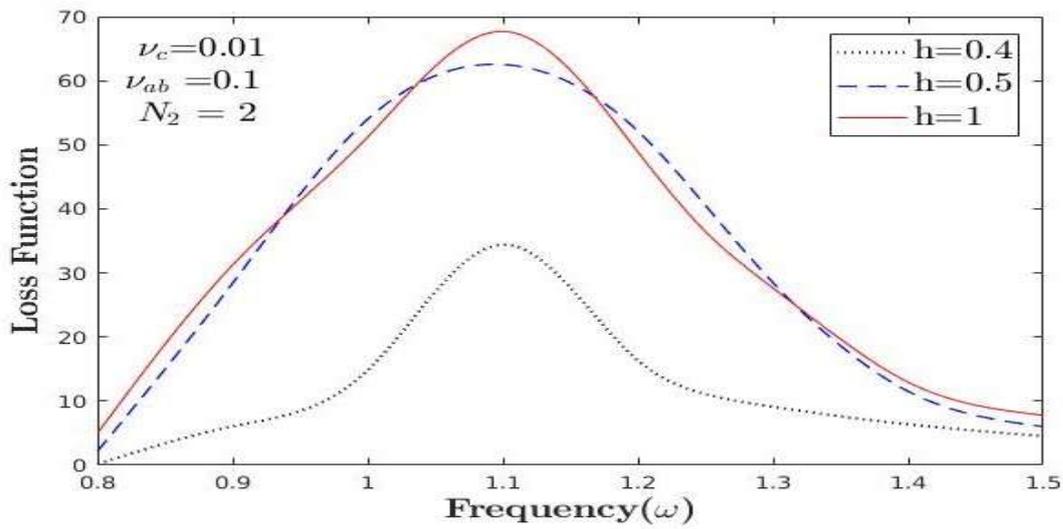
FIGURE: 5



Comparison between Vortex oscillation mode with exact numerical calculation for the loss function as a function of frequency ω with different values of ν_c, ν_{ab}, h and N_2 Loss-function

$$\nu_c = 0.01, \nu_{ab} = 0.1, h = 0.12, N_2 = 6$$

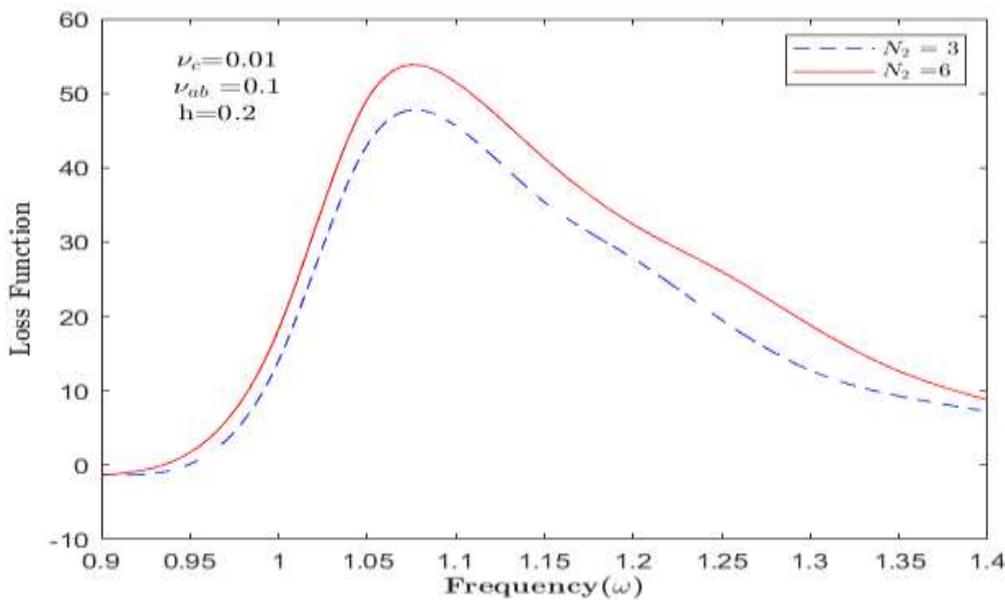
FIGURE :6



We have evaluated the frequency dependents loss function near josephson plasma resonance (JPR) frequency for different values of h and N_2 keeping value of $\nu_c = 0.01$ and $\nu_{ab} = 0.1$ Loss-function

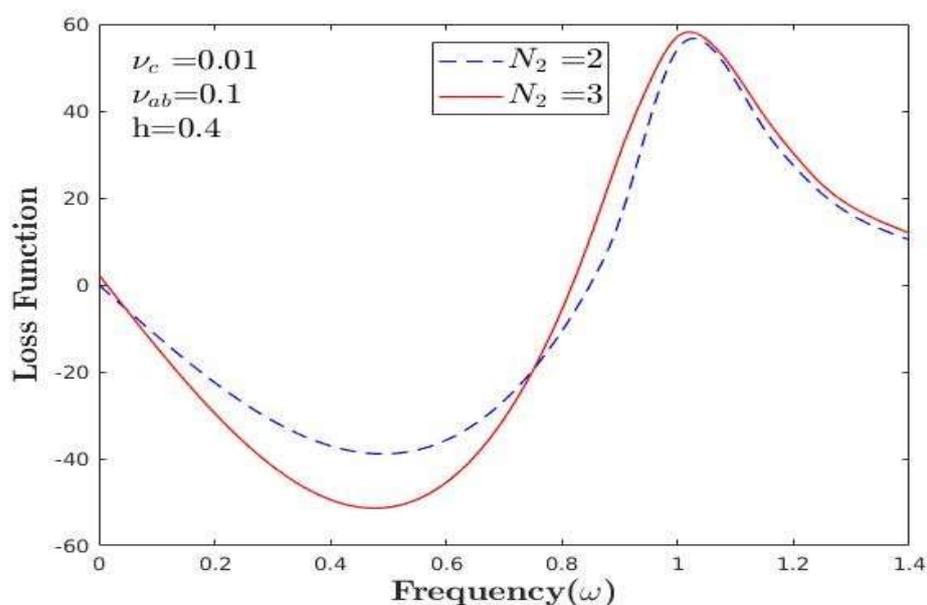
$$\nu_c = 0.01, \text{ and } \nu_{ab} = 0.1$$

FIGURE: 7



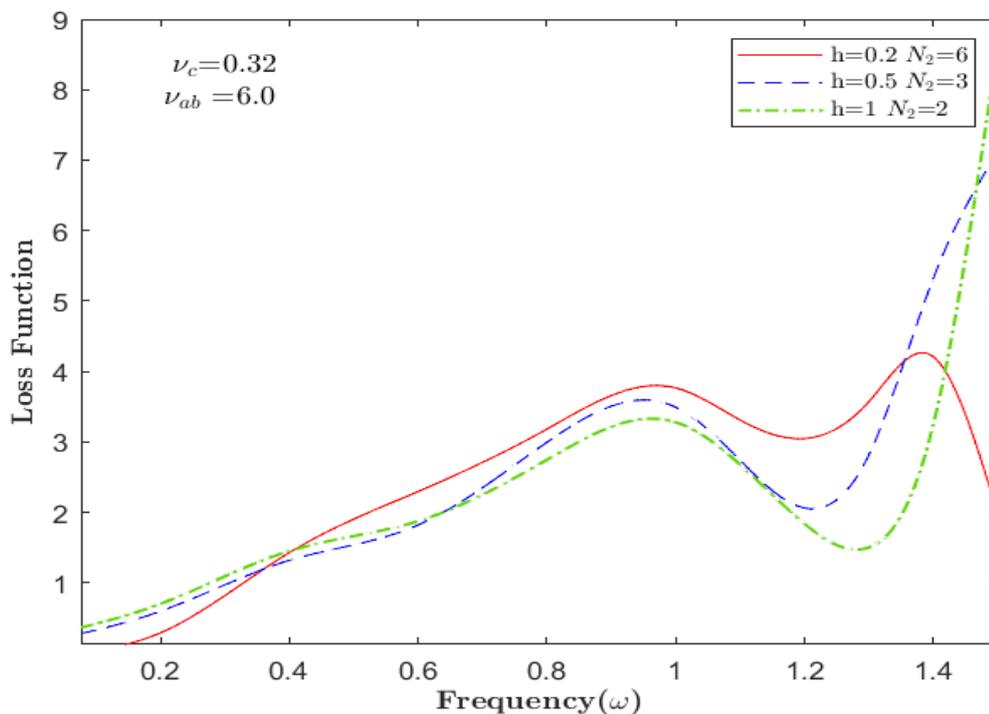
In this figure, we have evaluated loss function as a function of frequency ω with the same h and different N_2 for $\nu_c = 0.01$ and $\nu_{ab} = 0.1$ Loss-function for $h = 0.2$, $\nu_c = 0.01$, and $\nu_{ab} = 0.1$

FIGURE :8



In this figure, we have evaluated loss function as a function of frequency ω for the some $h=0.4$ and two different values of N_2 keeping $\nu_c = 0.01$, and $\nu_{ab} = 0.1$ Loss-function

FIGURE: 9

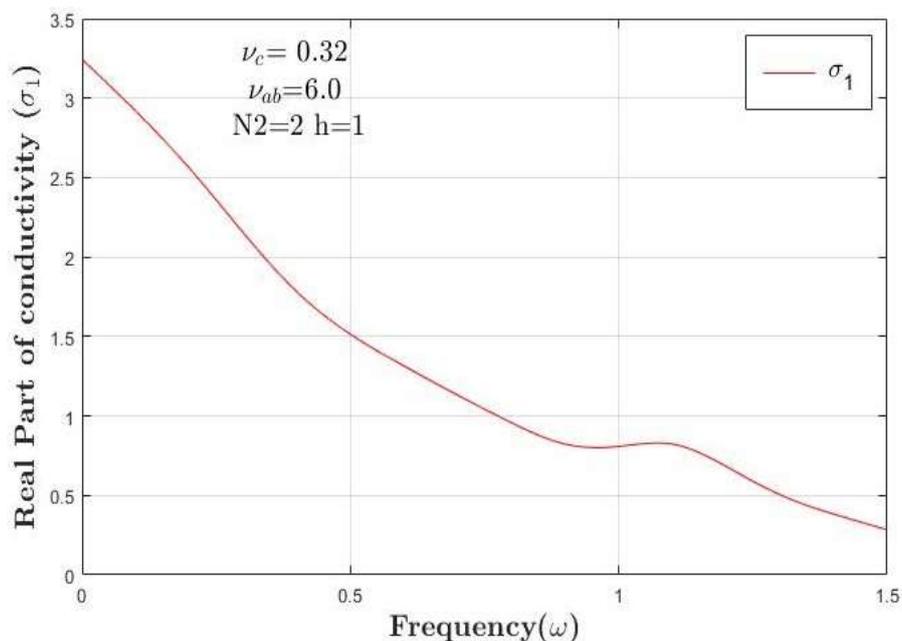


Evaluation of the frequency dependent loss-function at different fields for $\nu_c = 0.32$ and $\nu_{ab} = 6.0$ Loss-function

Table

In this table we have shown the evaluated values of the loss-function as a function of frequency (C_m^{-1}) with increasing in plane field for under doped

Frequency (c_m^{-1})	$T_c = 65\text{K}, T = 8\text{K}$ $E C, H \text{CuO}_2$		
	H=0T	H=2T	H=4 T
0	0.00	0.032	0.042
20	0.027	0.043	0.058
40	0.035	0.056	0.067
60	0.087	0.097	0.092
80	0.095	1.032	1.107
100	1.032	2.057	3.055
110	1.097	1.006	1.010
120	8.072	0.982	0.092
130	0.055	0.073	0.060
140	0.047	0.058	0.065

FIGURE: 10

Evaluated values of the frequency dependent of the loss-function, the real part of conductivity with $\nu_c = 0.32$ $\nu_{ab} = 6.0$, $N_2 = 2$, $h = 1$

CONCLUSION:

In this paper, we have obtained the frequency dependence of the loss-function of different magnetic fields including both of dilute and dense Josephson vortex lattice. We observed that in case of very strong in plane dissipation additional peak in the loss-function appears below the plasma frequency.

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