n-Step Iterative Algorithm for a System of General Variational Inequalities

[1] Monika

[1] Research Scholar
[1] Department of Mathematics,
[1] Maharshi Dayanand University, Rohtak India

Abstract : The purpose of this paper is to introduce n-step iterative algorithm for a system of general variational inequality. The idea is motivated from [10].

***IndexTerms* - Variational Inequality, monotone mapping, iteration, projection mapping.**

I. INTRODUCTION

Let H be a real Hilbert space whose inner product and norm are denoted by (.,.) and ||.||, respectively. Let C be a nonempty closed convex set in H. For given nonlinear operators T_1 , T_2, T_3,……T_n, g: H → H, consider the problem of finding u ∈ H, g(u) ∈ C such that

$\langle T_1u, g(v)-g(u)\rangle \geq 0,$
$\langle T_2u, g(v)-g(u)\rangle \geq 0,$
$\langle T_3u, g(v)-g(u)\rangle \geq 0,$
.
.
.
$\langle T_nu, g(v)-g(u)\rangle \geq 0,$ for all g(v) ∈ C. (I)

It will be called a system of n-general variational inequalities (n-SGVI).

Special Cases:

1. If $T_1 = T_2 = T_3 = \ldots\ldots = T_n$, the SGVI(I) collapses to find u ∈ H, g(u) ∈ C such that

$\langle Tu,g(v)-g(u)\rangle \geq 0$ for all g(v) ∈ C. (II)

2. For g = I, the identity operator, the general variational inequality reduces to find u ∈ C such that

$\langle Tu,v-u\rangle \geq 0$ for all v ∈ C, (III)

which is called the variational inequality (Stampacchia [11]).

3. Let C(u) be a closed convex-valued set in Hilbert space. Consider the problem of finding

u ∈ C(u) such that $\langle Tu,v-u\rangle \geq 0$ for all v ∈ C(u), (IV)

which is called quasi-variational inequality problem (Baiocchi and Capelo [1]).

The purpose of this paper is to develop the n-iterative algorithm to approximate the solution of the n-system of general Variational inequalities (n-SGVI).

Our result has a considerable improvement upon others and generalizes a number of iterative algorithms used earlier by many authors in the field of general variational inequalities (see [3, 4, 5, 7, 8]).

Section 1.1 contains basic definitions and idea about the n-step iteration scheme for the n-system of general variational inequalities and its convergence theorem. Our results generalize the results of Noor [5,8].

1.1. n-step iterative algorithm and its convergence analysis:

Definition 1.1. Let H be a real Hilbert space. An operator T: H →H is said to be:

A. **strongly monotone** if there exists a constant α > 0 such that $\langle Tu - Tv,u-v\rangle \geq \alpha \|u-v\|^2$ for all u, v ∈ H,

B. **Lipschitz continuous** if there exists a constant β > 0 such that $\|Tu-Tv\| \leq \beta \|u-v\|$ for all u, v ∈ H.

Definition 1.2.

(1)**Projection mapping** Let H be a real Hilbert space and C ⊂ H a nonempty closed convex set. If u ∈ H, by projection of u on C we mean the element Pc(u) ∈ C such that

$$\|u-P_C(u)\|_H \leq \|u-v\|_H \text{ for all } v \in C.$$

In other words, we can say that Pc (u) is the element of C closest to u.

Lemma 1.1. (Brezis [2]). Let C be a nonempty closed subset of H. For a given z ∈ H, u ∈ C satisfies the inequality

$\langle u-z,v-u\rangle \geq 0$ for all v ∈C (1.1)

if and only if

$u = P_Cz$ (1.2)

where ρ > 0 is a constant.

This property of the projection operator Pc plays an important role in obtaining our results.

We now prove the following lemma:

Lemma 1.2. The element u ∈ H is a solution of the SGVI(I) if and only u ∈ H satisfies the relation

$g(u) = P_C [g(u)-\rho_1T_1u],$
$g(u) = P_C [g(u)-\rho_2T_2u],$
$g(u) = P_C [g(u)-\rho_3T_3u],$

.
.
.
$g(u) = P_C [g(u)-\rho_n T_n u],$ (1.3)
where $\rho_1, \rho_2, \rho_3, \ldots..\rho_n > 0$ are some constants.
Proof. Let u be the solution of n-SGVI(I). Then for $g(u) \in C$, we have
$<T_1u, g(v)-g(u)> \geq 0,$
$<T_2u, g(v)-g(u)> \geq 0,$
$<T_3u,g(v)- g(u)> \geq 0,$
.
.
.
$<T_nu,g(v)- g(u)> \geq 0,$ for all $g(v) \in C$.
For any $\rho_1, \rho_2, \rho_3, \ldots..\rho_n > 0$, we have
$<g(u) - \{g(u) - \rho_1T_1u\}, g(v)-g(u)> \geq 0,$
$<g(u) - \{g(u) - \rho_2T_2u\}, g(v)-g(u)> \geq 0,$
$<g(u) - \{g(u) - \rho_3T_3u\}, g(v)-g(u)> \geq 0,$
.
.
.
$<g(u) - \{g(u) - \rho_nT_nu\}, g(v)-g(u)> \geq 0,$ for all $g(v) \in C$.
It follows from Lemma 1.1.1 that
$g(u) = P_C [g(u)-\rho_1T_1u],$
$g(u) = P_C [g(u)-\rho_2T_2u],$
$g(u) = P_C [g(u)-\rho_3T_3u],$
.
.
.
$g(u) = P_C [g(u)-\rho_nT_nu].$
Conversely, let $u \in H$ such that (1.1.3) holds, then it follows from Lemma 1. 1. 1 that $g(u) \in C$ and
$<g(u) - \{g(u) - \rho_1T_1u\}, g(v)-g(u)> \geq 0,$
$<g(u) - \{g(u) - \rho_2T_2u\}, g(v)-g(u)> \geq 0,$
$<g(u) - \{g(u) - \rho_3T_3u\}, g(v)-g(u)> \geq 0,$
.
.
.
$<g(u) - \{g(u) - \rho_nT_nu\}, g(v)-g(u)> \geq 0,$ for all $g(v) \in C$.
Thus,
$<T_1u, g(v)-g(u)> \geq 0,$
$<T_2u, g(v)-g(u)> \geq 0,$
$<T_3u,g(v)- g(u)> \geq 0,$
.
.
.
$<T_nu,g(v)- g(u)> \geq 0,$
and so u is a solution of n-SGVI(I).

II. Main Result

Based on n-SGVI(I) and equation (1.3), we are now in a position to propose the following general and unified new n-step iteration scheme for solving n-SGVI(I).

Algorithm 1.1. For a given $u_{1,0} \in H$, compute the approximate solution $\{u_{1,n}\}$ by the iterative scheme:

$$u_{1,n+1} = (1-\alpha_{1,n})u_{1,n} + \alpha_{1,n}\{u_{2,n}- g(u_{2,n}) + P_C[g(u_{2,n}) - \rho_1T_1u_{2,n}]\}, \quad (1.4)$$
$$u_{2,n} = (1-\alpha_{2,n})u_{1,n} + \alpha_{2,n}\{u_{3,n}- g(u_{3,n}) + P_C[g(u_{3,n}) - \rho_2T_2u_{3,n}]\}, \quad (1.5)$$
$$u_{3,n} = (1-\alpha_{3,n})u_{1,n} + \alpha_{3,n}\{u_{4,n}- g(u_{4,n}) + P_C[g(u_{4,n}) - \rho_3T_3u_{4,n}]\}, \quad (1.6)$$
.
.
.
$$u_{n,n} = (1-\alpha_{n,n})u_{1,n} + \alpha_{n,n}\{u_{1,n}- g(u_{1,n}) + P_C[g(u_{1,n}) - \rho_nT_nu_{1,n}]\}, \quad (1.7)$$

where $0 \leq \alpha_{1,n}, \alpha_{2,n}, \alpha_{3,n}, \ldots, \alpha_{n,n} \leq 1$ for all $n \geq 0$ and $\sum_{n=0}^{\infty} \alpha_{1,n}$ an diverges.

Special Cases:
For g =I, the identity operator, Algorithm 1.1.1 collapses to the following algorithm for a system of variational inequality, which appear to be a new one.

Algorithm 1.2. For a given $u_{1,0} \in C$, compute $\{u_{1,n}\}$ by the iterative scheme:
$u_{1,n+1} = (1-\alpha_{1,n})u_{1,n} + \alpha_{1,n}P_C[u_{2,n} - \rho_1T_1u_{2,n}],$
$u_{2,n} = (1-\alpha_{2,n})u_{1,n} + \alpha_{2,n}P_C[u_{3,n}- \rho_2T_2u_{3,n}],$
$u_{3,n} = (1-\alpha_{3,n})u_{1,n} + \alpha_{3,n}P_C[u_{4,n} - \rho_3T_3u_{4,n}],$

.
.
.
$u_{n,n} = (1-\alpha_{n,n})u_{1,n} + \alpha_{n,n}P_C[u_{1,n} - \rho_n T_n u_{1,n}]$,
where $0 \le \alpha_{1,n}, \alpha_{2,n}, \alpha_{3,n}, \ldots, \alpha_{n,n} \le 1$ for all $n \ge 0$ and $\sum_{n=0}^{\infty} \alpha_{1,n}$ an diverges.
If $T_1 = T_2 = T_3 = \ldots = T_n = T$, $\rho_1 = \rho_2 = \rho_3 = \ldots = \rho_n = \rho$, then Algorithm 1.1.2 reduces to:

Algorithm 1.3. For a given $u_{1,0} \in H$, compute the approximate solution $\{u_{1,n}\}$ by the iterative scheme:

$$u_{1,n+1} = (1-\alpha_{1,n})u_{1,n} + \alpha_{1,n}\{u_{2,n} - g(u_{2,n}) + P_C[g(u_{2,n}) - \rho T u_{2,n}]\}, \quad (1.8)$$
$$u_{2,n} = (1-\alpha_{2,n})u_{1,n} + \alpha_{2,n}\{u_{3,n} - g(u_{3,n}) + P_C[g(u_{3,n}) - \rho T u_{3,n}]\}, \quad (1.9)$$
$$u_{3,n} = (1-\alpha_{3,n})u_{1,n} + \alpha_{3,n}\{u_{4,n} - g(u_{4,n}) + P_C[g(u_{4,n}) - \rho T u_{4,n}]\}, \quad (1.10)$$
.
.
.
$$u_{n,n} = (1-\alpha_{n,n})u_{1,n} + \alpha_{n,n}\{u_{1,n} - g(u_{1,n}) + P_C[g(u_{1,n}) - \rho T u_{1,n}]\}, \quad (1.11)$$
$n = 0,1,2,3\ldots\ldots$,

which is known as near to n-generalized Ishikawa iteration process of rank n. Algorithm 1.3 was also suggested by Noor[72] for n=3 to approximate the solution of the general variational inequalities.
For $\alpha_{3,n}, \ldots, \alpha_{n,n} = 0$, Algorithm 1.3 reduces to:

Algorithm 1.4. For a given $u_{1,o} \in H$, compute the approximate solution $\{u_{n,n}\}$ by the iterative scheme:

$$u_{1,n+1} = (1-\alpha_{1,n})u_{1,n} + \alpha_{1,n}\{u_{2,n} - g(u_{2,n}) + P_C[g(u_{2,n}) - \rho T u_{2,n}]\}, \quad (1.12)$$
$$u_{2,n} = (1-\alpha_{2,n})u_{1,n} + \alpha_{2,n}\{u_{1,n} - g(u_{1,n}) + P_C[g(u_{1,n}) - \rho T u_{1,n}]\}, \quad (1.13)$$

which is known as the Ishikawa iterative scheme [45] for the general variational inequality.
If $\alpha_{1,n}, \alpha_{2,n}, \alpha_{3,n}, \ldots, \alpha_{n,n} = 0$ in Algorithm 1.3, we get the Mann iterative scheme [60] as below:

Algorithm 1.5. For a given $u_{1,o} \in H$, compute the approximate solution $\{u_{n,n}\}$ by the iterative scheme:

$$u_{1,n+1} = (1-\alpha_{1,n})u_{1,n} + \alpha_{1,n}\{u_{1,n} - g(u_{1,n}) + P_C[g(u_{1,n}) - \rho_1 T_1 u_{1,n}]\}. \quad (1.14)$$

We now study the convergence criteria of Algorithm 1.1.

Theorem 1.1. Let the operators $T_1, T_2, T_3, \ldots, T_n$, $g: H \to H$ be strongly monotone with constants $\sigma_1, \sigma_2, \sigma_3, \ldots, \sigma_n, \sigma_{n+1}$ and Lipschitz continuous with constants $\delta_1, \delta_2, \delta_3, \ldots, \delta_n, \delta_{n+1}$ respectively and $u \in H$ be the solution of n-SGVI(I) and the following conditions hold:

$$\left|\rho_i - \frac{\sigma_i}{\delta^2_i}\right| < \frac{\sqrt{\sigma^2_i - \delta^2_i k(2-k)}}{\delta^2_i}, \ \sigma_i > \delta_i \sqrt{k(2-k)}, \ k < 1 \quad (1.15)$$

where $\theta_i = k + (1 - 2\rho_i\sigma_i + \rho^2_i\delta^2_i)^{1/2}$, $k = 2\sqrt{1 - 2\sigma_{n+1} + \delta_4^2}$ and $i = 1,2,3,\ldots n+1$,

then the approximate solution $\{u_{n,n}\}$ obtained from Algorithm 1.1 converges strongly to the exact solution u in H of the n-SGVI(I).

Proof. Let $u \in H$ be the solution of n-SGVI(I). Then, using Lemma 1.2, we have

$$u = (1-\alpha_{1,n})u + \alpha_{1,n}\{u - g(u) + P_C[g(u) - \rho_1 T_1 u]\} \quad (1.16)$$
$$= (1-\alpha_{2,n})u + \alpha_{2,n}\{u - g(u) + P_C[g(u) - \rho_2 T_2 u]\} \quad (1.17)$$
$$= (1-\alpha_{3,n})u + \alpha_{3,n}\{u - g(u) + P_C[g(u) - \rho_3 T_3 u]\} \quad (1.18)$$
.
.
.
$$= (1-\alpha_{n,n})u + \alpha_{n,n}\{u - g(u) + P_C[g(u) - \rho_n T_n u]\} \quad (1.19)$$

From (1.4) and (1.16), we have

$$\|u_{1,n+1} - u\| = \|(1-\alpha_{1,n})(u_{1,n} - u) + \alpha_{1,n}(u_{2,n} - u - (g(u_{2,n}) - g(u)) + \alpha_{1,n}\{P_C(g(u_{2,n}) - \rho_1 T_1 u_{2,n}) - P_C(g(u) - \rho_1 T_1 u)\}\|$$
$$\le (1-\alpha_{1,n})\|u_{1,n} - u\| + \alpha_{1,n}\|u_{2,n} - u - (g(u_{2,n}) - g(u)\| + \alpha_{1,n}\|g(u_{2,n}) - g(u) - \rho_1(T_1u_{2,n} - T_1u)\|$$
$$\le (1-\alpha_{1,n})\|u_{1,n} - u\| + \alpha_{1,n}\|u_{2,n} - u - (g(u_{2,n}) - g(u)\| + \alpha_{1,n}\|u_{2,n} - u - g(u_{2,n}) - g(u)\| + \alpha_{1,n}\|u_{2,n} - u - \rho_1(T_1u_{2,n} - T_1u)\|$$
$$\le (1-\alpha_{1,n})\|u_{1,n} - u\| + 2\alpha_{1,n}\|u_{2,n} - u - (g(u_{2,n}) - g(u)\| + \alpha_{1,n}\|u_{2,n} - u - \rho_1(T_1u_{2,n} - T_1u)\|$$

Since

$$\|u_{2,n} - u - \rho_1(T_1u_{2,n} - T_1u)\|^2 = \|u_{2,n} - u\|^2 - 2\rho_1 \langle T_1u_{2,n} - T_1u, u_{2,n} - u\rangle + \rho^2_1\|T_1u_{2,n} - T_1u\|^2$$
$$\le \|u_{2,n} - u\|^2 - 2\rho_1\sigma_1\|u_{2,n} - u\|^2 + \rho_1^2\delta_1^2\|u_{2,n} - u\|^2$$

which follows that

$$\|u_{2,n} - u - \rho_1(T_1u_{2,n} - T_1u)\|^2 \le (1 - 2\rho_1\sigma_1 + \rho_1^2\delta_1^2)^{1/2}\|u_{2,n} - u\|.$$

Again

$$\|u_{2,n} - u - (g(u_{2,n}) - g(u))\|^2 = \|u_{2,n} - u\|^2 - 2\langle g(u_{2,n}) - g(u), u_{2,n} - u\rangle + \|g(u_{2,n}) - g(u)\|^2$$
$$\le \|u_{2,n} - u\|^2 - 2\sigma_{n+1}\|u_{2,n} - u\|^2 + \delta^2_{n+1}\|u_{2,n} - u\|^2$$
$$\le (1 - 2\sigma_{n+1} + \delta^2_{n+1})\|u_{2,n} - u\|^2,$$

it follows that

$$\|u_{2,n} - u - (g(u_{2,n}) - g(u))\|^2 = (1 - 2\sigma_{n+1} + \delta^2_{n+1})^{1/2}\|u_{2,n} - u\|.$$

Thus,

$$\|u_{1,n+1} - u\| \le (1-\alpha_{1,n})\|u_{1,n} - u\| + \alpha_{1,n}(2(1 - 2\sigma_{n+1} + \delta^2_{n+1})^{1/2} + (1 - 2\rho_1\sigma_1 + \rho_1^2\delta_1^2)^{1/2})\|u_{2,n} - u\|$$
$$\le (1-\alpha_{1,n})\|u_{1,n} - u\| + \alpha_{1,n}\theta_1\|u_{2,n} - u\|, \quad (1.20)$$

where $\theta_1 = 2(1 - 2\sigma_{n+1} + \delta^2_{n+1})^{1/2} + (1 - 2\rho_1\sigma_1 + \rho_1^2\delta_1^2)^{1/2} = k + (1 - 2\rho_1\sigma_1 + \rho_1^2\delta_1^2)^{1/2}$.
In a similar way from (1.5) and (1.17), we have

$$\|u_{2,n} - u\| \le (1-\alpha_{2,n})\|u_{1,n} - u\| + \alpha_{2,n}(2(1 - 2\sigma_{n+1} + \delta^2_{n+1})^{1/2} + (1 - 2\rho_2\sigma_2 + \rho_2^2\delta_2^2)^{1/2})\|u_{3,n} - u\|$$

$\leq (1-\alpha_{2,n})\,||u_{1,n}-u|| + \alpha_{2,n}\,\theta_2||u_{3,n}-u||.$ (1.21)

From (1.6) and (1.18), we have

$||u_{3,n}-u|| \leq (1-\alpha_{3,n})||u_{1,n}-u|| + \alpha_{3,n}(2(1-2\sigma_{n+1}+\delta^2_{n+1})^{1/2} + (1-2\rho_3\sigma_3+\rho_3{}^2\delta_3{}^2)^{1/2})\,||u_{4,n}-u||$

$\leq (1-\alpha_{3,n})\,||u_{1,n}-u|| + \alpha_{3,n}\,\theta_2||u_{4,n}-u||.$ (1.22)

When we take 2nd last step of the iteration, we get

$||u_{n-1,n}-u|| \leq (1-\alpha_{n-1,n})||u_{1,n}-u|| + \alpha_{n-1,n}(2(1-2\sigma_{n+1}+\delta^2_{n+1})^{1/2} + (1-2\rho_{n-1}\sigma_{n-1}+\rho_{n-1}{}^2\delta_{n-1}{}^2)^{1/2})\ ||u_{n,n}-u||$

$\leq (1-\alpha_{n-1,n})\,||u_{1,n}-u|| + \alpha_{n-1,n}\,\theta_{n-1}||u_{n,n}-u||.$ (1.23)

Similarly, from equation (1.7) and (1.19) {which are the nth step of the iteration}, we have

$||u_{n,n}-u|| \leq (1-\alpha_{n,n})||u_{1,n}-u|| + \alpha_{n,n}(2(1-2\sigma_{n+1}+\delta^2_{n+1})^{1/2} + (1-2\rho_n\sigma_n+\rho_n{}^2\delta_n{}^2)^{1/2})\,||u_{1,n}-u||$

$\leq (1-\alpha_{n,n})\,||u_{1,n}-u|| + \alpha_{n,n}\,\theta_n||u_{1,n}-u||.$ (1.24)

Then equation (1.23) gives

$||u_{n,n}-u|| \leq (1-\alpha_{n,n}+\alpha_{n,n}\,\theta_n||u_{1,n}-u||.$

$\leq (1-(1-\theta_n)\,\alpha_{n,n})\,||u_{1,n}-u||$

$\leq ||u_{1,n}-u||.$ (1.25)

From (1.23) and (1.24), we get

$||u_{n-1,n}-u|| \leq (1-\alpha_{n-1,n})||u_{1,n}-u|| + \alpha_{n-1,n}\theta_{n-1}||u_{1,n}-u||$

$\leq (1-(1-\theta_{n-1})\,\alpha_{n-1,n})\,||u_{1,n}-u||$

$\leq ||u_{1,n}-u||.$ (1.26)

Similarly, from (1.21) and (1.26) we have

$||u_{2,n}-u|| \leq (1-\alpha_{2,n})\,||u_{1,n}-u|| + \alpha_{2,n}\,\theta_2||u_{3,n}-u||.$

$\leq (1-\alpha_{2,n}+\alpha_{2,n}\,\theta_2)\,||u_{3,n}-u||.$

$\leq ||u_{1,n}-u||.$ (1.27)

From (1.20) and (1.27), we get

$||u_{1,n+1}-u|| \leq (1-\alpha_{1,n})\,||u_{1,n}-u|| + \alpha_{1,n}\,\theta_1||u_{2,n}-u||,$

$\leq (1-\alpha_{1,n}+\alpha_{1,n}\,\theta_1)||u_{1,n}-u||,$

$\leq (\,1-(1-\theta_1)\,\alpha_{1,n})\,||u_{1,n}-u||$

$||u_{1,n+1}-u|| \leq \prod_{j=0}^{n}\{\,1-(1-\theta_i)\alpha_j\}\ ||\,u_{1,0}-u||.$

Since $\sum_{n=0}^{\infty}\ \alpha_n$ diverges and $1-\theta_i > 0$, we have $\prod_{j=0}^{n}\{\,1-(1-\theta_i)\alpha_j\} = 0$. Consnequently, the sequence $\{u_{n,n}\}$converges strongly to u. From (1.25) , (1.26) and (1.27), it follows that the sequence $\{u_{2,n}\}$, $\{u_{3,n}\}$….and $\{u_{n,n}\}$ also converges strongly to u in H. This completes the proof.

As an immediate consequence of Theorem 1.1 is the following:

Corollary 2.1. [Theorem 3.2, Noor [72]] Let the operators T, g:H →H be strongly monotone with constants $\alpha > 0$, $\sigma > 0$ and Lipschitz continuous with constants $\beta > 0$, $\delta > 0$, respectively.

For a given $u_0 \in H$, compute the approximate solution $\{u_n\}$ by the iterative scheme:

$u_{n+1} = (1-\alpha_n)u_n + \alpha_n\{w_n - g(w_n) + P_C[g(w_n) - \rho Tw_n]\},$ (A1)

$w_n = (1-\beta_n)u_n + \beta_n\{y_n - g(y_n) + P_C[g(y_n) - \rho Ty_n]\},$ (A2)

$y_n = (1-\gamma_n)u_n + \gamma_n\{u_n - g(u_n) + P_C[g(u_n) - \rho Tu_n]\},$ (A3)

where $0 \leq \alpha_n, \beta_n, \gamma_n \leq 1$ for all $n \geq 0$ and $\sum_{n=0}^{\infty}\ \alpha_n$ diverges. If the following conditions hold:

$\left|\rho - \frac{\alpha}{\beta^2}\right| < \frac{\sqrt{\alpha^2-\beta^2 k(2-k)}}{\beta^2}$, $\alpha > \beta\sqrt{k(2-k)}$, $k < 1$, where $k=2\sqrt{1-2\sigma+\delta^2}$ then approximate solution $\{u_n\}$ defined by (Al), (A2), (A3) converges strongly to the exact solution u in H of the general variational inequality problem.

III. Acknowledgment

There is no funding agency..

References

[1] Baiocchi, C. and Capelo, A. 1984, Variational and quasivariational Inequalities J.Wiley and sons, New York.

[2] Brezis, H. 1973, Operateurs max1maux Monotone et semigroups de contractions dans les Espaces de Hilbert, North Holland, Amsterdam.

[3] Ishikawa, S., 1974. Fixed points by a new iteration method, Proc. Amer. Math. Soc., 44 (1974), 147-150.

[4] Mann, W.R., 1952, Mean value methods in iteration, Proc. Amer. Math. Soc., 4 (1952), 506-510.

[5] Noor, M. A., 2000, New approximation schemes for general variational inequalities, J. Math. Anal. Appl., 251 (2000), 217-229.

[6] Noor, M.A., 1988, General variational inequalities, Appl. Math. Lett., 1(1988), 119-121.

[7] Noor, M.A.,2000, Projection-splitting algorithm for monotone variational inequalities, J. Com put. Math. Appl. 39 (2000), 73-79.

[8] Noor, M.A., 2002, Two-step approximation schemes for multivalued quasivariational inclusions, Nonlinear Funct. Anal. Appl. 7 (1)(2002), 1-14.

[9] Sahu, D.R., 2003, On generalized Ishikawa iteration process and nonexpansive mapping in Banach Spaces, Demonstrative Mathematics, Vol., XXXVI (3).

[10] Sahu, D.R and Salhotra, R. 2007, Three step iterative algorithm for the system of general variational inequalities, Indian Journal of Mathematics and Mathematical Sciences. 3(2007).

[11] Stampacchia, G. 1964, Formes bilinearies coercitives sur les ensembles convexes, C.R. Acad. Sci. Paris Ser I. Math., 258(1964), 4413-4416.