# Domination in Graph and Some of its applications in various fields

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***Abstract :*** This paper focuses on the domination in graph with some of its application in various fields . Let G = (V,E) be a simple graph. A set of vertices S in a graph G is said to be a dominating set if every vertex $v \in V(G)$ is either an element of S or is adjacent to an element of S. A minimal dominating set in a graph G is a dominating set that contains no dominating set as a proper subset. The cardinality of minimum dominating set is called domination number and it is denoted by $\gamma(G)$ .Many different types of domination have been researched extensively this paper explores applications of dominating sets.

## I Introduction

Domination in graphs has been an extensively researched branch of graph theory. Graph theory is one of the most flourishing branches of modern mathematics and computer applications. Graph Theory is a branch of Mathematics which has become quite rich and interesting for several reasons. In last three decades hundreds of research article have been published in Graph Theory. There are several areas of Graph Theory which have received good attention from mathematicians. Some of these areas are Coloring of Graphs, Matching Theory, Domination Theory, Labeling of Graphs and areas related to Algebraic Graph Theory. It has a very wide range of applications to many fields like engineering, physical, social and biological sciences; linguistics etc., the theory of domination has been the nucleus of research activity in graph theory in recent times. Euler (1707-1782) became the father of Graph Theory as well as Topology when in 1736 he settled a famous unsolved problem of his day called the Konigsberg Bridge Problem.

It took 200 years before the first book on graph theory was written. This was “Theorie der endlichen und unendlichen Graphen” ( Teubner, Leipzig, 1936) by Konig [6] in 1936. Since then graph theory has developed into an extensive and popular branch of mathematics. The area of graph theory has experienced fast developments during the last 60 years. Among the huge diversity of concepts that appear while studying this subject one that has gained a lot of popularity is the concept of **domination**. **Domination** in graphs has been an extensively researched branch of graph theory.

The concept of domination in graph introduced by **Claude Berge**, in 1958. In a graph, a dominating set is a subset *S* of the vertices such that every vertex is either in *S* or adjacent to a vertex in *S*. **Historically**, the first domination-type problems came from chess. In the 1850s, several chess players were interested in the minimum number of queens such that every square on the chess board either contains a queen or is attacked by a queen (recall that a queen can move any number of squares horizontally, vertically, or diagonally on the chess board).

**Queens Problem:**

This problem was mentioned by **Ore** in [7]. According to the rules of chess a queen can move any number of squares horizontally, vertically, or diagonally on the chess board (assuming that no other chess figure is on its way). How to place a minimum number of queens on a chessboard so that each square is controlled by at least one queen? See one of the solutions in Figure 1.3.

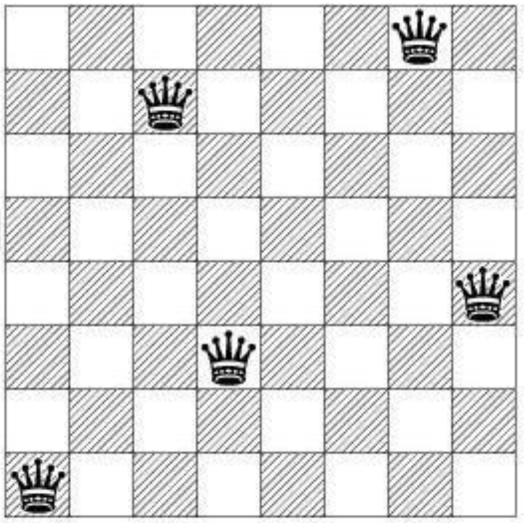


**Figure 1.3:- Queens dominating the chessboard**

Using graph theory to model this problem, the Queens graph is formed by representing each of the 64 (8X8) squares of the chessboard as a vertex of a graph G. Two vertices (squares) are adjacent in G if each square can be reached by a queen on the other square in a single move. Obviously, to solve the queens problem we are looking for the minimum number of queens that dominate all the squares of the chessboard that is domination number. (Note that many variations on this problem are formed by considering different chess pieces and/or different size chessboards).

Apart from chess, domination in graphs has applications to several other fields. Domination arises in facility location problems, where the number of facilities (e.g. hospitals, fire stations) is fixed and one attempts to minimize the distance that a person needs to travel to get to the closest facility. A similar problem occurs when the maximum distance to a facility is fixed and one attempts to minimize the number of facilities necessary so that everyone is serviced. Concepts from domination also appear in problems involving finding sets of representatives, in monitoring communication or electrical networks, and in land surveying.

The study of domination in graphs was further developed in the late 1950's and 1960's. **Claude Berge** [1] wrote a book on graph theory where he introduced the coefficient of external stability, which is now known as the domination number of a graph. In 1962**, Oystein Ore** [7] introduced the terms, dominating set and domination number in his book on graph theory. In 1969, Harary [5] used the notation $\alpha_\infty$(G) for absorption number of a graph G. In the late decades, Cockayne and Hedetniemi [2] published a survey paper, in which the notation ϒ(G) was used in place of $\alpha_\infty$(G) for representing the domination number of a graph G. Vizing [8 ,3] conjectured in 1963 that the domination number of the Cartesian product of two graphs is atleast the product of their domination number; this remains one of the biggest open problems in the study of domination in graphs. Several partial results has been proved, but the conjecture has yet to be proved in general.

In 1962, Ore [7] was the first to use the term **"domination"** for undirected graphs and he denoted the domination number by δ(G) and also he introduced the concepts of minimal and minimum dominating sets of vertices in a graph. In 1977, Cockayne and Hedetniemi [2] was introduced the accepted notation ϒ(G) to denote the domination number. In 1990, Hedetniemi and Laskar [4] had a survey of domination articles containing about 400 entries. This bibliography has grown to cover 1200 entries at the end of 1997. This clearly shows the growth of domination.

## II Applications of Domination in graph in various fields

Domination in graphs has applications to several fields. Domination arises in facility location problems, where the number of facilities (e.g., hospitals, fire stations) is fixed and one attempts to minimize the distance that a person needs to travel to get to the closest facility. A similar problem occurs when the maximum distance to a fality is fixed and one attempts to minimize the number of facilities necessary so that everyone is serviced. Concepts from domination also appear in problems involving finding sets of representatives, in monitoring communication or electrical networks, and in land surveying (e.g., minimizing the number of places a surveyor must stand in order to take height measurements for an entire region).

### *2.1.1 Locating Radar Stations Problem*

The problem was discussed by Berge . A number of strategic locations are to be kept under surveillance. The goal is to locate a radar for the surveillance at as few of these locations as possible. How a set of locations in which the radar stations are to be placed can be determined.

### *2.2.2 Facility Location Problems*

The dominating sets in graphs are natural models for facility location problems in operational research. Facility location problems are concerned with the location of one or more facilities in a way that optimizes a certain objective such as minimizing transportation cost, providing equitable service to customers and capturing the largest market share.

### *2.2.3 Modeling Social Networks*

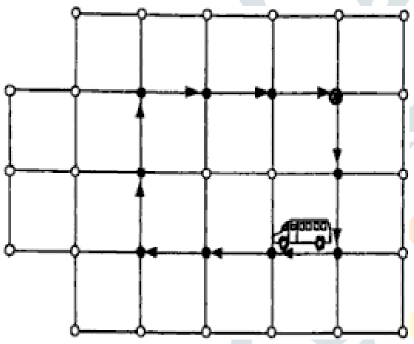
Dominating sets can be used in modeling social networks and studying the dynamics of relations among numerous individuals in different domains. A social network is a social structure made of individuals (or groups of individuals), which are connected by one or more specific types of interdependency. The choice of initial sets of target individuals is an important problem in the theory of social networks. In the work of Kelleher and Cozzens, social networks are modeled in terms of graph theory and it was shown that some of these sets can be found by using the properties of dominating sets in graphs.

### *2.2.4 Multiple Domination Problems*

An important role is played by multiple domination. Multiple domination can be used to construct hierarchical overlay networks in peer-to-peer applications for more efficient index searching. The hierarchical overlay networks usually serve as distributed databases for index searching, e.g. in modern file sharing and instant messaging computer network applications. Dominating sets of several kinds are used for balancing efficiency and fault tolerance as well as in the distributed construction of minimum spanning trees. Another good example of direct, important and quickly developing applications of multiple domination in modern computer networks is a wire- less sensor network.

### *2.2.5 School Bus Routing:*

Most school in the country provide school buses for transporting children to and from school Most also operate under certain rules, one of which usually states that no child shall have to walk farther than, say one quarter km to a bus pickup point. Thus, they must construct a route for each bus that gets with- in one quarter km of every child in its assigned area. No bus ride can take more than some specified number of minutes, and Limits on the number of children that a bus can carry at any one time. Let us say that the following figure represents a street map of part of a city, where each edge represents one pick up block. The school is located at the large vertex. Let us assume that the school has decided that no child shall have to walk more than two blocks in order to be picked up by a school bus. Construct a route for a school bus that leaves the school, gets within two blocks of every child and returns to the school.



### *2.2.6 Coding Theory*

The concept of domination is also applied in coding theory as discussed by Kalbfleisch, Stanton and Horton and Cockayne and Hedetniemi . If one defines a graph, the vertices of which are the n-dimensional vectors with coordinates chosen from{1,..., p}, p > 1, and two vertices are adjacent if they differ in one coordinate, then the sets of vectors which are (n, p)- covering sets, single error correcting codes, or perfect cover- ing sets are all dominating sets of the graph with determined additional properties.

## III. CONCLUSION

The main aim of this paper is to present the importance of graph theoretical ideas in various areas of Science & Engineering for researches that they can use Domination in graph theoretical concepts for the research

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