G*α-CLOSED SETS AND G*α- LOCALLY CLOSED SETS IN BITOPOLOGICAL SPACES

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Abstract : In this paper we introduce and study g*α-closed sets in topological and Bitopological spaces. Also we discuss some of their properties and investigate the relations between other closed sets in Bitopological Spaces.

I. INTRODUCTION

In the year 1963, Kelly initiated a systematic study of the concept of bitopological Spaces where X is the nonempty set and τ_i and τ_j are two topologies on X. In 1985, Fukutake introduced and studied the notions of generalized closed(g-closed) sets in bitopological spaces and after that several authors turned their attention towards generalization of various concepts of topology by considering bitopological spaces. We have introduced g*α-closed and g*α- locally Closed sets in Bitopological Spaces.

II. PRELIMINARIES:

Before entering into our work we recall the following definitions..

Definition 2.1:

1. a generalized closed set **[7]**(briefly g-closed set) if cl(A) ⊆ U whenever A ⊆ U and U is open in(X, τ).
2. a generalized star closed set**[11]** (briefly g*-closed set) if cl(A) ⊆ U whenever A ⊆ U and U is g-open in (X,τ)
3. g*α-closed if αcl(A) ⊆ U whenever A ⊆ U and U is g*-open in (X,τ) .

Definition 2.2 A subset A of a topology (X, τ_i, τ_j) is called

1. (i, j)-g-closed**[2]** if τ_j-cl(A) ⊆ U whenever A ⊆ U and U is open in τ_i.
2.(i,j)g*-closed**[1]**if τ_j-cl(A) ⊆ U whenever A ⊆ U and U is g-open in τ_i.

III.g*α-CLOSED SETS IN BITOPOLOGICAL SPACES

Definition 3.1:A subset A of a bitopological space (X,τ_1,τ_2) is said to be $\tau_1\tau_2$ -g*α closed if τ_2 - αcl(A) ⊆ U whenever A ⊆ U, U ∈ G*O(X,τ_1). The collection of all $\tau_1\tau_2$ -g*α closed sets of (X,τ_1,τ_2) are denoted by G*α(τ_1,τ_2).

Remark 3.2 By setting $\tau_1 = \tau_2$ in Definition 3.1, a $\tau_1\tau_2$ - g*α closed set is a g*α closed set.

Definition 3.3 A subset A of a bitopological space (X,τ_1,τ_2) is said to be $\tau_1\tau_2$ -g*α closed if τ_2 - αcl(A) ⊆ U whenever A ⊆ U, U ∈ G*O(X,τ_1). The collection of all $\tau_1\tau_2$ -g*α closed sets of (X,τ_1,τ_2) are denoted by G*α(τ_1,τ_2).

Remark 3.4 By setting $\tau_1 = \tau_2$ in Definition 3.1, a $\tau_1\tau_2$ - g*α closed set is a g*α closed set.

Proposition 3.5 If A is a τ_2 - α closed subset of (X,τ_1,τ_2), then A is $\tau_1\tau_2$ - g*α closed.

Proof Let U ∈ G*O(X,τ_1) be such that A ⊆ U. Then by hypothesis, τ_2 - αcl(A) ⊆ U and so A is $\tau_1\tau_2$ - g*α closed.

Example 3.6 Let X = {1,2,3,4} with τ_1 = {∅, X, {1}, {2}, {1,2}, {2,3}, {1,2,3}} andτ_2 = {∅, X, {3}, {4}, {3,4}}.Then the set A = {3,4} is $\tau_1\tau_2$ - g*α closed but not τ_2 - α- closed in (X,τ_1,τ_2).

Proposition 3.7 If A is both τ_1-g*open and $\tau_1\tau_2$-g*α closed then A is τ_2-α closed.

Proof If A ∈ G*O(X,τ_1), then by hypothesis τ_2 - αcl(A) ⊆ A. Therefore τ_2 - αcl(A) = A (ie) A is τ_2 - α closed.

Proposition 3.8 (i) Every (i,j) - g closed set is $\tau_1\tau_2$ - g*α closed.

(ii) Every (i,j) - g*closed set is $\tau_1\tau_2$ - g*α closed.

Proof follows from the Definition 3.1 and from the Definitions of (i,j)-g*closed and (i,j)- g closed sets.

Example 3.9 Let X = {1,2,3,4} with τ_1 = {∅, X, {1}, {1,2}, {1,3,4}} and τ_2 = {∅, X, {1}, {1,4}, {1,2,4}}. Here the subset {4} is $\tau_1\tau_2$ - g*α closed but not (1,2) - g closed.

Example 3.10 Let X = {1,2,3,4} with τ_1 = {∅, X, {2}} and τ_2 = {∅, X, {1}, {1,2}}. Here the subset {2,4} is $\tau_1\tau_2$ - g*α closed but not (1,2) - g*closed.

Proposition 3.11 If A, B ∈ G*α (X,τ_1, τ_2) then A ∪ B ∈ G*α (X,τ_1, τ_2).

Proof Let U be a τ_1 - g*open set containing A ∪ B. Since A, B ∈ G*α (X,τ_1,τ_2), we have τ_2 - αcl(A) ⊆ U and τ_2 - αcl(B) ⊆ U. Then τ_2 - αcl(A) ∪ τ_2 - αcl(B) = τ_2 - αcl(A ∪ B) ⊆ U. Hence A ∪ B ∈ G*α(X,τ_1,τ_2).

Remark 3.12 The intersection of two $\tau_1\tau_2$ - g*α closed sets is not a $\tau_1\tau_2$ - g*α closed set as seen from the following example.

Example 3.13 Let X = {1,2,3,4} be endowed τ_1 = {∅, X, {1}, {1,3}, {1,2,4}} and τ_2 = {∅, X, {1}, {2}, {1,2}}. Here the subset {1,2,3} and {1,3,4} are $\tau_1\tau_2$ - g*α closed sets but {1,2,3}∩ {1,3,4} = {1,3} is not a $\tau_1\tau_2$ - g*α closed set.

Remark 3.14 G*α(τ_1,τ_2) is generally not equal to G*α(τ_2,τ_1).

Example 3.15 Let X = {1,2,3} with τ_1 = {∅, X, {1}, {1,2}} and τ_2 = {∅, X, {1}}. Here G*α(τ_1,τ_2) ≠ G*α(τ_2,τ_1).

Proposition 3.16 If $\tau_1 \subseteq \tau_2$ in (X,τ_1,τ_2) then G*α(τ_2,τ_1) ⊆ G*α(τ_1,τ_2).

Proof Let A be a $\tau_2\tau_1$-g*α closed set and U be a τ_1 - g*open set containing A. Since $\tau_1 \subseteq \tau_2$, we have τ_2 - αcl(A) ⊆ τ_1- αcl(A) and G*O(X,τ_1) ⊆ G*α(X,τ_2). Since A ∈ G*α(τ_2,τ_1) ,τ_2 - αcl(A) ⊆ U, U is τ_1 - g*open. Thus A is $\tau_1\tau_2$ - g*α closed. Hence G*α(τ_2,τ_1) ⊆ G*α(τ_1,τ_2).

Proposition 3.17 For each x of a bitopological space (X,τ_1,τ_2), {x} is τ_1 - g*closed or $\{x\}^c$ is $\tau_1\tau_2$ - g*α closed.

Proof Suppose {x} is not τ_1 - g*closed. Then $\{x\}^c$ is not τ_1 - g*open. Therefore a τ_1- g*open set containing $\{x\}^c$ is X only. Also τ_2 - αcl($\{x\}^c$) ⊆ X. Hence $\{x\}^c$ is $\tau_1\tau_2$ - g*α closed.

IV.g*α-locallyclosedsets in BitopologicalSpaces

Definition 4.1 A subset A of a bitopological space (X,τ_1,τ_2) is called $\tau_1\tau_2$ - g*α locally closed (briefly $\tau_1\tau_2$ - g*αlc) set if A = S ∩ F where S is τ_1 - g*α open and F is τ_2 - g*α closed in (X,τ_1,τ_2).

Definition 4.2 A subset A of a bitopological space (X,τ_1,τ_2) is called $\tau_1\tau_2$ - g*α locally closed* (briefly $\tau_1\tau_2$ - g*αlc*) set if there exists τ_1 - g*α open set S and τ_2- closed set F of (X,τ_1,τ_2) such that A = S ∩ F.

Definition 4.3 A subset A of a bitopological space (X,τ_1,τ_2) is called $\tau_1\tau_2$ - g*α locally closed** (briefly $\tau_1\tau_2$ - g*αlc**) set if there exists τ_1 - open set S and τ_2 - g*α closed set F of (X,τ_1,τ_2) such that A = S ∩ F.

Remark 4.4 If $\tau_1 = \tau_2 = \tau$ in Definitions 4.1, 4.2, 4.3 then $\tau_1\tau_2$ - g*αlc(resp $\tau_1\tau_2$ - g*αlc*, $\tau_1\tau_2$ - g*αlc**) is a g*αlc (g*αlc*, g*αlc**) set in a topological space. The collection of all $\tau_1\tau_2$ - g*αlc (resp $\tau_1\tau_2$ - g*αlc*, $\tau_1\tau_2$ - g*αlc**) sets of (X,τ_1,τ_2) will be denoted by $\tau_1\tau_2$ - G*αLC (resp $\tau_1\tau_2$ - G*αLC*, $\tau_1\tau_2$ - G*αLC**).

Example 4.5 Let X = {1,2,3,4} with τ_1 = {∅, X, {1}, {2}, {1,2}, {1,2,3}} and τ_2 = {∅, X, {1}, {1,2}, {1,3,4}}. Then τ_1 - g*α open sets in (X,τ_1,τ_2) are { ∅, X, {1}, {2}, {3}, {1,2}, {1,3}, {2,3}, {1,2,3}, {1,2,4}}. τ_2 - g*α-closed sets in (X,τ_1,τ_2) are {∅, X, {2}, {3}, {4}, {2,3}, {2,4}, {3,4}, {1,2,3},{2,3,4}, {1,2,4}}.

(a) $\tau_1\tau_2$ - g*αlc sets in (X,τ_1,τ_2) are { ∅, X, {1}, {2}, {3}, {4}, {1,2}, {1,3}, {2,3}, {2,4}, {3,4}, {1,2,3}, {1,2,4}, {2,3,4}}.

(b) $\tau_1\tau_2$ - g*αlc* sets in (X,τ_1,τ_2) are { ∅, X, {1}, {2}, {3}, {4}, {1,2}, {1,3}, {2,3}, {2,4}, {3,4}, {1,2,3}, {1,2,4}, {2,3,4}}.

(c) $\tau_1\tau_2$ - g*αlc** sets in (X,τ_1,τ_2) are { ∅, X, {1}, {2}, {3}, {4}, {1,2}, {2,3}, {2,4}, {3,4}, {1,2,3}, {1,2,4}, {2,3,4}}.

Theorem 4.6 Let A be a subset of a space (X,τ_1,τ_2).

(i) If A ∈ (τ_1,τ_2) - LC(X) then A ∈ $\tau_1\tau_2$ - G*αLC, $\tau_1\tau_2$ - G*αLC*, $\tau_1\tau_2$ - G*αLC**.

(ii) If A ∈ $\tau_1\tau_2$ -G*αLC* then A ∈ $\tau_1\tau_2$ -G*αLC.

(iii) A ∈ $\tau_1\tau_2$ -G*αLC** then A ∈ $\tau_1\tau_2$ -G*αLC.

Proof Since every closed set is a g*α-closed set, the proof follows.

The converse of the above theorem need not be true in general as seen from the following examples.

Example 4.7 X = {1,2,3,4} with τ_1 = {∅, X, {1,3},{4}, {1,3,4}} and τ_2 = {∅, X, {2}}. Then the subset {1,4} ∈ $\tau_1\tau_2$ - G*αLC, $\tau_1\tau_2$ - G*αLC*, $\tau_1\tau_2$ - G*αLC** but{1,4} ∉ (τ_1,τ_2) - LC(X).

Example 4.8 X = {1,2,3} with τ_1 = {∅, X, {2},{3}, {1,3}, {2,3}}, τ_2 = {∅, X, {1,3}} . Then the subset {1,2} ∈ $\tau_1\tau_2$ - G*αLC but {1,2} ∉ $\tau_1\tau_2$ - G*αLC*.

Example 4.9 X = {1,2,3} with τ_1 = {∅, X, {2}, {2,3}}, τ_2 = {∅, X, {1,2}}. Then the subset {1} ∈ $\tau_1\tau_2$ - G*αLC but {1} ∉ $\tau_1\tau_2$ - G*αLC**.

Remark 4.10 Every $\tau_1\tau_2$ - g*α locally closed set in (X,τ_1,τ_2) are not τ_2 - closed in general as it can be seen from the following example.

Example 4.11 In example 4.7, {1,2,3} is $\tau_1\tau_2$ - g*α locally closed set in (X,τ_1,τ_2) but {1,2,3} is not τ_2-closed in (X,τ_1,τ_2).

Remark 4.12 Every $\tau_1\tau_2$ - g*α locally closed set in (X,τ_1,τ_2) are not τ_1-open in general as it can be seen from the following example.

Example 4.13 In example 4.7, {1,2} is $\tau_1\tau_2$ - g*α locally closed set in (X,τ_1,τ_2) but not τ_1-open in (X,τ_1,τ_2).

Theorem 4.14 (i) If A ∈ τ_2 -g*α-C(X,τ_1,τ_2) then A ∈ $\tau_1\tau_2$ - G*αLC

(ii) If A ∈ τ_1 - g*α-O(X,τ_1,τ_2) then A ∈ $\tau_1\tau_2$ - G*αLC.

Proof (i) Since A = A ∩ X and A is τ_2 - g*α closed and X is τ_1 - g*α open in (X,τ_1,τ_2), we have A ∈ $\tau_1\tau_2$ - G*αLC.

(ii) Since A = A ∩ X and A is τ_1- g*α open and X is τ_2-g*α closed in (X,τ_1,τ_2), we have A ∈ $\tau_1\tau_2$ - G*αLC.

The converses of (i) and (ii) of the above theorem need not be true in general as it can be seen from the following example.

Example 4.15 Let X = {1,2,3,4} with τ_1 = {∅, X, {1},{2,3}, {1,2,3}} and τ_2 = {∅, X, {1},{1,2,4}}. Then the subset {1}∈ $\tau_1\tau_2$ - G*αLC but {1}∉ τ_2 - g*α-C(X,τ_1,τ_2) and {2,3,4} ∈ $\tau_1\tau_2$ - G*αLC but {2,3,4} ∉ τ_1 -g*α-O(X,τ_1,τ_2).

V. $\tau_1\tau_2$ - g*α-Submaximal Spaces

Definition 5.1 A bitopological space (X,τ_1,τ_2) is

(i) $\tau_1\tau_2$ - g*α Submaximal space if every τ_1-dense subset of X is τ_2 - g*α open in X.

(ii) $\tau_2\tau_1$ - g*α Submaximal space if every τ_2-dense subset of X is τ_1 - g*α open in X.

Example 5.2 Let X = {1,2,3} with τ_1 = {∅, X, {1}} and τ_2 = {∅, X, {1}, {1,2}, {1,3}}. τ_2 - g*α open sets of (X,τ_1,τ_2) are {∅, X, {1}, {1,2},{1,3}}. Here every τ_1-dense subsets of (X,τ_1,τ_2) are τ_2 - g*α open sets. Hence it is $\tau_1\tau_2$ - g*α Submaximal.

Example 5.3 Let X = {1,2,3} with τ_1 = {∅, X, {1}, {1,2},{1,3}} and τ_2 = {∅, X, {1}, {1,2}}. τ_1 - g*α open sets of (X,τ_1,τ_2) are {∅, X, {1}, {1,2},{1,3}}. Here every τ_2-dense subsets of (X,τ_1,τ_2) are τ_1 - g*α open sets. Hence it is a $\tau_1\tau_2$ - g*α-Submaximal space.

Theorem 5.4 If (X,τ_1,τ_2) is $\tau_1\tau_2$ -Submaximal, then X is $\tau_1\tau_2$ - g*α Submaximal space but not conversely.

Proof Since (X,τ_1,τ_2) is $\tau_1\tau_2$ - Submaximal space, we have every τ_1 - dense subset of X is τ_2 - open in X. Since every τ_2 - open set in X is τ_2 - g*α open in X.Therefore (X,τ_1,τ_2) is $\tau_1\tau_2$ - g*α - Submaximal space.

Example 5.5 Let X = {1,2,3,4} with τ_1 = {∅, X, {2}, {1,2}} and τ_2 = {∅, X, {1,2}, {3,4}}. Then τ_1 - dense subsets of (X,τ_1,τ_2) are{∅, X, {2}, {1,2}, {2,3}, {2,4}, {1,2,3}, {1,2,4}, {2,3,4}}. τ_2 - g*α open sets of (X,τ_1,τ_2) are {∅, X, {1}, {2}, {3}, {4}, {1,2}, {1,3}, {1,4}, {2,3}, {2,4}, {3,4}, {1,2,3}, {1,3,4}, {2,3,4}, {1,2,4}}. Therefore (X,τ_1,τ_2) is $\tau_1\tau_2$ - g*α - Submaximal but not $\tau_1\tau_2$ - Submaximal space.

Theorem 5.6 A bitopological space (X,τ_1,τ_2) is $\tau_1\tau_2$ - g*α-Submaximal Space iff $\tau_2\tau_1$ - G*αLC* = P(X).

Proof Suppose that (X,τ_1,τ_2) is $\tau_1\tau_2$ - g*α - Submaximal space. Obviously $\tau_2\tau_1$- G*αLC* ⊆ P(X). Let A ∈ P(X) and U = A ∪ {X – [τ_1 - cl(A)]}. Since τ_1 -cl (U) = X, we have U is τ_1-dense subset of X. Since (X,τ_1,τ_2) is $\tau_1\tau_2$ - g*α - Submaximal space, we haveU is τ_2 - g*α-open in X. Since every τ_2 - g*α-open set in X is $\tau_2\tau_1$ - g*α locally set in (X,τ_1,τ_2), we have U ∈ $\tau_2\tau_1$ - G*αLC*.Therefore P(X) ⊆ $\tau_2\tau_1$ - G*αLC*. Hence $\tau_2\tau_1$ - G*αLC* = P(X).

Conversely suppose that $\tau_2\tau_1$- G*αLC* = P(X). Let A be the τ_1-dense subset of (X,τ_1,τ_2). Then A ∪ {X – [τ_1 - cl(A)]} = A ∪ [τ_1 cl(A)]^c = A. Therefore A ∈ $\tau_2\tau_1$ - G*αLC* implies that A is g*α- open in X (by theorem 7.3.18). Hence X is $\tau_1\tau_2$ - g*α - Submaximal Space.

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