Plasma Excitations in a pair of Metallic Carbon Nano Tube In Presence of External Transverse D.C Magnetic Field

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ABSTRACT : Plasma excitation in a pair of metallic carbon nano tube in presence of external transfer magnetic field are studied within the scope of classical electrodynamics. Excitation of electrons in each CNT surface are modeled by an infinitesimally thin layer of free electron gas which are studied with the help of linearized hydrodynamical model.

Key Words: Plasma, Carbon nano tube , Hydrodynamic model

Introduction: Plasma consists of electron and ions with total charge zero. Plasma is usually considered as an electron gas moving in back ground of positive ions. Plasma is a quasi neutral gas of ions, electrons and neutral particles, the term plasma was first used by Langmui

Among outstanding aspects about carbon nano tubes, the study of their collective electron excitation may be very important in understanding the electron interaction in carbon nano tubes as well as characteristics of their electronic structures. Collective excitations in the single walled and multi walled carbon nano tube have studied by several authors with various theoretical models. Among different theoretical models , hydrodynamical has found an important place because of their simplicity and physically intuition. Various versions of the hydrodynamical model of dielectric response of carbon valance electron are used to study collective excitation in such structure [1-9].The idea to apply a macroscopic hydrodynamic description to the collective dynamics of the many electron system was suggested by Bloch[10] as generalization of hydrostatic

Thomas- Fermi theory. Using this simple hydrodynamic model, Fetter found plasma oscillations and screening for electron layers.[11]

On the other hand, Schroter and Dereux [12], analyzed the propagation of plasmon on hollow metallic cylinder with a dielectric core , taking into account of retardation. Kaushawa and Djafari- Rouhani [13] used Green's function theory for calculating dispersion relations for coaxial and multiaxial structure with arbitrary dielectric constants with application to quantum wire and carbon nano tubes.

Planar and spherical geometries do not allow independent solutions for TM and TE modes with no angular dependence . The electromagnetic field propagating in such geometry are in general a liner combination of these two modes.

Recently , Lien and Lin [14] describe the low energy plosmon excitations in a pair of carbon nano tube with tight binding model . Also , Gumbs and Balassis [15] in the non retarded limit , studied collective excitations in a pair of parallel nonoverlapping cylindrical nano tubes by using random phase approximation and obtained a high frequency, corresponds to in phase longitudinal electron density oscillations along the axis of the nano tube and low frequency.

Her we study the plasma excitation in a pair of metallic carbon nano tube in presence of external transverse D.C magnetic field , using liner hydrodynamical model.

Formulation of The Problem:

Let us consider a pair of parallel nano tubes with radii r_1 and r_2 which density free electron fluid per unit area over each cylindrical surface is n_1^0 and n_2^0 respectively. The distance between the two axes will be d (d$> r_1 + r_2$) . The origin of the syndical coordinate x=(ρ,θ, z) be located at point z= 0 on the axis of first nono tube. Assuming that $n_j(x_i,t)$ be the pertubed density per unit area of homogeneous electron fluid on the j^{th} wall (j=1,2) due to propagation of plasma wave with frequency ω , along the z- axis.

Using linearized hydrodynamical model , we obtain the linearized continuity equation for each carbon nano tube,

$$\frac{\partial n_j}{\partial t}(x_j,t) + n_j^0 \nabla_j . u_j(x_j,t) = 0 \qquad (1)$$

and linearized momentum equation ,

$$\frac{\partial u_j}{\partial t}(x_j,t) = \frac{e}{m_e} E_{\parallel}^j(j,t) - \frac{\alpha_j}{n_j^0}\nabla_j(x_j,t) + \frac{\beta_j}{n_j^0}\nabla_j[\nabla_j^2 n_j(x_j,t)] \qquad (2)$$

Where $_j u_j(x_j,t)$ is the velocity of electrons residing on the jth shell and $\nabla_j = \hat{e_z}\left(\frac{\partial}{\partial z}\right) + a_j^{-1}\hat{e}\,\frac{\partial}{\partial \theta_j}$ differentiate only tangentially to the surface. $E_{\parallel}^j(j,t) = E_z^j\,\hat{e_z} + E_{\theta j}^j \widehat{e_\theta}$ is tangential component of electromagnetic field on jth cylindrical surface.

The first term on right hand side of equation (2) is the force on electron due to tangential component of electric field , evaluated at the surface of curbon nano tube r=r$_j$, the second and third term arise from the internal interaction force in the electron gas with $\alpha_j = \frac{\pi n_j^0}{m_e^2}\hbar^2$ is the square of speed of propagation of density disturbance in a uniform 2D homogeneous Thomas - Fermi electron fluid and $\beta = \frac{\hbar^2}{4m_e^2}$.

In the transverse magnetic wave , the magnetic wave component is not in the longitudinal direction ($B_z = 0$) but in transverse direction (B_r , $B_\theta \neq 0$) . Using Maxwell's equations we may obtain following solutions for the longitudinal electric field in different three regions,

$$E_z^1(x,t) = \sum_{m=-\infty}^{+\infty} A_m \frac{I_m(k\rho_1)}{I_m(kr_1)} e^{im\theta_1} e^{i(qz-\omega t)} \qquad \rho_1 < r_1 \qquad (3)$$

$$E_z^2(x,t) = \sum_{m=-\infty}^{+\infty} D_m \frac{I_m(k\rho_2)}{I_m(kr_2)} e^{im\theta_2} e^{i(qz-\omega t)} \qquad \rho_2 < r_2 \qquad (4)$$

and

$$E_z^3(x,t) = \sum_{m=-\infty}^{+\infty} [H_m \frac{k_m(k\rho_1)}{k_m(kr_1)} e^{im\theta_1} + Q_m \frac{k_m(k\rho_2)}{k_m(kr_2)} e^{im\theta_2}] e^{i(qz-\omega t)} \qquad (5)$$

for $\rho_1 < r_1$ and $\rho_2 < r_2$

where $I_m(x)$ and $K_m(x)$ are the modified Bessel functions. $k^2 = q^2 - \frac{\omega^2}{c^2}$ and c is the speed of light

After eliminating the velocity fluid $u_j(x_j,t)$ from equation (1) and

(2) and replacing the quantity n_j by expression of the form

$$n_j(x_j, t) = \sum_{m=-\infty}^{m=+\infty} N_{jm} e^{im\theta} e^{i(qz-\omega t)} \quad (6)$$

we find

$$N_{jm} = -i \frac{en_j^0}{m_{e\Omega_j}} q_m^j E^j \quad (7)$$

where $\Omega_j = \omega^2 - \alpha_j (k^2 + \frac{\omega^2}{c^2} + \frac{m^2}{a_j^2}) - \beta(k^2 + \frac{\omega^2}{c^2} + \frac{m^2}{a_j^2})^2$

and $q_m^j = q\,\widehat{e_z} + \left(\frac{m}{a_i}\right)\widehat{e_\theta}$

Now applying the boundary conditions [9] at the surface of first wall at $\rho_1 = r_1$ and express the term depending on ρ_2 and θ_2 in the outer cylinder in terms of ρ_1 and θ_1,using an addition theorem for modified Bessel function [16].

$$k_m(k\rho_2)e^{im\theta_2} = \sum_{l=-\infty}^{+\infty} k_{l+m}(kd) I_l(k\rho_1) e^{il\theta_1}$$

In low frequency electromagnetic wave region ($k \approx q \gg \frac{\omega}{c}$) at $\rho_1 = r_1$ we obtain the matrix form

$$H = SQ \quad (8)$$

Where H and Q are vectors whose components are H_m and Q_m , and matrix S has elements

$$S_{mn} = \left(k^2 + \frac{m^2}{r_1^2}\right) \frac{r_1^2 \omega_{1p}^2}{\omega^2 - \omega_1^2} \frac{K_{m+n}(kd) K_m(kr_1) I_m^2(kr_1)}{k_n(kr_2)} \quad (9)$$

In an analogous way , we used the boundary conditions at the surface of second wall at $\rho_2 = r_2$, we obtain

$$Q = TH \quad (10)$$

here we observe that the matrix T is obtain from S through permutation of the index 1 and 2 , where $\omega_{jp}^2 = \frac{e^2 n_j^0}{\epsilon_0 m_e r_j}$ and

$\omega_j^2(m,k)=$

$$\alpha_j\left(k^2+\frac{m^2}{r_j^2}\right)+\beta\left(k^2+\frac{m^2}{r_j^2}\right)^2+\omega_{jp}^2\alpha_j^2\left(k^2+\frac{m^2}{r_j^2}\right)I_m(kr_j)k_m(kr_j) \quad (11)$$

are the squares of the plasma dispersion on the cylinders j=1 and 2. from equations (8) and (10) one can find as

$$(ST-1)H = 0 \quad (12)$$

The zeros of determinant of matrix (ST-1) corresponds normal mode frequencies of the plosmon excitations on the surface of two coupled cylinders , to obtain a simple plasmon dispersion in term of interaction between the bare plosmon modes of the individual surfaces of the tubes and compare with two walled carbon nano tubes. we consider plasma wave which propagate parallel to an axial direction (z direction) of two parallel nano tube . So that from equation (12) by putting m=0 and n=0 we obtained two branches for defining the resonant frequencies of the Plasmon excitations which are clearly separated into a high frequency $\omega_+(0,k)$ and a low frequency $\omega_-(0,k)$.

$$\omega_\pm^2\,(0,k)=\frac{\omega_1^2+\omega_2^2}{2}\pm\left[\left(\frac{\omega_1^2-\omega_2^2}{2}\right)^2+\Delta^2\right]^{\frac{1}{2}} \quad (13)$$

where $\Delta^2=\omega_{1p}^2\omega_{2p}^2(kr_1)^2(kr_2)^2[K_0(kd)I_0(kr_1)I_0(kr_2)]^2$ (14)

gives the interaction between two parallel nanotubes. This interaction leads to shifts of plasmon energies between two plasmon modes. When distance between the two axes d , decreases the interaction will be strong and the splitting of the plasmon will be large . When $d\to\infty$ the oscillation are independent of each other with frequencies ω_1 and ω_2 as seen by eqn. (11)

In long wavelength limit , when each carbon nano tube has same radius and if neglect the retardation effect , we obtain

$$\omega_{\pm}^{2}(0,k\approx 0) \approx \frac{e^{2}rk^{2}}{2\epsilon_{0}m_{e}}(n_{1}^{0}+n_{2}^{0})\left|\mathrm{In}\frac{\mathrm{kr}}{2}\right| \pm \frac{e^{2}rk^{2}}{\epsilon_{0}m_{e}}\left[\frac{1}{4}(n_{1}^{0}-n_{2}^{0})^{2}\ \left|\mathrm{In}\frac{\mathrm{kr}}{2}\right|^{2} + n_{1}^{0}n_{2}^{0}\ \left|\mathrm{In}\frac{\mathrm{kd}}{2}\right|^{2}\right]^{\frac{1}{2}} \quad (15)$$

The lower energy plasmon exhibit a quasi acoustic linear dispersion that is similar to result obtained in the random phase approximation [15]. To illustrate the effect of two parallel walls on resonant frequency and compare with two walled carbon nano tubes. we choose an example of pair of carbon nano tube with radii r_1=6Δr and r_2 = 8Δr with d= 14Δr , wher Δr= 4.2A^0. To see clear behavior of two groups of resonant plasmon dispersion we plot dimensionless plasmon frequencies ω/ω_p versus dimensionless variable kΔr in figure1. Here $n_1^0 = n_2^0 = n_0$ and $\omega_p = (\frac{e^2 n_0}{\epsilon_0 m_e \Delta r})^{\frac{1}{2}}$. It is clear that in two walled carbon nao tube the splitting of plasmons is large as compared to pair of parallel carbon nano tubes.

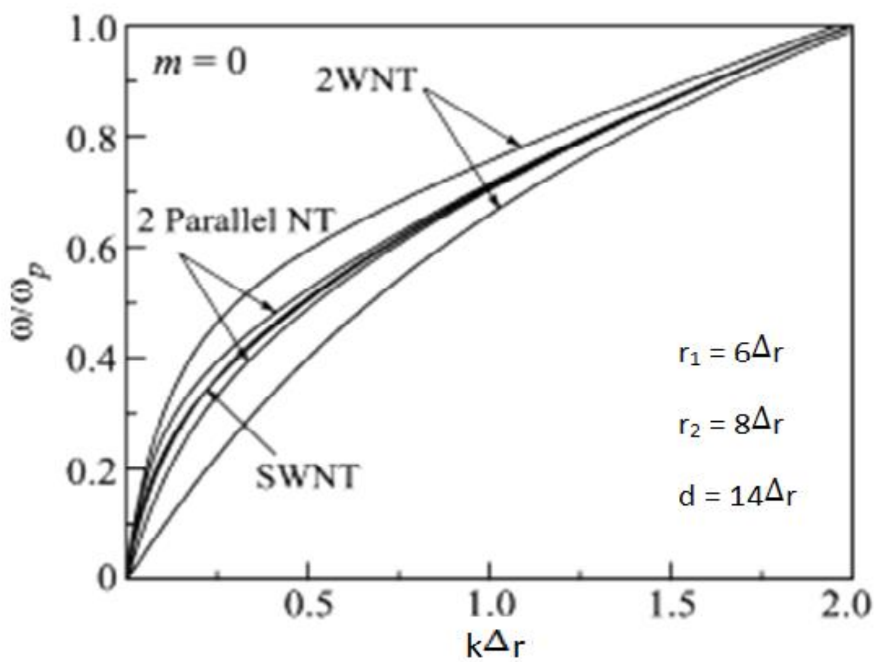

Figure:1[The dimensionless plasmon frequencies ω/ω_p versus dimensionless variable kΔr in a pair of parallel nano tubes with radii r_1=6Δr and r_2 = 8Δr with d= 14Δr are compared with two walled nano tube.**]**

In summary we used the linear hydrodynamical model in conjunction with Maxwell equations to describe the plosmonic response of a pair of metallic carbon nano tubes. Her we found analytically formulism of plasmon dispersion at low frequencies, in terms of interaction between the bare plasmon modes of the individual surfaces of the nano tubes . In long wavelength limit, if we ignore the retardation effect the result obtained is similar to random phase approximation.

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