Arc Coloring in Complement Fuzzy Graph

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Abstract Graph coloring is one of the most important problems of combinatorial optimization. Many problems of practical interest can be modeled as coloring problems. Two types of coloring namely node coloring and arc coloring are usually associated with any graph. Arc coloring is a function which assigns colors to the arcs so the incident arcs receive different colors. Let $G = (V, \mu, \sigma)$ be a simple connected undirected graph where V is a set of nodes and each nodes has a membership value μ and each edge has a membership value σ. Minimum number of color needed to color the graph is known as chromatic number of the graph. In this paper, we introduce an algorithm to find the complement of any fuzzy graph with order of n^2 time and also coloring this complement fuzzy graph using α cut.

Keywords *Complement fuzzy graph, arc coloring and chromatic number.*

I. INTRODUCTION

Graphs are simple model of relation. A graph is a convenient way of representing information involving relationship between objects. The object is represented by nodes and relations by arcs. When there is vagueness in the description of the objects or in its relationships or in both we need to design fuzzy graph model.

One of the most important properties of fuzzy graph model is fuzzy graph coloring which is used to solve problems of combinatorial optimization like traffic light control, exam-scheduling, register allocation etc. Two types of coloring namely node coloring and arc coloring are usually associated with any graph.

This paper consist of four sections. Introductory concepts are clearly given in the first section. In the second section, the preliminaries are provided, which are very much support to consecutive sections. In section three, a new algorithm is introduced and discussed to arc coloring in the complement fuzzy graph. Finally conclusions are given in the fourth section.

II. PRELIMINARIES

Definition 2.1

The node coloring of a graph is an assignment of labels or colors to each node of a graph such that no arc connects two identically colored nodes.

Definition 2.2

A proper arc coloring of a simple graph G = (V, E) is defined as a arc coloring from set of colors such that no two adjacent arcs share a same color.

Definition 2.3

The α cut of fuzzy graph defined as $G_\alpha = (V_\alpha, E_\alpha)$ where $V_\alpha = \{v \in V \mid \sigma \geq \alpha\}$ and $E_\alpha = \{e \in E \mid \mu \geq \alpha\}$.

Definition 2.4

The complement of a fuzzy graph $G:(\sigma,\mu)$ is $G^C:(\sigma^C,\mu^C)$, the advantage of this defined was that, for every fuzzy graph G. $(G^C)^C = G$, where $\sigma^C = \sigma$ and $\mu^C(x,y) = (\sigma(x) \wedge \sigma(y)) - \mu(x,y)$.

Definition 2.5

The minimum number if colors required to color the nodes of the given fuzzy graph is known as chromatic number . The chromatic number is denoted by, $\chi(G)$. $\chi(G) = \{(x_\alpha, \alpha)\}$ where x_α is the chromatic number of G_α and α values are the same different membership value of nodes and arcs of graph G = (V_F, E_F).

III. EDGE COLORING IN COMPLEMENT FUZZY GRAPH

We have calculated the complement of the fuzzy graph. Consider α value from different membership value of nodes and arcs in the complement fuzzy graph. In this, we have to done the procedures into three stages.

In stage 1, we take a fuzzy graph (G) which has 6 nodes and 6 arcs. All the nodes and arcs have fuzzy membership value.

In stage 2, we find the complement of this fuzzy graph (G^C).

In stage 3, we define the arc coloring function to color the complement fuzzy graph. We have taken a fuzzy graph between which have 6 arcs . To color the arcs in G we follow

Stage 1:

We consider a fuzzy graph G which have 6 nodes V_1,V_2,V_3,V_4,V_5,V_6 and corresponding membership value 0.45, 0.55, 0.65, 0.75, 0.85, 0.95.

Fuzzy graph G consist of 6 arcs e_1,e_2,e_3,e_4,e_5,e_6 with their corresponding membership value 0.1, 0.2, 0.3, 0.4, 0.5, 0.6.

$$\mu_1 = \begin{array}{c} \\ V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{array} \begin{array}{c} \begin{array}{cccccc} V_1 & V_2 & V_3 & V_4 & V_5 & V_6 \end{array} \\ \begin{bmatrix} 0.0 & 0.1 & 0.0 & 0.0 & 0.0 & 0.6 \\ 0.1 & 0.0 & 0.2 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.2 & 0.0 & 0.3 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.3 & 0.0 & 0.4 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.4 & 0.0 & 0.5 \\ 0.6 & 0.0 & 0.0 & 0.0 & 0.5 & 0.0 \end{bmatrix} \end{array}$$

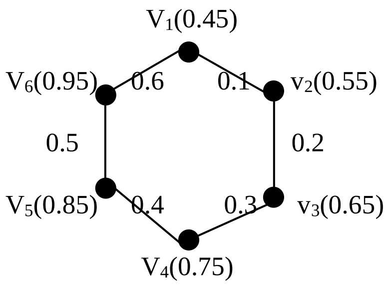

Fig-1 G (Fuzzy graph)

Let G = (V_F,E_F) be a fuzzy graph where V_F = {(v_1,0.45), (v_2,0.55), (v_3,0.65), (v_4,0.75), (v_5,0.85), (v_6,0.95)} and E_F = {(e_1,0.1), (e_2,0.2), (e_3,0.3), (e_4,0.4), (e_5,0.5), (e_6,0.6)}.

Stage 2:

To find the complement graph of the fuzzy graph (G^C)

$$\mu_2 = \begin{array}{c} \\ V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{array} \begin{array}{c} \begin{array}{cccccc} V_1 & V_2 & V_3 & V_4 & V_5 & V_6 \end{array} \\ \begin{bmatrix} 0.0 & 0.0 & 0.3 & 0.2 & 0.1 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.6 & 0.5 & 0.4 \\ 0.3 & 0.0 & 0.0 & 0.0 & 0.7 & 0.8 \\ 0.2 & 0.6 & 0.0 & 0.0 & 0.0 & 0.9 \\ 0.1 & 0.5 & 0.7 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.4 & 0.8 & 0.9 & 0.0 & 0.0 \end{bmatrix} \end{array}$$

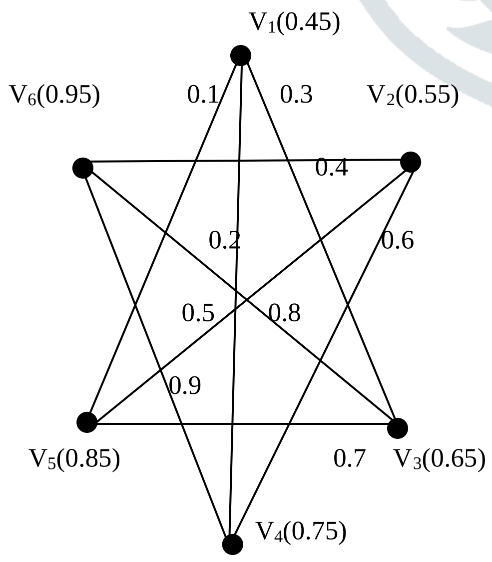

Fig-2 G^c Complement of fuzzy graph

Stage 3:

Graph G^C we have α values {0.1, 0.2, 0.3, 0.4, 0.5, 0.6,0.7, 0.8 0.9, 0.95}. By the Definition, we have colored the arcs of G_α^C for each α belongs to the above set and also find its chromatic number by the definition.

For $\alpha = 0.1$, the fuzzy graph G_α^C where $\sigma = \{0.45, 0.55, 0.65, 0.75, 0.85, 0.95\}$ and the adjacent matrix is

$$\mu_3 = \begin{array}{c} \\ V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{array} \begin{array}{c} \begin{array}{cccccc} V_1 & V_2 & V_3 & V_4 & V_5 & V_6 \end{array} \\ \begin{bmatrix} 0.0 & 0.0 & 0.3 & 0.2 & 0.1 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.6 & 0.5 & 0.4 \\ 0.3 & 0.0 & 0.0 & 0.0 & 0.7 & 0.8 \\ 0.2 & 0.6 & 0.0 & 0.0 & 0.0 & 0.9 \\ 0.1 & 0.5 & 0.7 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.4 & 0.8 & 0.9 & 0.0 & 0.0 \end{bmatrix} \end{array}$$

For $\alpha = 0.1$, we found the fuzzy graph $G_{0.1}^C$ (Fig-3). Then we use the proper coloring to color the arcs of the fuzzy graph G_α^C and the chromatic number of this graph 5.

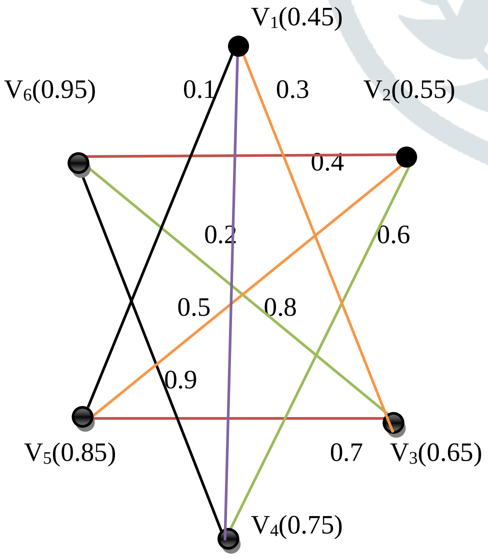

Fig-3 $\chi_{(0.1)} = 5$

For $\alpha = 0.2$, the fuzzy graph $G_{0.2}^C$ where $\sigma = \{0.45, 0.55, 0.65, 0.75, 0.85, 0.95\}$ and the adjacent matrix is

$$\mu_4 = \begin{array}{c|cccccc} & V_1 & V_2 & V_3 & V_4 & V_5 & V_6 \\ \hline V_1 & 0.0 & 0.0 & 0.3 & 0.2 & 0.0 & 0.0 \\ V_2 & 0.0 & 0.0 & 0.0 & 0.6 & 0.5 & 0.4 \\ V_3 & 0.3 & 0.0 & 0.0 & 0.0 & 0.7 & 0.8 \\ V_4 & 0.2 & 0.6 & 0.0 & 0.0 & 0.0 & 0.9 \\ V_5 & 0.0 & 0.5 & 0.7 & 0.0 & 0.0 & 0.0 \\ V_6 & 0.0 & 0.4 & 0.8 & 0.9 & 0.0 & 0.0 \end{array}$$

For $\alpha = 0.2$, we find the graph $G_{0.2}^{C}$ (Fig-4). Then we proper color all the arc of this graph and the chromatic number of this graph is 5.

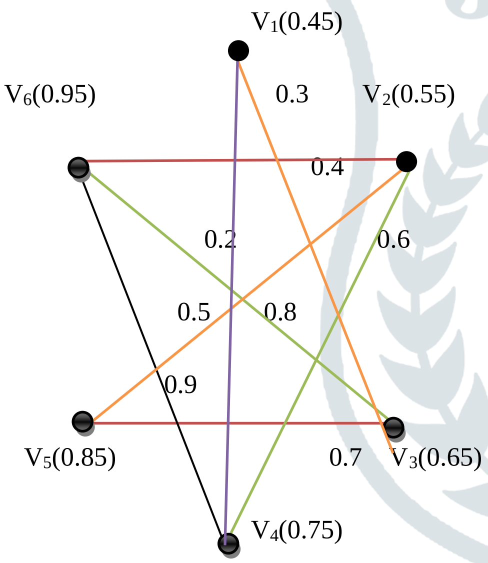

Fig-4 $\chi_{(0.2)} = 5$

For $\alpha = 0.3$, the fuzzy graph $G_{0.3}^{C}$ where $\sigma = \{0.45, 0.55, 0.65, 0.75, 0.85, 0.95\}$ and the adjacent matrix is

$$\mu_5 = \begin{array}{c|cccccc} & V_1 & V_2 & V_3 & V_4 & V_5 & V_6 \\ \hline V_1 & 0.0 & 0.0 & 0.3 & 0.0 & 0.0 & 0.0 \\ V_2 & 0.0 & 0.0 & 0.0 & 0.6 & 0.5 & 0.4 \\ V_3 & 0.3 & 0.0 & 0.0 & 0.0 & 0.7 & 0.8 \\ V_4 & 0.0 & 0.6 & 0.0 & 0.0 & 0.0 & 0.9 \\ V_5 & 0.0 & 0.5 & 0.7 & 0.0 & 0.0 & 0.0 \\ V_6 & 0.0 & 0.4 & 0.8 & 0.9 & 0.0 & 0.0 \end{array}$$

For $\alpha = 0.3$ we find the graph $G_{0.3}^{C}$ (Fig-5). Then we proper color all the arc of this graph and the chromatic number of this graph is 4.

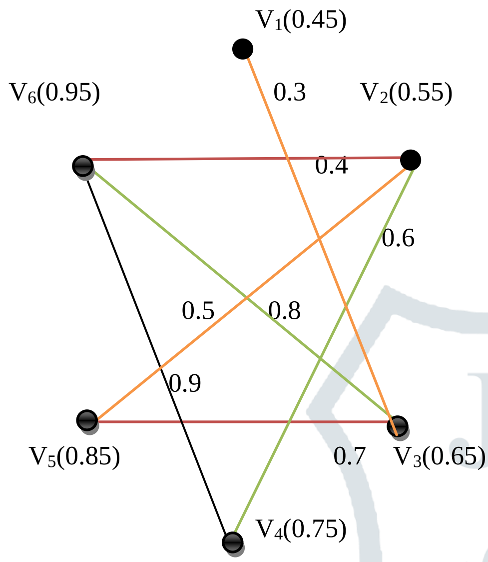

Fig-5 $\chi_{(0.3)} = 4$

For $\alpha = 0.4$, the fuzzy graph $G_{0.4}^{C}$ where $\sigma = \{0.45, 0.55, 0.65, 0.75, 0.85, 0.95\}$ and the adjacent matrix is

$$\mu_6 = \begin{array}{c} \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_5 \end{array} \begin{array}{c} \begin{array}{ccccc} V_2 & V_3 & V_4 & V_5 & V_6 \end{array} \\ \begin{bmatrix} 0.0 & 0.0 & 0.6 & 0.5 & 0.4 \\ 0.0 & 0.0 & 0.0 & 0.7 & 0.8 \\ 0.6 & 0.0 & 0.0 & 0.0 & 0.9 \\ 0.5 & 0.7 & 0.0 & 0.0 & 0.0 \\ 0.4 & 0.8 & 0.9 & 0.0 & 0.0 \end{bmatrix} \end{array}$$

For $\alpha = 0.4$, we find the graph $G_{0.4}^{C}$ (Fig-6). The we use the proper coloring to color of arc of graph G and the chromatic number of this graph is 4.

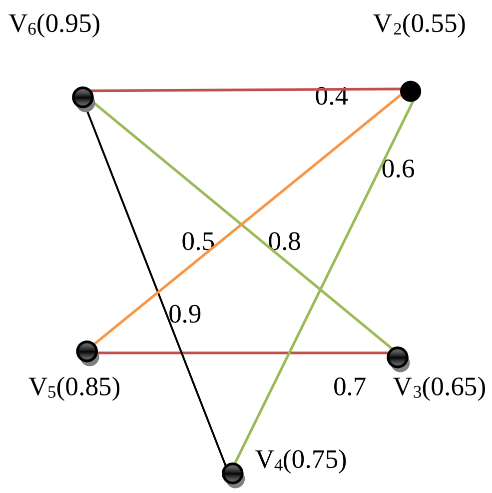

Fig-6 $\chi_{(0.4)} = 4$

For $\alpha = 0.5$, the fuzzy graph $G_{0.5}^{C}$ where $\sigma = \{0.55, 0.65, 0.75, 0.85, 0.95\}$ and the adjacent matrix is

$$\mu_7 = \begin{array}{c} \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_5 \end{array} \begin{array}{c} \begin{array}{ccccc} V_2 & V_3 & V_4 & V_5 & V_6 \end{array} \\ \begin{bmatrix} 0.0 & 0.0 & 0.6 & 0.5 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.7 & 0.8 \\ 0.6 & 0.0 & 0.0 & 0.0 & 0.9 \\ 0.5 & 0.7 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.8 & 0.9 & 0.0 & 0.0 \end{bmatrix} \end{array}$$

For $\alpha = 0.5$, we find the graph $G_{0.5}^{C}$ (Fig-7). Then we proper color all the arc of this graph and the chromatic number of this graph is 4.

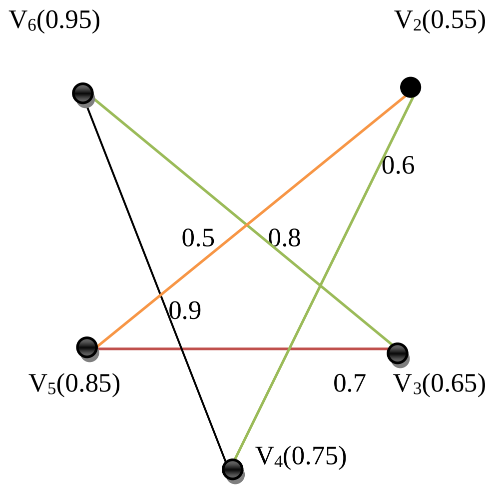

Fig-7 $\chi_{(0.5)} = 4$

For $\alpha = 0.6$, the fuzzy graph $G^C_{0.6}$, where $\sigma = \{0.65, 0.75, 0.85, 0.95\}$

$$\mu_8 = \begin{array}{c} \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_5 \end{array} \begin{array}{c} \begin{array}{ccccc} V_2 & V_3 & V_4 & V_5 & V_6 \end{array} \\ \begin{bmatrix} 0.0 & 0.0 & 0.6 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.7 & 0.8 \\ 0.6 & 0.0 & 0.0 & 0.0 & 0.9 \\ 0.0 & 0.7 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.8 & 0.9 & 0.0 & 0.0 \end{bmatrix} \end{array}$$

For $\alpha = 0.6$, we find the graph $G^C_{0.6}$ (Fig-8). Then we use the proper coloring to color the arc of graph G and the chromatic number of this graph is 3.

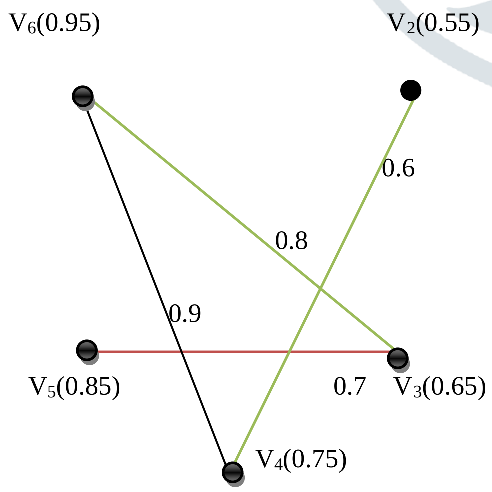

Fig-8 $\chi_{(0.6)} = 3$

For $\alpha = 0.7$, the fuzzy graph $G^C_{0.7}$, where $\sigma = \{0.75, 0.85, 0.95\}$

$$\mu_9 = \begin{array}{c} \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{array} \begin{array}{c} \begin{array}{cccc} V_3 & V_4 & V_5 & V_6 \end{array} \\ \begin{bmatrix} 0.0 & 0.0 & 0.7 & 0.8 \\ 0.0 & 0.0 & 0.0 & 0.9 \\ 0.7 & 0.0 & 0.0 & 0.0 \\ 0.8 & 0.9 & 0.0 & 0.0 \end{bmatrix} \end{array}$$

For $\alpha = 0.7$, we find the graph $G^C_{0.7}$ (Fig-9). Then we use the proper coloring to color the arc of graph and chromatic number of this graph is 3.

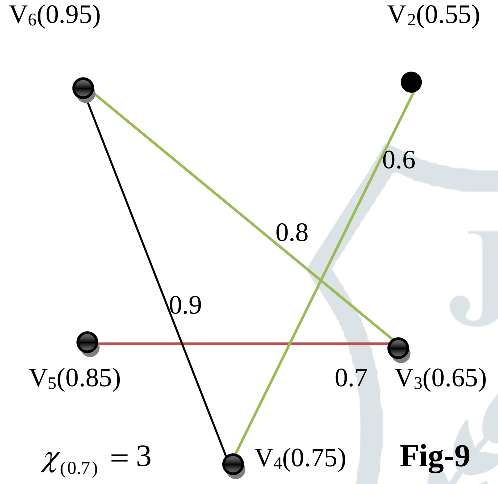

Fig-9

For $\alpha = 0.8$, the fuzzy graph $G^C_{0.8}$ (Fig-10). Where $\sigma = \{0.85, 0.95\}$ and use the proper coloring the arc and chromatic number of graph is 2.

$$\mu_{10} = \begin{array}{c} \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{array} \begin{array}{c} \begin{array}{cccc} V_3 & V_4 & V_5 & V_6 \end{array} \\ \begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.8 \\ 0.0 & 0.0 & 0.0 & 0.9 \\ 0.0 & 0.0 & 0.0 & 0.0 \\ 0.8 & 0.9 & 0.0 & 0.0 \end{bmatrix} \end{array}$$

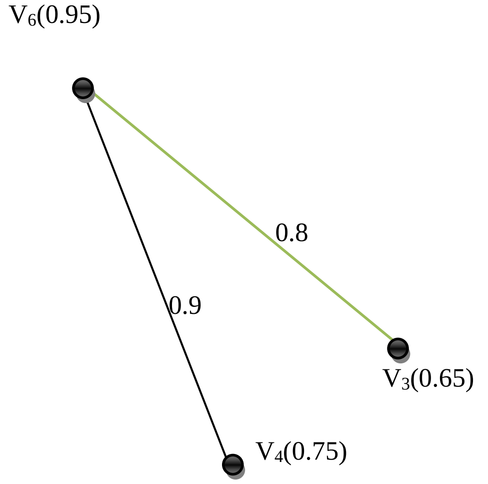

Fig-10 $\chi_{(0.8)} = 2$

For $\alpha = 0.9$, the fuzzy graph $G^C_{0.9}$ (Fig-11). Where $\sigma = \{0.95\}$ and use the proper coloring to color the arc and chromatic number is 1.

$$\mu_{11} = \begin{array}{c|cc} & V_4 & V_6 \\ \hline V_4 & 0.0 & 0.9 \\ V_6 & 0.9 & 0.0 \end{array}$$

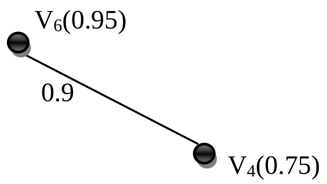

Fig-11 $\chi_{(0.9)} = 1$

$\alpha = 0.95$, the fuzzy graph $G^C_{0.95}$ (Fig-12). Then use the proper coloring to color the node and we have $\chi_{(0.95)} = 0$

$$\mu_{12} = \begin{array}{c|cc} & V_4 & V_6 \\ \hline V_4 & 0.0 & 0.0 \\ V_6 & 0.0 & 0.0 \end{array}$$

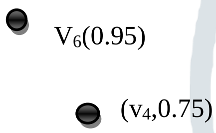

Fig-12 $\chi_{(0.95)} = 0$

Theorem:

Let ξ be a fuzzy graph. If $0 \le \alpha \le \beta \le 1$, then $\xi^\beta \subseteq \xi^\alpha$.

Proof:

Let $\xi = (V, \sigma, \mu)$ be a fuzzy graph and $0 \le \alpha \le \beta \le 1$. Now $\xi^\alpha = (V^\alpha, E^\alpha)$ where $V^\alpha = \{u \in V \mid I_v \ge \alpha\}$ and $E^\alpha = \{(u,v), u, v, \in V \mid I_{(u,v)} \ge \alpha\}$. Also, $\xi^\beta = (V^\beta, E^\beta)$ where $V^\beta = \{u \in V \mid I_v \ge \beta\}$ and $E^\beta = \{(u,v), u, v, \in V \mid I_{(u,v)} \ge \beta\}$. Let x be any element of V^β. Then, $I_x \ge \beta \ge \alpha$.

Therefore, $x \in V^\alpha$. Similarly, for any element $(x, y) \in E^\beta$.

Therefore, $\xi^\beta \subseteq \xi^\alpha$.

Hence proved.

Theorem:

Let ξ be a fuzzy graph. If $0 \le \alpha \le 1$, then $\xi^{\beta} \subseteq \xi^{\alpha}$.

Proof:

Let $\xi = (V, \sigma, \mu)$ be a fuzzy graph. $\xi_{\alpha} = (V_{\alpha}, E_{\alpha})$ such that $V_{\alpha} = \{u \in V \mid \sigma(u) \ge \alpha\}$ and $E_{\alpha} = \{(u,v), u, v, \in V \mid \mu(u,v) \ge \alpha\}$.

Again $\xi^{\alpha} = (V^{\alpha}, E^{\alpha})$ such that $V^{\alpha} = \{u \in V \mid I_v \ge \alpha\}$ and $E^{\alpha} = \{(u,v), u, v, \in V \mid I_{(u,v)} \ge \alpha\}$.

Let $x, y \in V_{\alpha}$ and $(x,y) \in E_{\alpha}$. Therefore $\sigma(x) \ge \alpha, \sigma(y) \ge \alpha$ & $\mu(x,y) \ge \alpha$. This results along with $\alpha \le 1$ implies that $\frac{\mu(x,y)}{\sigma(x) \wedge \sigma(y)} \ge \alpha$. Hence $I_{(x,y)} \ge \alpha$. So, $(x,y) \in E^{\alpha}$. Thus for every edges of ξ_{α}, their exist an edge in ξ^{α}. Now, clearly from the definitions of strength of nodes, $V_{\alpha} \subseteq V^{\alpha}$. Hence the results $\xi_{\alpha} \subseteq \xi^{\alpha}$ is true.

Hence proved.

IV.CONCLUSION

In this paper, we computed chromatic number for the arc coloring of a complement fuzzy graph using α -cut. We took α value from the different membership values of nodes and arcs in G. we found the chromatic number (arc coloring) for the different α value of complement fuzzy graph G follows $\chi(G) = (5,0.1),(5,0.2),(4,0.3),(4,0.4),(4,0.5),(3,0.6)(3,0.7),(2,0.8),(1,0.9),(0,0.95)$

In future, we shall try to desigen an algorithm on arc coloring function to any fuzzy graph.

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