# RELIABILITY ANALYSIS FOR LIQUEFACTION POTENTIAL ASSESSMENT BASED ON STANDARD PENETRATION TEST

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***Abstract:*** In engineering field no one can owe to have 100% accuracy. It is mainly due to errors during data collection or during the analysis due to manual errors or instrumental errors. Because of these errors, the analysis may go wrong. So at least the probability for which the analysis holds well should be known. Liquefaction analysis has a lot of uncertainties and hence its traditional analysis hasn't hold well in many case histories. Thus the reliability analysis has to be done. In this work the deterministic and probabilistic analysis of liquefaction of soil using EXCEL Software is attempted.

**1. Introduction:** In engineering field, 100% accuracy of result cannot be assured as the expected load calculations, equations and graphs used in the analysis are developed with limited database available at that point of time. Therefore there will be limitations on the design or on the result obtained. In addition to the above, there will be some random decisions taken during the execution which adds uncertainty to the result obtained. All these collectively causes considerable error on the designed product or on the result obtained. Hence, there are considerable chances for the deviations in the result obtained or the product designed. If a structure is to be designed, we need to know the load that the structure has to bear. As the exact load that will be imposed on the structure cannot be measured, there will be approximations in the load calculations and in the designing of the structure and hence uncertainty in the structure designed. In the case of geotechnical engineering, even in a limited working area, there will be variability in the soil used and will also be varying with time. The variability in soil with time will be due to variation of ground water levels. In the case of liquefaction analysis, all the variables used in the analysis will have their own uncertainties. These uncertainties are due to sampling errors or due to lab testing errors. In addition to these, as liquefaction mainly depends on the earthquake magnitude, which is hard to predict, there are quite good number of chances for the failure of the analysis. Hence uncertainties in the engineering fields are unavoidable. Therefore, at least the probability of failure of analysis needs to be calculated, which emerges the need of reliability analysis.

## 2. Literature Review

Liquefaction is a state where there the saturated soil losses its strength because of the excess pore water pressure developed due to dynamic loading resulting in the reduction of effective stress of soil. It is a phenomenon in which the stiffness of the soil is reduced by earthquake shaking.

### 2.1 Theory of Liquefaction

The shear strength of soil mainly depends on cohesion and frictional resistance of the soil particles. The inter-molecular attraction and the frictional resistance of soil particles results in the shear strength of soil. When an earthquake occurs, due to severe vibrations, there will be sudden rise in the pore water pressures. This sudden rise of pore water pressure causes a reduction in shear strength. This loss of strength due to transfer of inter granular stress from soil grains to pore water, due to dynamic load is known as "liquefaction".

### 2.2 Deterministic Liquefaction Analysis

Liquefaction analysis involves ascertaining whether the soil subjected to earthquake will liquefy or not. For the liquefaction analysis, a dimensionless parameter CSR (Cyclic Stress Ratio) is introduced by Seed and Idriss (1971) and is defined as

$$CSR = r_d . (\sigma_{v0} / \sigma_{v0'})( a_{max} / g) \quad \text{Eq.1}$$

Where, $a_{max}$ is maximum horizontal acceleration at ground surface (m/sec$^2$)
g is acceleration due to gravity
$\sigma_{v0}$ is total vertical stress at a particular depth where liquefaction analysis is done
$\sigma_{v0'}$ is vertical effective stress at the same depth
$r_d$ is stress reduction factor that accounts for the flexibility of soil column
Liquefaction resistance of soil is represented by the Cyclic Resistance Ratio (CRR). If an earthquake occurs and if the cyclic stress ratio (CSR) caused by that earthquake is greater than the cyclic resistance ratio (CRR) of that particular insitu soil, and then liquefaction will occur. Thus, the factor of safety (FS) against liquefaction may be defined as

$$FS = CRR/CSR \quad \text{Eq.2}$$

The higher the factor of safety, the more resistant is the soil to liquefaction. In general if the FS ≤1, then the soil will liquefy otherwise the soil is safe against liquefaction.

### 2.2.1 Evaluation of Cyclic Stress Ratio

Seed et al (1975) modified the equation given by Seed and Idriss (1971) and suggested the following equation:

$$CSR = 0.65\, r_d.\, (\sigma_{v0} / \sigma_{v0'})( a_{max} / g) \quad \text{Eq.3}$$

To account for earthquake magnitudes of smaller or larger than 7.5, Seed and Idriss (1982) introduced correction factors called Magnitude Scaling Factors (MSFs) and revised the CSR equation given by Seed et al as

$$CSR = 0.65\, r_d.\, (\sigma_{v0} / \sigma_{v0'})( a_{max} / g)(1/MSF) \quad \text{Eq.4}$$

Various researchers like Liao and Whiteman (1986), Kayen et al., (1992), Blake (1996), Robertson and Wride (1998) worked on the calculation of $r_d$ with respect to varying depths. Idriss (1999) performed several hundred parametric site response analyses and concluded that for the conditions of most practical interest, the parameter $r_d$ could be expressed as a function of depth and earthquake magnitude (M). The following equations derived by Idriss (1999) are used in the present study.

$$\ln (r_d) = \alpha (z) + \beta (z)*M \quad \text{Eq.5}$$

$$\text{where } \alpha (z) = -1.012 - 1.126 \sin \left(\frac{z}{11.73} + 5.133\right) \quad \text{Eq.6}$$

$$\beta (z) = 0.106 + 0.118 \sin \left(\frac{z}{11.28} + 5.142\right) \quad \text{for } Z \le 34 \text{ m} \quad \text{Eq.7}$$

where Z is the depth in m and M is the moment magnitude.

$$\text{For } Z > 34 \text{ m},\ r_d = 0.12*\exp(0.22M) \quad \text{Eq.8}$$

Researchers like Seed and Idriss (1982), revised Idriss Factors (1988), Ambraseys (1988), Arango (1996), Andrus and Stokoe (1997), Youd and Noble (1997) worked on the development of magnitude scaling factors. Based on the results of cyclic tests on high quality samples obtained by frozen sampling techniques, the relations proposed by Idriss (1999) are used in the present analysis.

$$MSF = 6.9 \exp(-M/4)-0.058 \quad \text{Eq.9}$$

MSF is limited to a maximum of 1.8

### 2.2.2 Evaluation of Cyclic Resistance Ratio

The cyclic resistance ratio represents the liquefaction resistance of the soil. The most commonly used method for determining the liquefaction resistance is to use the data obtained from the standard penetration test (SPT). Many researchers like Seed and Idriss (1982), Seed et al (1985), and Rauch (1998) suggested equations for calculating CRR based on SPT number and fines content. Cetin et al (2000) expanded the SPT case history database by including an additional 67 cases of liquefaction/no-liquefaction in 12 earthquakes and suggested the following equation to calculate CRR.

$$CRR = \exp\left\{\frac{(N_1)_{60cs}}{14.1} + \left[\frac{(N_1)_{60cs}}{126}\right]^2 - \left[\frac{(N_1)_{60cs}}{23.6}\right]^3 + \left[\frac{(N_1)_{60cs}}{25.4}\right]^4 - 2.8\right\} \quad \text{Eq.10}$$

where $(N_1)_{60cs}$ is the SPT number corrected to 5% fines.
The equation suggested by Cetin et al (2000) is used in the present analysis.
$(N_1)_{60cs}$ is calculated by using these equations.

$$(N_1)_{60cs} = (N_1)_{60} + \Delta(N_1)_{60} \quad \text{Eq.11}$$

$$\Delta(N_1)_{60} = \exp\left[1.63 + \frac{9.7}{FC} - \left(\frac{15.7}{FC}\right)^2\right] \quad \text{Eq.12}$$

$$(N_1)_{60} = C_N\,(N)_{60} \quad \text{Eq.13}$$

where $(N)_{60}$ is the observed SPT number,
$C_N$ is the overburden stress correction factor.

Based on theoretical and experimental data for SPT, Boulanger and Idriss (2004) suggested the following equation for overburden stress correction factor and are used in the present analysis.

$$C_N = \left(\frac{P_a}{\sigma'_{vo}}\right)^{\alpha} \le 1.7 \quad \text{Eq.14}$$

$$\alpha = 0.784 - 0.0768\sqrt{(N_1)_{60}} \quad \text{Eq.15}$$

where $(N_1)_{60}$ is limited to a maximum of 46. Solving for $C_N$ requires iteration because $(N_1)_{60}$ depends on $C_N$ and $C_N$ depends on $(N_1)_{60}$.

### 2.2.3 Adjustment of CRR for the Effect of Overburden Stress and Sloping ground conditions

Since CRR of cohesion less soils varies with effective confining stress and is affected by the presence of static driving shear stresses, Seed (1983) introduced correction factors Kα and Kσ to extrapolate the simplified procedure to large overburden pressure and static shear stress conditions. Hence the Factor of safety becomes

$$F.S = \frac{CRR}{CSR} * K\alpha * K\sigma \quad \text{Eq.16}$$

where Kσ is the overburden correction factor and Kα is the static shear stress correction.
The equations suggested by Idriss and Boulanger (2004) for Kσ are used in the present analysis.

$$K_\sigma = 1- C_\sigma \ln\left(\frac{\sigma'_{vo}}{P_a}\right) \le 1.0 \quad \text{Eq.17}$$

$$C_\sigma = \frac{1}{18.9-2.55\sqrt{(N_1)_{60}}} \quad \text{Eq.18}$$

$$C_\sigma \le 0.3 \quad \text{Eq.19}$$

Where $(N_1)_{60}$ is limited to a maximum of 37.
Harder and Boulanger (1997) reviewed past publications, test results, and analyses of *Kα* and noted a wide range of *Kα* values indicating a lack of convergence and a need for continuing research. Hence Kα is taken as unity in the present analysis.

The above equations are used for the liquefaction analysis of the following borehole data shown in the table 1.

**Table 1: SPT data for Liquefaction analysis (Rathod 2011)**

| Depth (m) | Unit weight γ (kN/m$^3$) | Observed SPT no (N) | Fines (%) |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1.5 | 16.87 | 9 | 7 |
| 3 | 17.75 | 6 | 8 |
| 5 | 17.75 | 12 | 10 |
| 6 | 18.35 | 14 | 4 |
| 8 | 18.35 | 21 | 1 |
| 10 | 19.13 | 24 | 3 |
| 12 | 19.62 | 20 | 2 |
| 15 | 19.72 | 27 | 5 |
| 18 | 19.72 | 28 | 1 |
| 20 | 19.82 | 31 | 8 |
| 22 | 19.92 | 31 | 5 |

The EXCEL sheet showing deterministic analysis is shown in the figure 1.

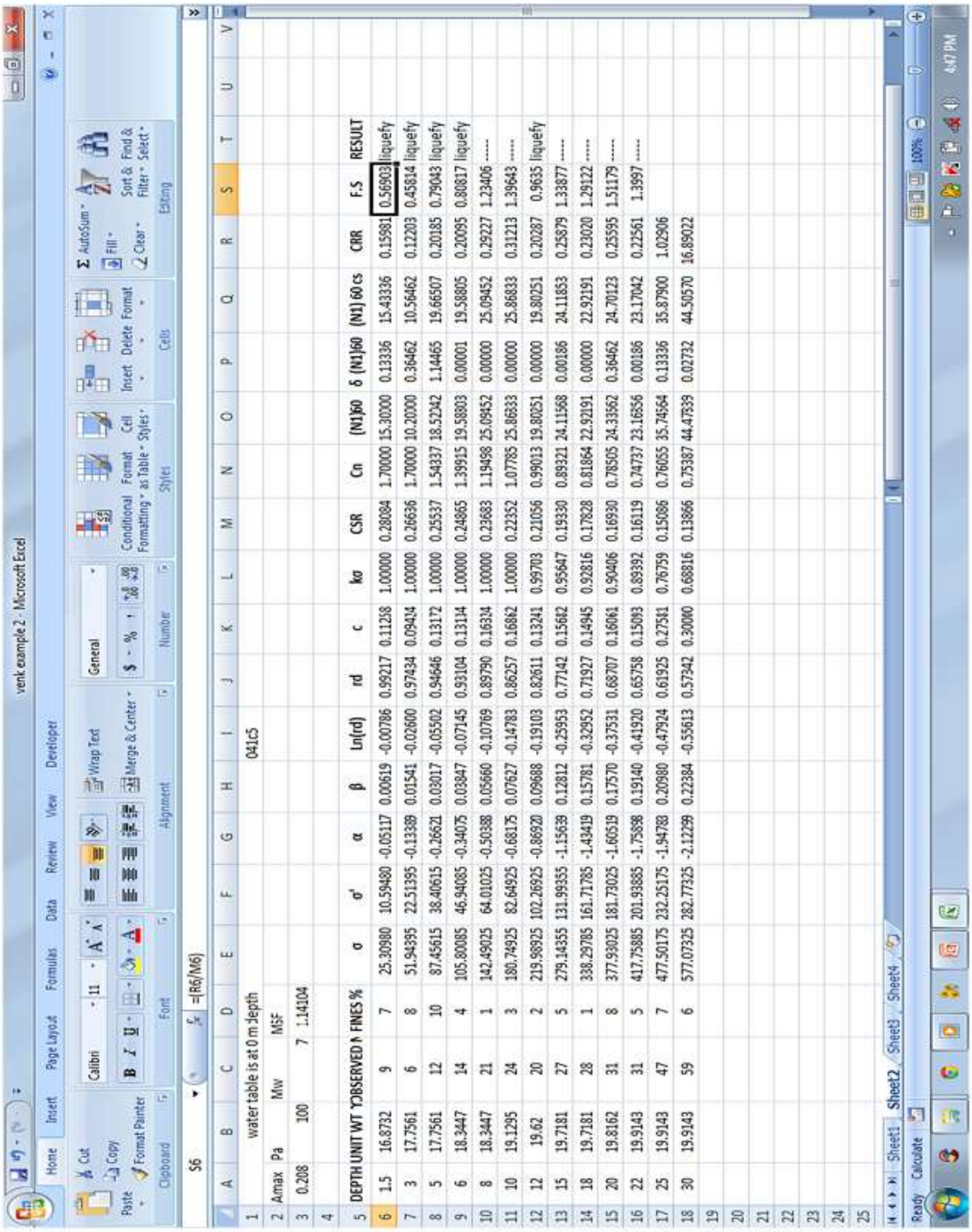

**Figure 1: EXCEL sheet showing deterministic analysis**

## 3. Methodology:

### 3.1 Reliability analysis of liquefaction of soil

The first step in evaluating the reliability is to decide on specific performance criteria and the relevant parameters, called the basic variables $X_i$, and the functional relationships among them corresponding to each performance criterion. For liquefaction analysis, if S denotes the CSR and R denotes the CRR, the performance function for liquefaction analysis can be defined as $Z = R - S$. If $Z = R - S < 0$, the performance function fails and liquefaction occurs and if $Z = R - S > 0$, the performance function is safe and liquefaction will not occur. If $Z = R - S = 0$, the performance function is at the limiting state.

Distributions: The parameter in the liquefaction analysis seems to follow different distributions like normal, lognormal, beta, extreme, weibull etc. Earlier studies on reliability analysis assumed all variables to follow normal distribution which is not true. As the exact distributions of the variables in liquefaction are not yet known, in the present study a generalized spread sheet is prepared so that analysis can be done for different distributions.

Correlation: It can be simply related to intersection used in probability. Mathematically correlation coefficient is expressed as

$$\rho = \frac{COV(X,Y)}{\sigma_X \sigma_Y} \qquad \text{Eq.20}$$

where $COV(X,Y)$ is the covariance, $\sigma_X$ and $\sigma_Y$ are standard deviations of X and Y.

Safety index (β) is calculated is calculated from the performance function, which is the shortest distance from the origin to the curve. The minimum distance point on the limit state surface is called the design point or the checking point. In order to obtain the design point, First Order Reliability Method I (FORM 1) is used for the present analysis.

### 3.2 Reliability Analysis of Liquefaction using FORM 1

The main aim of the reliability analysis is to determine the safety index or reliability index 'β'. The probability of failure ($P_f$) in terms of safety index is given by the equation

$$P_f = \phi(-\beta) = 1 - \phi(\beta) \qquad \text{Eq.21}$$

β is found by Hasofer – Lind method (1974). Veneziano (1974) and Ditlevsen (1981) formulated the Hasofer – Lind method to matrix form and proposed the following equation for β.

$$\beta = \min_{x \epsilon F} \sqrt{\left[\frac{x_i - \mu_i}{\sigma_i}\right]^T [R]^{-1} \left[\frac{x_i - \mu_i}{\sigma_i}\right]} \qquad \text{Eq.22}$$

where $x_i$ is a random variable, $\sigma_i$ is the standard deviation of the $i^{th}$ variable, $\mu_i$ is the mean value of the $i^{th}$ variable and R is the correlation matrix. The above equation is used for the variables which follows normal distribution. As all variables in liquefaction analysis will not follow the normal distribution, they have to be converted to standard normal form. Rackwitz Fiessler (1978) suggested the following equations to convert non normal to standard normal form.

$$\sigma^N = \frac{\Phi\{\Phi^{-1}[F(x)]\}}{f(x)} \qquad \text{Eq.23}$$

$$\mu^N = x - \sigma^N \Phi^{-1} [F(x)] \qquad \text{Eq.24}$$

where x is the original non normal variate, $\Phi^{-1}[.]$ is the inverse of the cumulative probability (CDF) of a standard normal distribution, F(x) is the original non normal CDF evaluated at x, $\phi\{.\}$ is the probability density function (PDF) of the standard normal distribution, and f(x) is the original non normal probability density ordinate at x. The above mathematical expressions are the basic principles used for the conversion of non-normal to standard normal form. It can be done in spreadsheet using VBA code. This can only be done in a macros enabled work sheet. In order to obtain the design point, optimization of $\beta$ has to be done. For optimization of $\beta$ the solver parameter can be used in spread sheet. In solver parameter the target cell is given as $\beta$, constraint is the performance function and the iterating variable is $x_i^N$.

The EXCEL sheet showing reliability analysis by FORM 1 is shown in the figure 2.

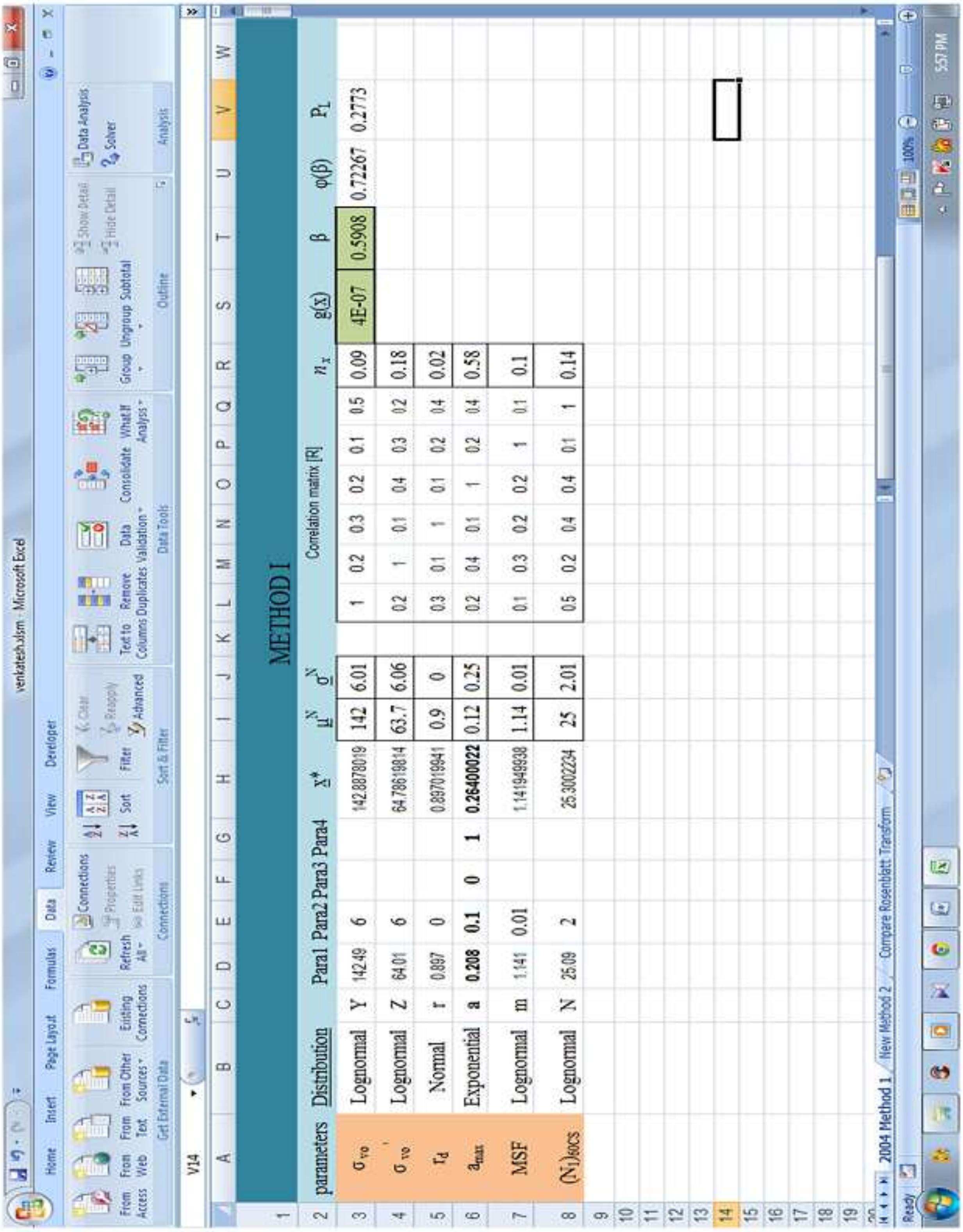


**FIGURE 2: EXCEL sheet showing reliability analysis by FORM 1**

**4. Results and Discussions:**

In the deterministic analysis at 8 m depth, the factor of safety obtained was 1.234, which is greater than one and hence should not liquefy. In the reliability analysis the safety index obtained is 0.5908. The probability of liquefaction (PL) obtained from reliability analysis is 0.27. Though from the deterministic analysis the F.S is 1.234, there are 27% chances to liquefy due to parameter uncertainties.

**5. Conclusions:**

Based on the above study the following conclusions can be drawn:

- Equations and correction factors used are from recent studies and are based on enlarged database. Thus the results obtained in the deterministic analysis have higher degree of accuracy.
- As the uncertainties are not considered in deterministic analysis, there is a possibility that the analysis will not hold good.
- As shown in the present study, though the factor of safety is greater than one it had 27% chances for liquefaction because of parameter uncertainties.

- The first-order reliability method is shown to be able to estimate accurately the reliability index β and the corresponding probability of liquefaction (PL).
- As the exact distributions of the variables in liquefaction are not yet known, in the present study a generalized spread sheet is prepared so that analysis can be done for different distributions.

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