GROSS LIFT - OFF MASS OPTIMIZATION FOR SPACE LAUNCH VEHICLES

[1]R.D.V.Prasad, [2]N.Manikanta, [3]S.Mohana Sangeetha, [4]P.Pavani, [5]P. Bhargavi, [6]P.Anirudh

[1]Asst Professor, [2,3,4,5,6]B.Tech

[1,2,3,4,5,6]Deparment of Mechanical Engineering,

[1] Anil Neerukonda Institute Of Technology & Sciences (A), Sagivalasa, India

Abstract : Rockets have multiple stages because the effectiveness of a rocket is inversely proportional to its mass and number of stages allows us to reduce the mass of the rockets as it operates. The single stage has a lot of empty fuel tank mass that we need to carry with us. The multi-stage has dropped its empty fuel tank and become a smaller; more effective rocket. The lower power stage will be flying up with empty tanks at the expense of fuel of a new tank. Each and every kilogram out will be beneficial. The masses of the empty tanks would not be nice to keep the engines and let go the tanks. We are doing a mathematical optimization of gross lift off mass for a required burnout velocity and payload ,optimal weight distribution for arbitrary number of stages like 2,3,4 and 5 having different structural ratios and specific impulses in each stage, staging optimization gives a quick insight about vehicle performance capability prior to trajectory design. Finally with this optimization technique we compared to gross lift off mass variation for two launch vehicles (i.e Ariane-1 and Proton m) for various burnout velocities and payloads at different number of stages.

***IndexTerms* – Payload, Stages , structural ratio, burnout velocity ,trajectory losses, LaGranger optimization**

I. Introduction to Rockets

The study of rockets is an excellent way for students to learn the basics of forces and the response of an object to external forces. All rockets use the thrust generated by a propulsion system to overcome the weight of the rocket. For full scale satellite launchers, the weight of the payload is only a small portion of the lift-off weight. Most of the weight of the rocket is the weight of the propellants. As the propellants are burned off during poweredascent, a larger proportion of the weight of the vehicle becomes the near-empty tankage and structure that was required when the vehicle was fully loaded.

II. LITERATURE REVIEW

H.H.Hallet.al[1] The optimum weight distribution for multistage rockets having different specific impulses and structural factors in each stage is derived. Minimization of gross weight for a given required burnout velocity and payload is the criterion of optimization used. A method is suggested for including in first approximation the effects of gravity, drag and turn.

EzgiCivek-Coskunet.al[2] The staging optimization problem for multistage rockets which carry payloads from the Earth's surface into the Earth orbits. In the early design phases, requirements are not so strict, there are many unknowns and problem arises as to what is the optimum staging to achieve the given mission. Therefore, designers need simplified tools providing a quick insight on the vehicle performance with minimum basic vehicle data. For this purpose, a Mat lab® based computer program has been written to determine staging parameters (number of stages, mass distribution between stages, and the propellant and structural masses for each individual stage) which minimize the gross lift-off mass of the launch vehicle for a specific mission. In this study, staging optimization problem has been formulated based on Delta-V equations and solved by method of Lagrange Multipliers. The problem has been stated in a general form to handle launch vehicles having arbitrary number of stages and with various configurations involving serial, parallel and clustered stages; and with different structural ratios and propellant exhaust velocities in each stage. Staging optimization program developed in this study has been verified for different missions using available data of existing launch vehicles. Thus, a quick and effective tool to find optimal vehicle configurations in the conceptual design phase of a generic multistage launch vehicle has been achieved.

David N. Burgheset.al[3] In this paper the fundamental characteristics of rocket staging are described. The equation of motion of a rocket is derived, and it is demonstrated that single stage rockets are not able to launch earth satellites successfully. Two-stage rockets are analyzed and the optimum choice of the rocket stage masses is found for maximum final speed. Multistage rockets are then considered and again individual stage masses are found for maximum final speed with constant total mass. This maximum final speed is evaluated for varying number of stages, and it is shown that the optimum choice for n is 2 or 3 for most earth satellite launching operations.

III. Basic Rocket Equation For Velocity Increment In A Time Interval

(From impulse–momentum principle)

For launch vehicle

:$[\overrightarrow{\Delta v}]_{ideal} = |\overrightarrow{cj}| \ln[\wedge]$

$\wedge = \frac{mi}{mf}$ = mass ratio

cj = velocity of exhaust gas with respect to nozzle

For multistage rockets having n stages the

Net ideal velocity increment

$|\Delta v|_{vehicle} = \sum_{k=1}^{n} |cj_k| \ln[\wedge_k]$

Serial staging

For each stage :

$Cj_k = Isp_k(g)$

Isp_k = Specific impulse of k th stage

Mass ratio $= \wedge_k = \frac{m_i,k}{m_f,k} = \frac{mo,k}{ms,k+mpl,k}$

Structure ratio $\varepsilon_k \Rightarrow \frac{ms,k}{ms,k+mp,k}$

Payload ratio = $\lambda_k \Rightarrow \frac{mpl,k}{ms,k+mp,k}$

$\wedge_k = \frac{1+\lambda_k}{\varepsilon_k+\lambda_k}$

3.3 Parallel staging:

For parallel staging an equivalent serial staging has determined and all parallel boosters along with the propellant of core stage which burnt along with boosters is considered as zero th stage

$M_{bk} = mass\ of\ booster\ stage$

M_{p01} = mass of propellant of core stage consumed along with boosters

$M_b = mass\ of\ all\ parallel\ boosters$

$$\wedge_o = \frac{m_{kb+m_{01}+m_{p01}}}{m_{sb} + m_{o1}}$$

$$\varepsilon_o = \frac{m_{sb}}{m_{kb} + m_{po1}}$$

$$\lambda_o = \frac{m_{o1}}{mk_b + m_{p1o}}$$

For equivalent zero th stage

$$[I_{sp}]_{avg} = \frac{[I_{sp}]_b m_{pb} + [I_{sp}]_c m_{p1o}}{m_{pb} + m_{p1o}}$$

$$[c_{sp}]_o = [I_{sp}]_{avg} \times g$$

Orbital velocity

$$v_{orbital} = \sqrt{GM\left[\frac{2}{rp} - \frac{1}{a}\right]}$$

a =semi major axis

r_p =payload fairing radial location (focal point)

GM =Earth gravitational parameter

Velocity gain due to earth rotation

A_0= launch azimuth ; i=orbital inclination

Ø= latitude angle of launch site ω_e = earth angular velocity

$$A_o = \sin^{-1}\left[\frac{\cos i}{\cos \delta_o}\right]$$

Linear velocity due to earth rotation at launch site, r_o= radius of earth

$V_{r,\emptyset} = \omega_e r_o \cos \delta_o$

Required velocity ΔV_{req} =

$$\sqrt{\left[v_{orbital} \sin A_o - V_{(r,\emptyset)}\right]^2 + [V_{orbit} cos A_o]^2}$$

$\Delta V_{gain} = \Delta V_{orbit} - \Delta V_{req}$

Trajectory losses

Gravity losses arise because part of the rockets energy is wasted in holding it aloft and in pushing it against the relentless pull of earth's gravity. The gravity loss equation

$$\int_{t_o}^{t_f} gsin\gamma\ dt$$

γ =flight path angle

G= local acceleration due to gravity

The drag loss is caused by the friction between the rocket and the ambient air. It can be expressed as

$$\int_{t_o}^{t_f} \frac{D}{m} dt$$

D= Drag force acting on vehicle at that instant

m =mass of vehicle at that instant

The steering loss arises because the instantaneous thrust vector is not always parallel to the current velocity vector. This small mismatch is necessary otherwise, we could not steer the rocket along an optimal trajectory as it flies into space. The steering loss can be evaluated from the following expression:

$$\int_{t_f}^{t_f} \frac{F}{m}(1 - cos \propto)dt$$

Where F is the current thrust of the rocket m is the current mass, and α is the steering angle, the angle between the thrust vector and the current velocity vector.

In order calculate these losses we need have trajectory of vehicle according to the requirement

From study of loftus and texeria approximate equations obtained for gravity and drag losses with thrust to weight ratio by digitalizing a graph

Velocity loss due to gravity [m/s]

V_g = 81.006*TW^2 – 667.62*TW + 1505.4;

Velocity loss due to aerodynamic drag [m/s]

V_d = -32.962*TW^2 + 258.86*TW – 226.57;

$\Delta v_{mission} = \Delta v_{orbital} + \Delta v_{gravity} + \Delta v_{drag} + \Delta v_{propolsive} - \Delta v_{gain} + \Delta v_{performance\ margin}$

Total mass

$m_o = \left[\sum_{k=1}^{n}(m_s, k + m_p, k)\right]$ +mpl.

$and\ \frac{m_o}{mpl} =$

$\frac{m_{o,1}}{mpl,2} \times \frac{mpl,1}{mpl,2} \times \frac{mpl,2}{mpl,3} \times \cdots\cdots \times \frac{mpl_{n-1}}{mpl}$

(GLOM) $m_o = \left[\prod_{k=1}^{n} \frac{m_{o},k}{mpl,k}\right]$ mpl

$m_o = \left[\prod_{k=1}^{n} \frac{(1-\varepsilon_k)\wedge_k}{(1-\varepsilon_{k\wedge_k})}\right]$mpl

By using LaGrange's multipliers we find minimum (GLOM) M_O .

f^*=ln$[f]$ + P g

$f^* = \Delta v_{vehicle} = \sum_{k=1}^{n} c_j, k \ln(\wedge k)$

$\ln\left[\prod_{k=1}^{n}\left[\frac{(1-\varepsilon_k)\Delta_k}{(1-\varepsilon_{k\Delta_k})}\right] mpl\right]$ + P$\left[\sum_{k=1}^{n} c_j, k \ln \Delta k\right]$ - $\Delta v_{vehicle}]$

P = Lagrange multiplier

$f^* = \sum_{k=1}^{n} \ln\left[\frac{(1-\varepsilon_k)\wedge k}{(1-\varepsilon_{k\wedge_k})}\right] + \ln[mpl] + \mathrm{P}\ [\sum_{k=1}^{n} cj, k \ln[\wedge k] - \Delta v_{mission}]$

$f^* = \sum_{k=1}^{n}\left[\ln\left[\frac{(1-\varepsilon_k)\wedge_k}{1-\varepsilon_{k\wedge_k}}\right] + Pcj_k \ln[\wedge k]\right] + \ln[mpl] - P\ \Delta v_{mission}$

For stationary point

$\frac{\partial f^*}{\partial \wedge_k} = 0 \quad \frac{(1-\varepsilon_k)}{\wedge_k(1-\varepsilon_k)} + \frac{\varepsilon_k}{(1-\varepsilon_{k\wedge_k})} + \mathrm{P}\,\frac{cj_k}{\wedge k} = 0$

$[1 + Pcj_k] = \frac{\varepsilon_k}{\left[\varepsilon_k - \frac{1}{\wedge k}\right]}$

$\wedge k = \frac{1+Pcj_k}{P\varepsilon_k cj_k}$

(constrained function)

By Newton raphson (technique) method y=f(P)

P will be the root of equation function y

$\Delta v = \sum_{k=1}^{n} cj_k \ln\left[\frac{1+Pcj_k}{P\varepsilon_k cj_k}\right]$

Let Y= $\Delta v - \sum_{k=1}^{n} cj_k \ln\left[\frac{1+Pcj_k}{P\varepsilon_k cj_k}\right] = 0$

For initial guess of root

For $\ln\left[\frac{(1-\varepsilon_k)\wedge k}{(1-\varepsilon_k\wedge_k)}\right]$

$\varepsilon_k \wedge_k < 1$; and $1 < \wedge_k < \frac{1}{\varepsilon_k}$

$1 < \frac{1+Pcj_k}{P\,cj_k\varepsilon_k} < \frac{1}{\varepsilon_k}$ (since p may or may not be positive)

P< 0 and And => $\frac{1+Pcj_k}{Pcj_{k\varepsilon_k}} > 1$

⇨ $P < \frac{-1}{cj_k(1-\varepsilon_k)}$

By Newton raphson method Since y = $f(P)$

Tangent equation at point (y_1, P)

y-$y_1 = f^1(P)(\lambda - \lambda_1)$

P_1 be any trail value it meet P axis at y=0

P =$P_1 - \frac{f(P_1)}{f^1(P_1)}$

Then for all stages $\wedge$k will obtain

$\wedge k = \frac{1+Pcj_k}{Pcj_k\varepsilon_k}$

Since $\lambda_k = \frac{(1-\wedge k\varepsilon k)}{(\wedge k-1)}$

$for\ the\ last\ stage\ payload\ is\ the\ mission\ payload\ \lambda_n = \frac{mpl}{m_{k,n}},$

$m_{k,n} = \frac{mpl}{\lambda_n}$

$\lambda_n = \frac{m_{k,n}}{m_s n+mpl}$

From this structural mass of nth stage is obtained.
By using structural ratio we can get propellant mass of the nth stage

$$\varepsilon_n = \frac{ms_n}{ms_n + mp_n}$$

From the above values payload mass for n-1 stage(i,e. initial mass of the nth stage) can be obtained by using this we can get stage mass ,structural mass and propellant mass for n-1[th] stage ,

$$\lambda_{n-1} = \frac{m_o, n}{m_k, n-1}$$

$$mo_{n-1} = \frac{m_o, n}{\lambda_{n-1}}$$

Similarly In this manner we can get the initial mass of the first stage it is the optimized gross lift off mass for this mission

IV. RESULTS AND DISCUSSIONS

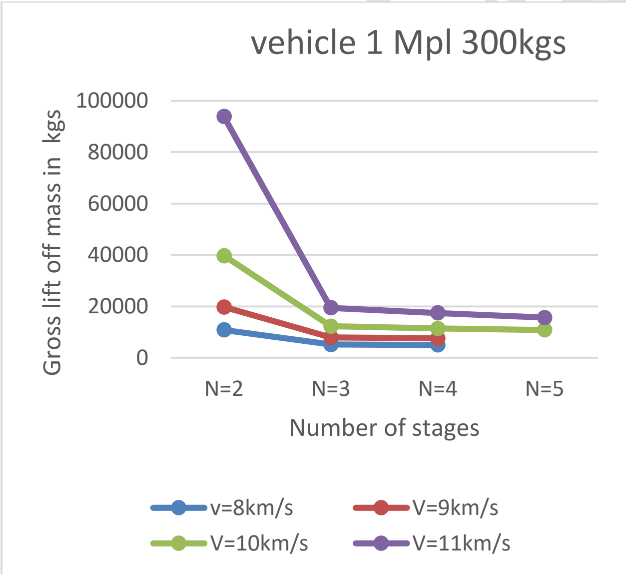
Vehicle 1 Ariane:

For payload mass 300 kgs

	N=2	N=3	N=4	N=5
v=8km/s	10799.7	5119.18	4962.73	
V=9km/s	19731.8	7858.58	7482.37	
V=10km/s	39659.1	12243.2	11368.7	10800.82
V=11km/s	93857.7	19422.3	17427.2	15599

Vehicle 2 Proton M Britz

	N=2	N=3	N=4	N=5
V=8km/s	5753.27	5219.84	5116.71	
V=9km/s	8862.14	7671.87	7438.24	
V=10km/s	14049.3	11384.4	10876.2	
V=11km/s	23159	17084	16006	15023



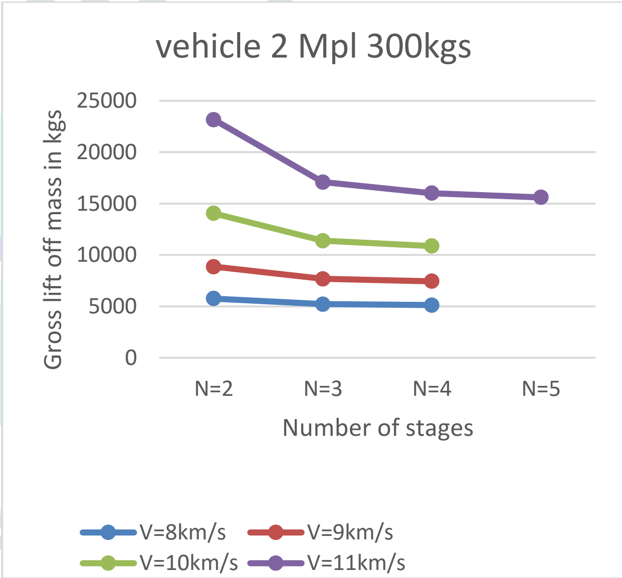


Fig 1: GLOM vs stages at 300kgs payload for various burnout velocity for both vehicles

FOR PAYLOAD MASS 300KGS:
To attain velocity 8km/s,
For vehicle 1 with the above specifications, the gross lift of mass for 2[nd] stage is 10799.7 and for 3[rd] stage is 5119.18(where it decreased to 50%), for 4rth stage the gross lift off mass is 4962.3(where the decrease is less than 3%) So 3[rd] stage gives the optimal solution for gross lift off mass.
To attain velocity 9km/s,
For vehicle 1 with the above specifications, the gross lift of mass for 2[nd] stage is 19731.8 and for 3[rd]stage is 7858.58 (where it decreased to 60%), for 4rth stage the gross lift off mass is 7482.37(where the decrease is less than 3%) So 3[rd] stage gives the optimal solution for gross lift off mass.
To attain velocity 10km/s,
For vehicle 1 with the above specifications, the gross lift of mass for 2[nd] stage is 39659.1 and for 3[rd] stage is 12243.2(where it decreased to 50%), for 4rth stage the gross lift off mass is 11368.7 (the decrease is 8%) and for 5[th] stage the gross lift off mass is 10800.5(the decrease is less than 5%).So 4rth stage gives the optimal solution for gross lift off mass.
To attain velocity 11km/s,
For vehicle 1 with the above specifications, the gross lift of mass for 2[nd] stage is 93857.7and for 3[rd] stage is 19422.3(where it decreased to 50%), for 4rth stage the gross lift off mass is 17427.2 (the decrease is 8%) and for 5[th] stage the gross lift off mass is 15599(the decrease is less than 5%). So 4rth stage gives the optimal solution for gross lift off mass.

FOR VEHICLE 2 AND FOR PAYLOAD MASS 300KGS:

To attain velocity 8km/s,
with the above specifications, the gross lift of mass for 2nd stage is 5753.27 and for third stage is 5219.84(where it decreased to 10%), for 4rth stage the gross lift off mass is 5116.71(where the decrease is less than 2%) So 3rd stage gives the optimal solution for gross lift off mass.
To attain velocity 9km/s,
with the above specifications, the gross lift of mass for 2nd stage is 8862.14and for third stage is 7671.87 (where it decreased to 14%), for 4rth stage the gross lift off mass is7438.4 (where the decrease is less than 4%) So 3rd stage gives the optimal solution for gross lift off mass.
To attain velocity 10km/s,
with the above specifications, the gross lift of mass for 2nd stage is 21095.3 and for third stage is 19139.1(where it decreased to 10%), for 4rth stage the gross lift off mass is 10876.2 (where the decrease is less than 2%) So 3rd stage gives the optimal solution for gross lift off mass.
To attain velocity 11km/s,
For vehicle 1 with the above specifications, the gross lift of mass for 2nd stage is 23159 and for 3rd stage is 17084 (where it decreased to 27%), for 4rth stage the gross lift off mass is 16006 (the decrease is 7%) and for 5th stage the gross lift off mass is 15023 (the decrease is less than 3%). So 4rth stage gives the optimal solution for gross lift off mass
.
For same mission parameters on comparing both launch vehicles for 2 stages optimum GLOM for vehichle1 is more than vehichle2 because its having lesser specific impulse and more structural ratio than vehicle 2 for this 2 stages
And for 3 stages the GLOM difference between them is less because the 3rdstage of vehicle 1 having I_{sp}=443 sec which is more than I_{sp} of 2nd vehicle (i.e. I_{sp}=325s) even though structural ratio of 2nd vehicle is lesser than 1st vehicle but specific Impulse plays a prominent role in optimal GLOM .due to this in 4th and 5th stages the optimal GLOM values varies for both vehichles and the change is less when compared to the change occurs in between them at 2 stages

Similarly comparison made between 2 launch vehicles at payload mass 700kg and 900kg by varying number of stages and burnout velocities

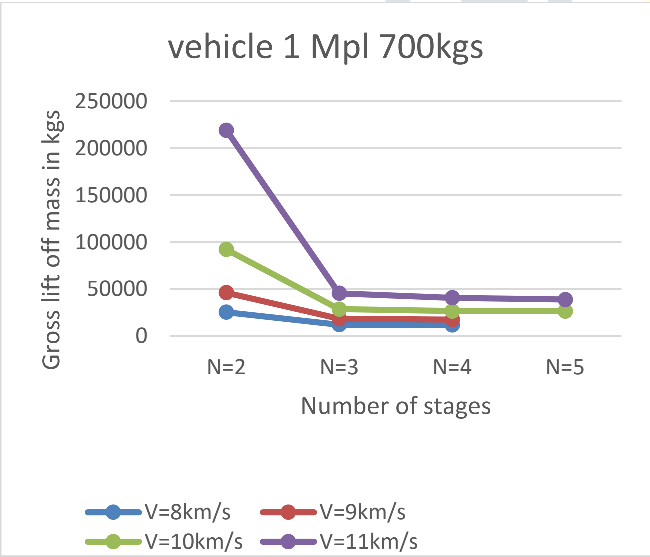
FOR PAYLOAD MASS=700KGS

VEHICHLE1

	N=2	N=3	N=4	N=5
V=8km/s	25199.4	11944.8	11579.7	
V=9km/s	46040.9	18336.7	17458.9	
V=10km/s	92097.8	28567.5	26526.9	26400.5
V=11km/s	219001	45318.8	40663.4	38731.5

VEHICHLE2

	N=2	N=3	N=4	N=5
V=8km/s	13424.3	12179.6	11939	
V=9km/s	20678.3	17901	17355.9	
V=10km/s	32781.7	26563.6	25377	
V=11km/s	54037.8	39863	37346	36110.2



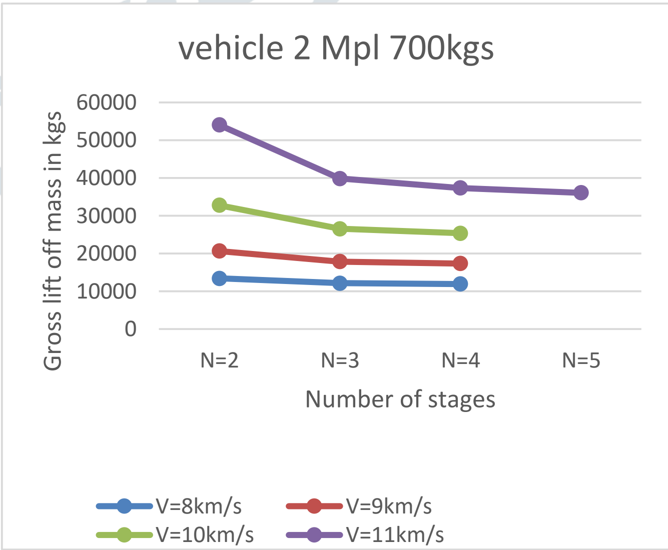


Fig 2: GLOM vs stages at 700kgs payload for various burnout velocities.

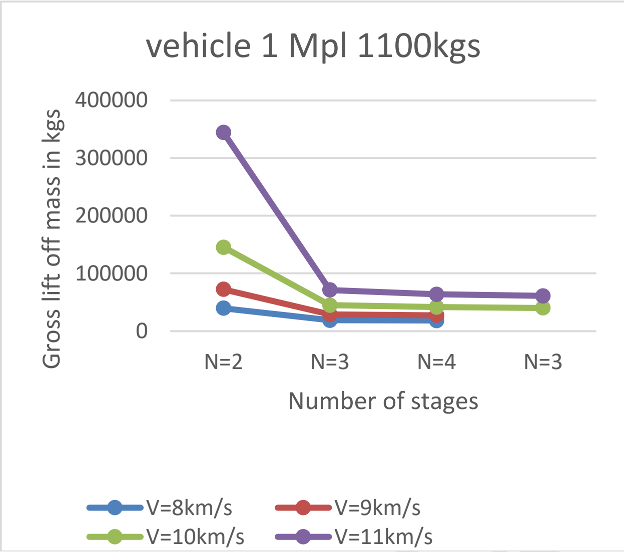
FOR PAYLOAD MASS=1100KGS,

	N=2	N=3	N=4	N=5
V=8km/s	39599.1	18770.3	18196.7	
V=9km/s	72349	28814.8	27435.4	
V=10km/s	145417	44891.2	41685.2	40266.5
V=11km/s	344145	71215.2	63899.7	60863

VEHICHLE 1

	N=2	N=3	N=4	N=5
V=8km/s	21095.3	19139.1	18761.3	
V=9km/s	32494.5	28130.2	27273.5	
V=10km/s	51514.1	41742.8	39879.2	
V=11km/s	84916.6	62642.4	58691.6	56744.6

VEHICHLE 2



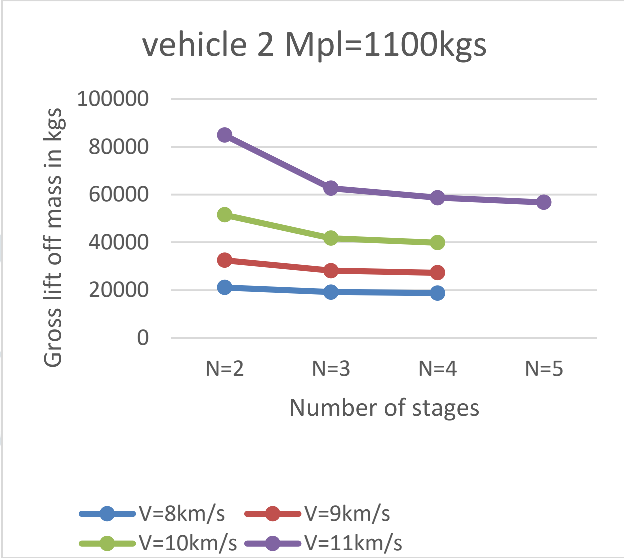


Fig3:GLOM vs stages at 1100kgs payload for various burnout velocity.

V. CONCLUSION

For the optimization of gross lift mass of space launch vehicles, a mat lab script written so that the optimal vehicle configuration in conceptual design phase is obtained and the variation for gross lift mass with other parameters is plotted for the requirement i.e, during a predesigned phase of a space launch vehicle which is carrying a certain amount of payload from one particular position on earth surface to a desired orbital injection position, a launch vehicle is designed such that the gross lift of mass is optimized. We also took into account about the number of stages that are to be kept in order to get optimized solution. As lift of mass is minimized cost also minimized automatically. we used Lagrange multiplier and Newton raphson method for getting the optimization solution.npt only optimization but also the mass is distributed in each stage such that we got the optimal lift of mass and how the lift of mass varies with velocity ,payload mass and number of stages and by this we got the optimum solution

REFERENCES

[1] H.H.Hall (1958) ON THE OPTIMIZATION OF MULTISTAGE ROCKETS, Journal Of Jet Propulsion,Vol.28,No.7,pp.463-465

[2] Ezgi Civek-Coskun (2014) MULTISTAGE LAUNCH VEHICLE DESIGN WITH THRUST PROFILE AND TRAJECTORY OPTIMIZATION.

[3] David N. Burghes (2006) OPTIMUM STAGING OF MULTISTAGE ROCKETS, International Journal Of Mathematical Education In Science And Technology

[4] www.astronautix.com/

[5] https://www.mathworks.com/products/matlab.html