Comparison of Reliability of a System under the Exponential and Hypoexponential Failure Distribution

S.Kalaiarasi[1], E.Venkatesan[2], G.Venkatesan[3], A.Merceline Anita[4]
[1,2,3,4] PG and Research Department of Mathematics,
Sacred Heart College, Tirupattur - 635601,
Vellore, Tamil Nadu, India.

ABSTRACT. In this paper we derived the reliability of a system with two workstations and one file server with hypoexponential failure distribution and we study the comparison of this reliability with exponential failure distribution. Numerical results for both distributions are inserted to verify the results.
Keywords: Reliability, Fileserver, Workstation, Hypoexponential distribution, Exponential distribution, failure rate.

1. INTRODUCTION

The hypoexponential distribution is continuous probability distribution as the sum of independent random variables of exponential distribution with different parameters, this kind of sum of random variables is also known as convolution of exponential random variable. Akkouchi [9] used analytical method in convolution to find hypoexponential distribution. In many process
in nature can be divided into sequential phases. If the time the process spends in each phase in independent and exponentially distributed, then it can be shown that the overall time is hypoexponentially distributed[2]. The hypoexponential distriution plays on important role of modeling in many domain such as in communication system and computer science [2], queuing theory and markov processes[10].

In this paper we derived the reliability of a system with two workstations and one file server are exponential and hypoexponentially distributed. Then we compare the reliability for the both failure distributions.

This paper will set definition in section 2, Reliability of a system using exponential and hypoexponential distribution in section 3, and Comparison for Numerical results in section 4, finally we conclude the results in section 5

2. SOME BASIC DEFINITION

Definition 2.1 (Failure Rate). Failure rate in the frequency with which an engineered system or component fails expressed in failure per unit of time. It is often denoted by the Greek letter β and is highly used in reliability engineering.

Definition 2.2 (Server). A server is an application or device that performs service for connected clients as part of client server architecture.
Example: webserver, application server etc.

Definition 2.3 (work station). A computer that is used to power application such as graphic art, 3-D design, video editing, or other CPU/RAM intensive software .

Example: Video and Audio station.

Definition 2.4 (Hypoexponential distribution). [2] Many process in nature can be divided into sequential phases. It the time the process spends in each phase in independent and exponentially distributed, then it can be shown that the overall time is hypoexponential distributed. A two-stage hypoexponential random variable x with parameter β_1 and β_2 ($\beta_1 \neq \beta_2$)

$$f(t) = \frac{\beta_1\beta_2}{\beta_2 - \beta_1}(e^{-\beta_1 t} - e^{-\beta_2 t}), t > 0 \qquad (1)$$

3. RELIABILITY OF A SYSTEM USING EXPONENTIAL AND HYPOEXPONENTIAL DISTRIBUTION

In this section we consider a system consisting of p work stations and q file servers. The network connecting these devices is assumed to be fault free.

The system is considered to be operational so long as at least k workstations and l file servers are operational. $R_w(t)$ and $R_f(t)$ denote the reliability of a single workstation and file server respectively.

Assuming that all devices fail independently of each other, system reliability is

$$R(t) = \sum_{j=k}^{p}\binom{p}{j}[R_w(t)]^j[1-R_w(t)]^{p-j}\sum_{j=l}^{q}\binom{q}{j}[R_f(t)]^j[1-R_f(t)]^{q-j} \qquad (2)$$

The times to failure for the WFS are exponentially distributed, then find the R(t) and MTTF for the system when p = 2 for workstations and q = 1 for the file server and k=l=1, that is the system is up so long as a work station and file server are up.

Let β_w and β_f represent the failure rates of each work station and file server.

The system reliability is

$$R(t) = 2e^{-(\beta_w+\beta_f)t} - e^{-(2\beta_w+\beta_f)t} \qquad (3)$$

Hence the mean time to failure for the given system is

$$MTTF = \left[\frac{2}{(\beta_w + \beta_f)} - \frac{1}{(2\beta_w + \beta_f)}\right] \qquad (4)$$

Next, we consider the times to failure for the WFS are hypoexponential distributed. We find the R(t) and MTTF for the system when p = 2 for work station and q = 1 for file server and k = l = 1. Let β_{1w}, β_{2w}, β_{1f} and β_{2f} represent the failure rates of each work station and file server respectively.

Then the system reliability is

$$R(t)=\frac{\beta_{2w}\beta_{2f}}{(\beta_{2w}-\beta_{1w})(\beta_{2f}-\beta_{1f})}\left(2e^{-(\beta_{1w}+\beta_{1f})t}-e^{-(2\beta_{1w}+\beta_{1f})t}\right)-\frac{\beta_{2w}\beta_{1f}}{(\beta_{2w}-\beta_{1w})(\beta_{2f}-\beta_{1f})}\left(2e^{-(\beta_{1w}+\beta_{2f})t}-e^{-(2\beta_{1w}+\beta_{2f})t}\right)$$
$$-\frac{\beta_{1w}\beta_{2f}}{(\beta_{2w}-\beta_{1w})(\beta_{2f}-\beta_{1f})}\left(2e^{-(\beta_{2w}+\beta_{1f})t}-e^{-(2\beta_{2w}+\beta_{1f})t}\right)$$
$$+\frac{\beta_{1w}\beta_{1f}}{(\beta_{2w}-\beta_{1w})(\beta_{2f}-\beta_{1f})}\left(2e^{-(\beta_{2w}+\beta_{2f})t}-e^{-(2\beta_{2w}+\beta_{2f})t}\right) \quad (5)$$

Hence the mean time to failure for the given system

$$MTTF=\frac{\beta_{2w}\beta_{2f}}{(\beta_{2w}-\beta_{1w})(\beta_{2f}-\beta_{1f})}\left(\frac{2}{\beta_{1w}+\beta_{1f}}-\frac{1}{2\beta_{1w}+\beta_{1f}}\right)$$
$$-\frac{\beta_{2w}\beta_{1f}}{(\beta_{2w}-\beta_{1w})(\beta_{2f}-\beta_{1f})}\left(\frac{2}{\beta_{1w}+\beta_{2f}}-\frac{1}{2\beta_{1w}+\beta_{2f}}\right)$$
$$-\frac{\beta_{1w}\beta_{2f}}{(\beta_{2w}-\beta_{1w})(\beta_{2f}-\beta_{1f})}\left(\frac{2}{\beta_{2w}+\beta_{1f}}-\frac{1}{2\beta_{2w}+\beta_{1f}}\right)$$
$$+\frac{\beta_{1w}\beta_{1f}}{(\beta_{2w}-\beta_{1w})(\beta_{2f}-\beta_{1f})}\left(\frac{2}{\beta_{2w}+\beta_{2f}}-\frac{1}{2\beta_{2w}+\beta_{2f}}\right) \quad (6)$$

4. COMPARISON FOR NUMERICAL RESULTS

In this section, we present numerical experiment to reliability of a system with exponential and hypoexponential failure distribution. The results for various performance measures are shown by developing a program in 'MATLAB' software.

Fig.1 display the trends of system reliability against time for different values of failure rates $\beta_w = 0.3, \beta_f = 0.4$ in (3)

TABLE 1

T	0	1	2	3	4	5	6	7	8
R	1	0.6253	0.3529	0.1951	0.1033	0.0537	0.0275	0.0140	0.0071

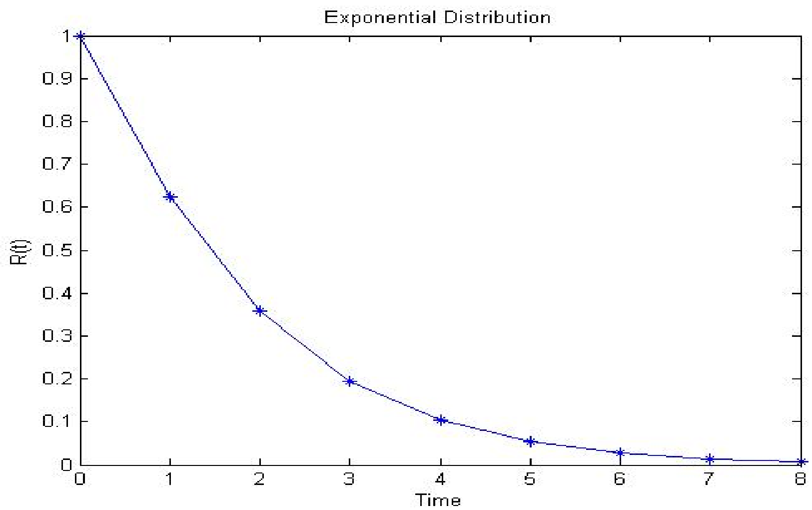

FIGURE 1. System Reliability Vs Time

Fig.2 display the trends of system reliability against time for different values of failure rates $\beta_{1w} = 0.1, \beta_{2w} = 0.2, \beta_{1f} = 0.3, \beta_{2f} = 0.4$ in (5)

TABLE 2

T	0	1	2	3	4	5	6	7	8
R	1.0000	2.8756	2.7440	2.1595	1.5867	1.1330	0.7991	0.5604	0.3918

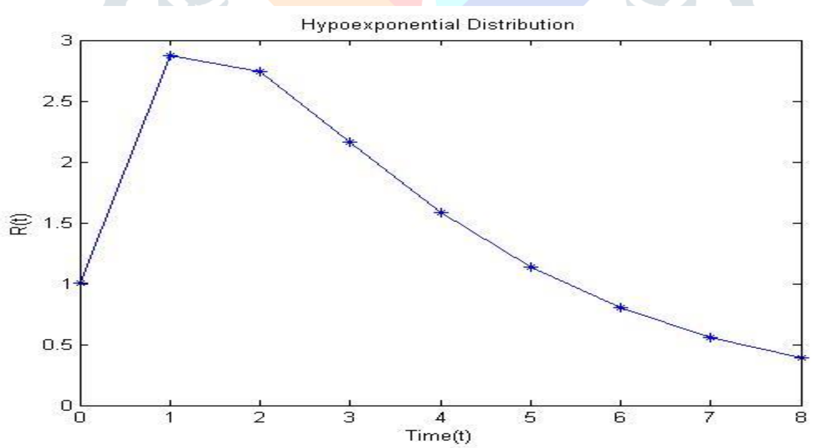

FIGURE 2. System Reliability Vs Time

5. CONCLUSION

In the present study, we have concluded that the system Reliability decreases when time increases by using exponential failure distribution. System Reliability is first increase after it decreases when time increases by hypoexponential failure distribution.

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