Solution of Abel's Integral Equation Using Mahgoub Transform Method

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ABSTRACT: Abel's integral equation is an important singular integral equation and generally appears in many branches of sciences such as atomic scattering, mechanics, radio astronomy, physics, electron emission, X-ray radiography and seismology. In this paper, we use Mahgoub transform method to solve Abel's integral equation and some numerical applications in application section are given to demonstrate the effectiveness of Mahgoub transform method to solve Abel's integral equation.

KEYWORDS: Abel's integral equation, Mahgoub transforms method, Inverse Mahgoub transform, Convolution theorem.

AMS SUBJECT CLASSIFICATION 2010: 44A05, 44A35, 45E10.

I.INTRODUCTION: Niels Henrik Abel [1-2] discussed the motion of particle on smooth curve lying on a vertical plane using Abel's integral equation in mathematical form as

$$f(x) = \int_0^x \frac{1}{\sqrt{x-t}} u(t)\, dt \tag{1}$$

Here the kernel of integral equation, $K(x,t) = \frac{1}{\sqrt{x-t}}$ becomes ∞ at $t = x$, the function $f(x)$ is known function and the function $u(t)$ is unknown function.

Integral transforms are widely used mathematical techniques for solving advanced problems of science and engineering which mathematically express in terms of differential equations, partial differential equations, integral equations, partial integro-differential equations, integro-differential equations etc. Many researchers used different integral transforms (Laplace transform [3-4], Fourier transform [3], Hankel transform [3], Kamal transfom [5, 16-19, 38], Mahgoub transform [8-12, 24], Elzaki transform [6-7, 30-31], Mohand transform [20-22, 36-37, 39-40], Aboodh transform [13-15, 23, 32-35], Sumudu transform [41-42], Wavelet transform [3]) for solving many problems of science and engineering. Aggarwal and others [25-29] discussed the comparative study between these transforms.

The Mahgoub transform of the function $F(t)$ for all $t \geq 0$ is defined as [43]:

$$M\{F(t)\} = v\int_0^\infty F(t)e^{-vt}dt = K(v), 0 < k_1 \leq v \leq k_2, \tag{2}$$

where the operator M is called the Mahgoub transform operator.

The Mahgoub transform of the function $F(t)$ for $t \geq 0$ exist if $F(t)$ is piecewise continuous and of exponential order. These conditions are only sufficient conditions for the existence of Mahgoub transforms of the function $F(t)$. Aggarwal and Sharma [44] gave the solution of Abel's integral equation by Aboodh transform method. Aggarwal and Gupta [45] discussed Sumudu transform for the solution of Abel's integral equation. Application of Kamal transform for solving Abel's integral equation was given by Aggarwal and Sharma [46]. Aggarwal et al. [47] gave a new application of Mohand transform for handling Abel's integral equation.

In this paper, we are giving the solution of Abel's integral equation using Mahgoub transform method and explain all procedure by giving some numerical applications in application section.

II. SOME USEFUL PROPERTIES OF MAHGOUB TRANSFORM:

2.1 Linearity property of Mahgoub transforms [11-12, 24]:

If Mahgoub transform of functions $F_1(t)$ and $F_2(t)$ are $K_1(v)$and $K_2(v)$ respectively then Mahgoub transform of $[aF_1(t) + bF_2(t)]$ is given by $[aK_1(v) + bK_2(v)]$, where a, b are arbitrary constants.

2.2 Change of scale property of Mahgoub transforms:

If Mahgoub transform of function $F(t)$ is $K(v)$ then Mahgoub transform of function $F(at)$is given by $K\left(\frac{v}{a}\right)$.

2.3 Shifting property of Mahgoub transform:

If Mahgoub transform of function $F(t)$ is $K(v)$ then Mahgoub transform of function $e^{at}F(t)$is given by $\left[\frac{v}{v-a}\right]K(v-a)$.

2.4 Mahgoub transform of the derivatives of the function $F(t)$[8-10, 12]:

If $M\{F(t)\} = K(v)$then

a) $M\{F^{'}(t)\} = vK(v) - vF(0)$
b) $M\{F^{''}(t)\} = v^2K(v) - v^2F(0) - vF'(0)$
c) $M\{F^{(n)}(t)\} = v^nK(v) - v^nF(0) - v^{n-1}F'(0) - \cdots \ldots - vF^{(n-1)}(0)$.

2.5 Convolution theorem for Mahgoub transforms [9-11, 24]:

If Mahgoub transform of functions $F_1(t)$ and $F_2(t)$are $K_1(v)$and $K_2(v)$ respectively then Mahgoub transform of their convolution $F_1(t) * F_2(t)$ is given by $M\{F_1(t) * F_2(t)\} = \frac{1}{v}M\{F_1(t)\}M\{F_2(t)\}$

$\Rightarrow M\{F_1(t) * F_2(t)\} = \frac{1}{v}\, K_1(v)K_2(v)$, where $F_1(t) * F_2(t)$ is defined by

$$F_1(t) * F_2(t) = \int_0^t F_1(t-x)\, F_2(x)dx = \int_0^t F_1(x)\, F_2(t-x)dx$$

III. MAHGOUB TRANSFORM OF FREQUENTLY ENCOUNTERED FUNCTIONS [8-12, 24]:

Table: 1

S.N.	$F(t)$	$M\{F(t)\} = K(v)$
1.	1	1
2.	t	$\frac{1}{v}$
3.	t^2	$\frac{2!}{v^2}$
4.	$t^n, n \in N$	$\frac{n!}{v^n}$
5.	$t^n, n > -1$	$\frac{\Gamma(n+1)}{v^n}$
6.	e^{at}	$\frac{v}{v-a}$
7.	$sinat$	$\frac{av}{v^2+a^2}$
8.	$cosat$	$\frac{v^2}{v^2+a^2}$
9.	$sinhat$	$\frac{av}{v^2-a^2}$
10.	$coshat$	$\frac{v^2}{v^2-a^2}$
11	$J_0(t)$	$\frac{v}{\sqrt{(1+v^2)}}$

12	$J_1(t)$	$v-\frac{v^2}{\sqrt{(1+v^2)}}$

IV. INVERSE MAHGOUB TRANSFORM [8-12, 24]:

If $K(v)$ is the Mahgoub transforms of $F(t)$ then $F(t)$ is called the inverse Mahgoub transform of $K(v)$and in mathematical terms, it can be expressed as $F(t)=M^{-1}\{K(v)\}$, where M^{-1} is an operator and it is called as inverse Mahgoub transform operator.

V. LINEARITY PROPERTY OF INVERSE MAHGOUB TRANSFORMS:

If $M^{-1}\{H(v)\}=F(t)$ and $M^{-1}\{I(v)\}=G(t)$ then $M^{-1}\{aH(v)+bI(v)\}=aM^{-1}\{H(v)\}+bM^{-1}\{I(v)\}$

$\Rightarrow M^{-1}\{aH(v)+bI(v)\}=aF(t)+bG(t)$, where a,b are arbitrary constants.

VI. INVERSE MAHGOUB TRANSFORM OF FREQUENTLY ENCOUNTERED FUNCTIONS [8-12, 24]:

Table: 2

S.N.	$T(v)$	$F(t)=M^{-1}\{K(v)\}$
1.	1	1
2.	$\frac{1}{v}$	t
3.	$\frac{1}{v^2}$	$\frac{t^2}{2!}$
4.	$\frac{1}{v^n}, n\in N$	$\frac{t^n}{n!}$
5.	$\frac{1}{v^n}, n>-1$	$\frac{t^n}{\Gamma(n+1)}$
6.	$\frac{v}{v-a}$	e^{at}
7.	$\frac{v}{v^2+a^2}$	$\frac{sinat}{a}$
8.	$\frac{v^2}{v^2+a^2}$	$cosat$
9.	$\frac{v}{v^2-a^2}$	$\frac{sinhat}{a}$
10.	$\frac{v^2}{v^2-a^2}$	$coshat$
11.	$\frac{v}{\sqrt{(1+v^2)}}$	$J_0(t)$
12.	$v-\frac{v^2}{\sqrt{(1+v^2)}}$	$J_1(t)$

VII. MAHGOUB TRANSFORM METHOD FOR SOLVING ABEL'S INTEGRAL EQUATION: In this section, we present Mahgoub transform method for the solution of Abel's integral equation.

Taking Mahgoub transform of both sides of (1), we have

$$M\{f(x)\} = M\left\{\int_0^x \frac{1}{\sqrt{x-t}} u(t)\, dt\right\}$$

$$\Rightarrow M\{f(x)\} = M\{x^{-1/2} * u(x)\} \quad (3)$$

Applying convolution theorem of Mahgoub transform in (3), we have

$$M\{f(x)\} = \frac{1}{v} M\{x^{-1/2}\} A\{u(x)\}$$

$$\Rightarrow M\{f(x)\} = \frac{1}{v}\left(\sqrt{\pi} v^{1/2}\right) M\{u(x)\}$$

$$\Rightarrow M\{u(x)\} = \frac{v^{1/2}}{\sqrt{\pi}} M\{f(x)\}$$

$$\Rightarrow M\{u(x)\} = \frac{v}{\pi}\left[\frac{1}{v}\left(\sqrt{\pi} v^{1/2}\right) M\{f(x)\}\right]$$

$$\Rightarrow M\{u(x)\} = \frac{v}{\pi}\left[\frac{1}{v} M\{x^{-1/2}\} M\{f(x)\}\right]$$

$$\Rightarrow M\{u(x)\} = \frac{v}{\pi} M\{x^{-1/2} * f(x)\}$$

$$\Rightarrow M\{u(x)\} = \frac{v}{\pi}\left[M\left\{\int_0^x \frac{1}{\sqrt{x-t}} f(t)\, dt\right\}\right]$$

$$\Rightarrow M\{u(x)\} = \frac{v}{\pi} M\{F(x)\} \quad (4)$$

where $F(x) = \int_0^x \frac{1}{\sqrt{x-t}} f(t)\, dt$ (5)

Now applying the property, Mahgoub transform of derivative of the function, on (5), we have

$$M\{F'(x)\} = vM\{F(x)\} - vF(0)$$

$$\Rightarrow M\{F'(x)\} = vM\{F(x)\}$$

$$\Rightarrow M\{F(x)\} = \frac{1}{v} M\{F'(x)\} \quad (6)$$

Now from (4) and (6), we have

$$M\{u(x)\} = \frac{1}{\pi} M\{F'(x)\} \quad (7)$$

Applying inverse Mahgoub transform on both sides of (7), we get

$$u(x) = \frac{1}{\pi} F'(x) = \frac{1}{\pi} \frac{d}{dx} F(x) \quad (8)$$

Using (5) in (8), we have

$$u(x) = \frac{1}{\pi}\left[\frac{d}{dx}\int_0^x \frac{1}{\sqrt{x-t}} f(t)\, dt\right] \quad (9)$$

which is the required solution of (1).

VIII. APPLICATIONS: In this section, we present some numerical applications to demonstrate the effectiveness of Mahgoub transform method to solve Abel's integral equation**.**

8.1 Consider the Abel's integral equation:

$$x = \int_0^x \frac{1}{\sqrt{x-t}} u(t)\, dt \quad (10)$$

Taking Mahgoub transform of both sides of (10), we have

$$M\{x\} = M\left\{\int_0^x \frac{1}{\sqrt{x-t}} u(t)\, dt\right\}$$

$$\Rightarrow \frac{1}{v} = M\{x^{-1/2} * u(x)\} \quad (11)$$

Applying convolution theorem of Mahgoub transform in (11), we have

$$\frac{1}{v} = \frac{1}{v} M\{x^{-1/2}\} M\{u(x)\}$$

$$\Rightarrow \frac{1}{v} = \frac{1}{v}\left(\sqrt{\pi} v^{1/2}\right) M\{u(x)\}$$

$$\Rightarrow M\{u(x)\} = \frac{v^{-1/2}}{\sqrt{\pi}} \quad (12)$$

Applying inverse Mahgoub transform on both sides of (12), we get

$$u(x) = \frac{1}{\sqrt{\pi}} M^{-1}\{v^{-1/2}\}$$

$$\Rightarrow u(x) = \frac{2}{\pi} x^{1/2} \quad (13)$$

which is the required solution of (10).

8.2 Consider the Abel's integral equation:

$$1 + x + x^2 = \int_0^x \frac{1}{\sqrt{x-t}} u(t)\, dt \quad (14)$$

Taking Mahgoub transform of both sides of (14), we have

$$M\{1\} + M\{x\} + M\{x^2\} = M\left\{\int_0^x \frac{1}{\sqrt{x-t}} u(t)\, dt\right\}$$

$$\Rightarrow 1 + \frac{1}{v} + \frac{2}{v^2} = M\{x^{-1/2} * u(x)\} \quad (15)$$

Applying convolution theorem of Mahgoub transform in (15), we have

$$1 + \frac{1}{v} + \frac{2}{v^2} = \frac{1}{v} M\{x^{-1/2}\} M\{u(x)\}$$

$$\Rightarrow 1 + \frac{1}{v} + \frac{2}{v^2} = \frac{1}{v}\left(\sqrt{\pi} v^{1/2}\right) M\{u(x)\}$$

$$\Rightarrow M\{u(x)\} = \frac{1}{\sqrt{\pi}}\left[v^{1/2} + v^{-1/2} + 2v^{-3/2}\right] \quad (16)$$

Applying inverse Mahgoub transform on both sides of (16), we get

$$u(x) = \frac{1}{\sqrt{\pi}} M^{-1}\{v^{1/2} + v^{-1/2} + 2v^{-3/2}\}$$

$$\Rightarrow u(x) = \frac{1}{\sqrt{\pi}}\left[M^{-1}\{v^{1/2}\} + M^{-1}\{v^{-1/2}\} + 2M^{-1}\{v^{-3/2}\}\right]$$

$$\Rightarrow u(x) = \frac{1}{\pi}\left[x^{-1/2} + 2x^{1/2} + \frac{8}{3}x^{3/2}\right] \quad (17)$$

which is the required solution of (14).

8.3 Consider the Abel's integral equation:

$$3x^2 = \int_0^x \frac{1}{\sqrt{x-t}}u(t)\,dt \quad (18)$$

Taking Mahgoub transform of both sides of (18), we have

$$3M\{x^2\} = M\left\{\int_0^x \frac{1}{\sqrt{x-t}}u(t)\,dt\right\}$$

$$\Rightarrow \frac{6}{v^2} = M\{x^{-1/2} * u(x)\} \quad (19)$$

Applying convolution theorem of Mahgoub transform in (19), we have

$$\frac{6}{v^2} = \frac{1}{v}M\{x^{-1/2}\}M\{u(x)\}$$

$$\Rightarrow \frac{6}{v^2} = \frac{1}{v}\left(\sqrt{\pi}v^{1/2}\right)M\{u(x)\}$$

$$\Rightarrow M\{u(x)\} = \frac{6}{\sqrt{\pi}}v^{-3/2} \quad (20)$$

Applying inverse Mahgoub transform on both sides of (20), we get

$$u(x) = \frac{6}{\sqrt{\pi}}M^{-1}\{v^{-3/2}\}$$

$$\Rightarrow u(x) = \frac{8}{\pi}x^{3/2} \quad (21)$$

which is the required solution of (18).

8.4 Consider the Abel's integral equation:

$$\frac{4}{3}x^{3/2} = \int_0^x \frac{1}{\sqrt{x-t}}u(t)\,dt \quad (22)$$

Taking Mahgoub transform of both sides of (22), we have

$$\frac{4}{3}M\{x^{3/2}\} = M\left\{\int_0^x \frac{1}{\sqrt{x-t}}u(t)\,dt\right\}$$

$$\Rightarrow \sqrt{\pi}v^{-3/2} = M\{x^{-1/2} * u(x)\} \quad (23)$$

Applying convolution theorem of Mahgoub transform in (23), we have

$$\sqrt{\pi}v^{-3/2} = \frac{1}{v}M\{x^{-1/2}\}M\{u(x)\}$$

$$\Rightarrow \sqrt{\pi}v^{-3/2} = \frac{1}{v}\left(\sqrt{\pi}v^{1/2}\right)M\{u(x)\}$$

$$\Rightarrow M\{u(x)\} = \frac{1}{v} \quad (24)$$

Applying inverse Mahgoub transform on both sides of (24), we get

$$u(x) = M^{-1}\left\{\frac{1}{v}\right\}$$

$$\Rightarrow u(x) = x \tag{25}$$

which is the required solution of (22).

8.5 Consider the Abel's integral equation:

$$2\sqrt{x} + \frac{8}{3}x^{3/2} = \int_0^x \frac{1}{\sqrt{x-t}} u(t)\, dt \tag{26}$$

Taking Mahgoub transform of both sides of (26), we have

$$2M\{x^{1/2}\} + \frac{8}{3}M\{x^{3/2}\} = M\left\{\int_0^x \frac{1}{\sqrt{x-t}} u(t)\, dt\right\}$$

$$\Rightarrow \sqrt{\pi}v^{-1/2} + 2\sqrt{\pi}v^{-3/2} = M\{x^{-1/2} * u(x)\} \tag{27}$$

Applying convolution theorem of Mahgoub transform in (27), we have

$$\sqrt{\pi}v^{-1/2} + 2\sqrt{\pi}v^{-3/2} = \frac{1}{v}M\{x^{-1/2}\}M\{u(x)\}$$

$$\Rightarrow \sqrt{\pi}v^{-1/2} + 2\sqrt{\pi}v^{-3/2} = \frac{1}{v}\left(\sqrt{\pi}v^{1/2}\right)M\{u(x)\}$$

$$\Rightarrow M\{u(x)\} = 1 + \frac{2}{v} \tag{28}$$

Applying inverse Mahgoub transform on both sides of (28), we get

$$u(x) = M^{-1}\{1\} + 2M^{-1}\left\{\frac{1}{v}\right\}$$

$$\Rightarrow u(x) = 1 + 2x \tag{29}$$

which is the required solution of (26).

8.6 Consider the Abel's integral equation:

$$\frac{3}{8}\pi x^2 = \int_0^x \frac{1}{\sqrt{x-t}} u(t)\, dt \tag{30}$$

Taking Mahgoub transform of both sides of (30), we have

$$\frac{3}{8}\pi M\{x^2\} = M\left\{\int_0^x \frac{1}{\sqrt{x-t}} u(t)\, dt\right\}$$

$$\Rightarrow \frac{3}{4}\left(\frac{\pi}{v^2}\right) = M\{x^{-1/2} * u(x)\} \tag{31}$$

Applying convolution theorem of Mahgoub transform in (31), we have

$$\frac{3}{4}\left(\frac{\pi}{v^2}\right) = \frac{1}{v}M\{x^{-1/2}\}M\{u(x)\}$$

$$\Rightarrow \frac{3}{4}\left(\frac{\pi}{v^2}\right) = \frac{1}{v}\left(\sqrt{\pi}v^{1/2}\right)M\{u(x)\}$$

$$\Rightarrow M\{u(x)\} = \frac{3}{4}\sqrt{\pi}v^{-3/2} \tag{32}$$

Applying inverse Mahgoub transform on both sides of (32), we get

$$u(x) = \frac{3}{4}\sqrt{\pi} M^{-1}\{v^{-3/2}\}$$

$$\Rightarrow u(x) = x^{3/2} \quad (33)$$

which is the required solution of (30).

IX. CONCLUSION: In this paper, we have successfully discussed Mahgoub transform method for the solution of Abel's integral equation. The given numerical applications in the application section explain the complete procedure for the solution of Abel's integral equation using Mahgoub transform method. The results show that Mahgoub transform method is a powerful integral transform method for the solution of Abel's integral equation. In the future, Mahgoub transform method can be used for solving other singular integral equations.

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