ANALYSIS OF FUZZY RETRIAL QUEUE WITH UNRELIABLE SERVER USING PENTAGONAL FUZZY NUMBERS

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Abstract : In this study we propose a procedure to find the various performance measures in terms of crisp values for M/M/1 retrial queuing system with breakdowns, in which the arrival rate, service rate, retrial rate, failure rate and repair rate are Fuzzy numbers. DSW algorithm is used to define membership functions of the performance measures of retrial queuing system with unreliable server. The algorithm is based on the α – cut representation of fuzzy sets in a standard interval analysis. Numerical example is also illustrated to check the validity of the model.

***Index Terms* - Retrial queues, α – cuts, Pentagonal fuzzy number, DSW algorithm.**

I. INTRODUCTION

Retrial queuing models have been widely applied to many practical problems in telecommunication networks, telephone switching systems and computers competing to gain service from a central processing unit. Retrial queues are also used as mathematical models for several computer systems: Packet switching networks, shared bus local area networks operating under the carrier-sense multiple access protocol and collision avoidance star local area networks, etc.

A nonlinear programming approach for a fuzzy queue with an unreliable server was studied by Ashok Kumar[1]. Gautam Choudhury *et al*[2] considered the N-policy for an unreliable server with delaying repair and two phases of service. Retrial queues with breakdown and repair was investigated by Kulkarni, Choi[3]. Fuzzy queue with Erlang service model using DSW algorithm was done by Mohammed Shapique[4]. A.A.Noora *et al*[5] studied discrete time multi server retrial queue with finite population and fuzzy parameters. An M/M/1 retrial queue with unreliable server under fuzzy environment was studied by S. Shanmugasundaram and B.Venkatesh[6]. R. Srinivasan[7] studied Fuzzy queueing model using DSW algorithm. K. Usha Madhuri and K. Chandan[8] studied fuzzy queues with pentagon fuzzy number using α cuts. S. Upadhyaya[9] considered Bernoulli vacation policy for a bulk retrial queue with Fuzzy parameters. V.Visalakshi and V. Suvitha[10] studied the fuzzy queues using pentagonal fuzzy numbers.

Many queuing situations have the feature that customers who find all the servers busy will have to leave the service area and repeat their request after some random time known as retrial time. During trials, the blocked customers join a pool of unsatisfied customers called orbit.

In general queuing systems use perfect or reliable servers. But in real life systems, the server may experience an unpredictable breakdowns. Therefore, queuing models with server breakdowns provide a realistic representation of such systems.

In this paper, we develop an approach that can provide the system characteristics of fuzzy retrial queues with an unreliable server with five fuzzy variables, namely fuzzified exponential arrival rate, service rate, retrial rate, failure rate and repair rate. Through the α-cuts and Zadeh's extension principle, we transform the fuzzy queue with an unreliable server into a family of crisp queues with an unreliable server. As α -varies, the family of crisp queues are then described and solved by using DSW algorithm.

II. PRELIMINARIES

2.1 Definition

Let Z denote a universe of discourse. A fuzzy set $\tilde{A}$ in Z is determined by a membership function mapping elements of a domain space or universe of discourse Z to the unit interval $[0,1]$.

(i.e.) $\tilde{A}=\left\{\left(x,\eta_{\tilde{A}}(x)\right);x\in Z\right\}$

Here $\eta_{\tilde{A}}:Z\rightarrow[0,1]$ is a mapping called the degree of membership function of the fuzzy set $\tilde{A}$ and $\eta_{\tilde{A}}(x)$ is called the membership value of $x\in Z$ of the fuzzy set $\tilde{A}$. Thus, the function value $\eta_{\tilde{A}}(x)$ is termed the grade of membership of x in $\tilde{A}$.

2.2 Definition

A fuzzy set $\tilde{A}$ in the universe of discourse Z is a fuzzy number if and only if it satisfies the following conditions

(i) $Z=\mathcal{R}$;
(ii) $\tilde{A}$ is normal ;
(iii) $\tilde{A}$ is convex ;
(iv) The membership function $\eta_{\tilde{A}}$ is piecewise continuous ;
(v) There exists one and only one x $\in\mathcal{R}$, such that $\eta_{\tilde{A}}(x)=1$.

2.3 Definition

A pentagonal fuzzy number $\tilde{A}(x)$ can be represented by $\tilde{A}(a_1, a_2, a_3, a_4, a_5;\ 1)$ with membership function $\eta_{\tilde{A}}(x)$ given by

$$\eta_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1}, & a_1 \le x \le a_2 \\ \dfrac{x-a_2}{a_3-a_2}, & a_2 \le x \le a_3 \\ 1, & x = a_3 \\ \dfrac{x-a_4}{a_3-a_4}, & a_3 \le x \le a_4 \\ \dfrac{x-a_5}{a_5-a_4}, & a_4 \le x \le a_5 \\ 0, & otherwise. \end{cases}$$

2.4 DEFINITION

The α - cut of a fuzzy number $\tilde{A}$ is defined as $\tilde{A}_\alpha = \{x : \eta_{\tilde{A}}(x) \ge \alpha, x \in Z$ and $\alpha \in [0,1]\}$

III. RETRIAL FUZZY QUEUE MODEL WITH UNRELIABLE SERVER

Consider a queuing system in which customers join the retrial orbit only when they are interrupted by server breakdown. These interrupted customers access the server only when it is operational and idle and repeat service. Retrial customers do not join the normal queue. The fuzzified exponential arrival rate $\tilde{\lambda}$, service rate $\tilde{\mu}$, retrial rate $\tilde{\theta}$, failure rate $\tilde{\sigma}$, repair rate $\tilde{\beta}$ are all fuzzy numbers. This model will hereafter be denoted by FM/FM/1/FCFS. Suppose arrival rate, service rate, retrial rate, failure rate and repair rate are approximately known and can be represented by convex fuzzy sets.

Let $\eta_{\tilde{\lambda}}(a)$, $\eta_{\tilde{\mu}}(b)$, $\eta_{\tilde{\theta}}(c)$, $\eta_{\tilde{\sigma}}(d)$ and $\eta_{\tilde{\beta}}(e)$ denote the membership functions of the arrival rate, service rate, retrial rate, failure rate and repair rate respectively.

$$P = \{(a,\ \eta_{\tilde{\lambda}}(a)) \mid a \in A\}$$
$$Q = \{(b,\ \eta_{\tilde{\mu}}(b)) \mid b \in B\}$$
$$R = \{(c,\ \eta_{\tilde{\theta}}(c)) \mid c \in C\}$$
$$S = \{(\mathrm{d}, \eta_{\tilde{\sigma}}(\mathrm{d})) \mid d \in D\}$$
$$T = \{(\mathrm{e},\ \eta_{\tilde{\beta}}(\mathrm{e})) \mid e \in E\}$$

where A, B, C, D and E are the crisp universal sets of the arrival rate, service rate, retrial rate, failure rate and repair rate respectively.

The α – cut of inter arrival time and service time are represented as

$$P(\alpha) = \{p \in A / \eta_P(p) \ge \alpha\}$$
$$Q(\alpha) = \{q \in B / \eta_Q(q) \ge \alpha\}$$
$$R(\alpha) = \{r \in C / \eta_R(r) \ge \alpha\}$$
$$S(\alpha) = \{s \in D / \eta_S(s) \ge \alpha\}$$
$$T(\alpha) = \{t \in E / \eta_T(t) \ge \alpha\}$$

Let f (p, q, r, s, t) denote the system performance of interest. Hence p, q, r, s, t are all fuzzy numbers and likewise f (p, q, r, s, t) are fuzzy numbers.

IV. INTERVAL ANALYSIS ARITHMETIC

Let I_1 and I_2 be two interval numbers defined by order of real numbers with lower and upper bounds. Then, $I_1 = [a, b]$, $a \le b$ and $I_2 = [c, d]$, $c \le d$. Define a general arithmetic property with the symbol $* = [+, -, \times, \div]$ symbolically the operation $I_1 * I_2 = [a, b] * [c, d]$ represents another interval. The interval calculation depends on the magnitudes and signs of the elements a, b, c, d.

$[a, b] + [c, d] = [a + c, b + d]$
$[a, b] - [c, d] = [a - d, b - c]$
$[a, b] \times [c, d] = [\min(ac, ad, bc, bd), \max(ac, ad, bc, bd)]$
$[a, b] \div [c, d] = [a, b] \times [1/d, 1/c]$, provided that $0 \notin [c, d]$.

$$\alpha[a,b] = \begin{cases} [\alpha a, \alpha b] & for\ \alpha > 0 \\ [\alpha b, \alpha a] & for\ \alpha < 0 \end{cases}$$

V. DSW ALGORITHM

The DSW algorithm consists of the following steps:

1) Select a α – cut value where $0 \le \alpha \le 1$.
2) Find the intervals in the input membership functions that correspond to this α.
3) Using standard binary interval operations compute the interval for the output membership function for the selected α – cut level.
4) Repeat steps 1-3 for different values of α to compute α – cut representation of the solution.

VI. NUMERICAL EXAMPLE

The arrival rate λ, service rate μ, retrial rate θ, failure rate σ, repair rate β are all pentagonal fuzzy numbers represented by $\tilde{\lambda} = [3, 4, 5, 6, 7]$, $\tilde{\mu} = [25, 26, 27, 28, 29]$, $\tilde{\theta} = [14, 15, 16, 17, 18]$ $\tilde{\sigma} = [36, 37, 38, 39, 40]$ and $\tilde{\beta} = [46, 47, 48, 49, 50]$ respectively. The system manager wants to evaluate the performance measures of the system such as the expected number of customers in the queue, expected number of customers in the system and the expected orbit size.

Therefore, we have,

$$E[Q] = \frac{\lambda\left[\mu\sigma(\mu+\sigma)+\lambda(\beta+\sigma)^2\right]}{\mu(\beta+\sigma)\left[\beta(\mu+\sigma)-\lambda(\beta+\sigma)\right]}$$

$$E[N] = \frac{\lambda\left[\mu\sigma+(\beta+\sigma)^2\right]}{(\beta+\sigma)\left[\beta\mu-\lambda(\beta+\sigma)\right]} + \frac{\sigma\lambda(\beta+\sigma)}{\theta\left[\beta\mu-\lambda(\beta+\sigma)\right]}$$

$$E[R] = \frac{\beta\lambda\sigma\left[\mu(\mu+\sigma-\lambda)+\lambda(\beta+\sigma)\right]}{\mu\left[\beta\mu-\lambda(\beta+\sigma)\right]\left[\beta(\mu+\sigma)-\lambda(\beta+\sigma)\right]} + \frac{\lambda\sigma(\beta+\sigma)}{\theta\left[\beta\mu-\lambda(\beta+\sigma)\right]}$$

$\tilde{\lambda} = [3+2\alpha, 7-2\alpha]$, $\tilde{\mu} = [25+2\alpha, 29-2\alpha]$, $\tilde{\theta} = [14+2\alpha, 18-2\alpha]$, $\tilde{\sigma} = [36+2\alpha, 40-2\alpha]$ and $\tilde{\beta} = [46+2\alpha, 50-2\alpha]$.

Table - 1. The α – cuts of E[Q] at α values

α	$\tilde{\lambda}^L$	$\tilde{\lambda}^U$	$\tilde{\mu}^L$	$\tilde{\mu}^U$	$\tilde{\theta}^L$	$\tilde{\theta}^U$	$\tilde{\sigma}^L$	$\tilde{\sigma}^U$	$\tilde{\beta}^L$	$\tilde{\beta}^U$	$E[Q]^L$	$E[Q]^U$
0	3	7	25	29	14	18	36	40	46	50	0.0269	0.2146
0.1	3.2	6.8	25.2	28.8	14.2	17.8	36.2	39.8	46.2	49.8	0.0306	0.1959
0.2	3.4	6.6	25.4	28.6	14.4	17.6	36.4	39.6	46.4	49.6	0.0346	0.1787
0.3	3.6	6.4	25.6	28.4	14.6	17.4	36.6	39.4	46.6	49.4	0.0389	0.1629
0.4	3.8	6.2	25.8	28.2	14.8	17.2	36.8	39.2	46.8	49.2	0.0437	0.1484
0.5	4	6	26	28	15	17	37	39	47	49	0.0489	0.1351
0.6	4.2	5.8	26.2	27.8	15.2	16.8	37.2	38.8	47.2	48.8	0.0546	0.1228
0.7	4.4	5.6	26.4	27.6	15.4	16.6	37.4	38.6	47.4	48.6	0.0608	0.1116
0.8	4.6	5.4	26.6	27.4	15.6	16.4	37.6	38.4	47.6	48.4	0.0676	0.1012
0.9	4.8	5.2	26.8	27.2	15.8	16.2	37.8	38.2	47.8	48.2	0.0750	0.0917
1.0	5	5	27	27	16	16	38	38	48	48	0.0830	0.0830

Figure - 1. Membership Function Graph of E[Q]

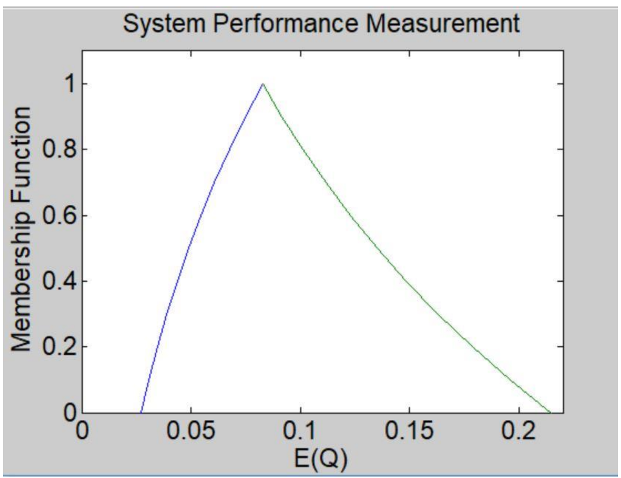


Table - 2. The α – cuts of E[N] at α values

α	$\tilde{\lambda}^L$	$\tilde{\lambda}^U$	$\tilde{\mu}^L$	$\tilde{\mu}^U$	$\tilde{\theta}^L$	$\tilde{\theta}^U$	$\tilde{\sigma}^L$	$\tilde{\sigma}^U$	$\tilde{\beta}^L$	$\tilde{\beta}^U$	$E[N]^L$	$E[N]^U$
0	3	7	25	29	14	18	36	40	46	50	0.6197	4.9817
0.1	3.2	6.8	25.2	28.8	14.2	17.8	36.2	39.8	46.2	49.8	0.6931	4.4414
0.2	3.4	6.6	25.4	28.6	14.4	17.6	36.4	39.6	46.4	49.6	0.7729	3.9731
0.3	3.6	6.4	25.6	28.4	14.6	17.4	36.6	39.4	46.6	49.4	0.8597	3.5642
0.4	3.8	6.2	25.8	28.2	14.8	17.2	36.8	39.2	46.8	49.2	0.9544	3.2049
0.5	4	6	26	28	15	17	37	39	47	49	1.0577	2.8875
0.6	4.2	5.8	26.2	27.8	15.2	16.8	37.2	38.8	47.2	48.8	1.1707	2.6057
0.7	4.4	5.6	26.4	27.6	15.4	16.6	37.4	38.6	47.4	48.6	1.2944	2.3544
0.8	4.6	5.4	26.6	27.4	15.6	16.4	37.6	38.4	47.6	48.4	1.4303	2.1293
0.9	4.8	5.2	26.8	27.2	15.8	16.2	37.8	38.2	47.8	48.2	1.5798	1.9270
1.0	5	5	27	27	16	16	38	38	48	48	1.7447	1.7447

Figure - 2. Membership Function Graph of E[N]

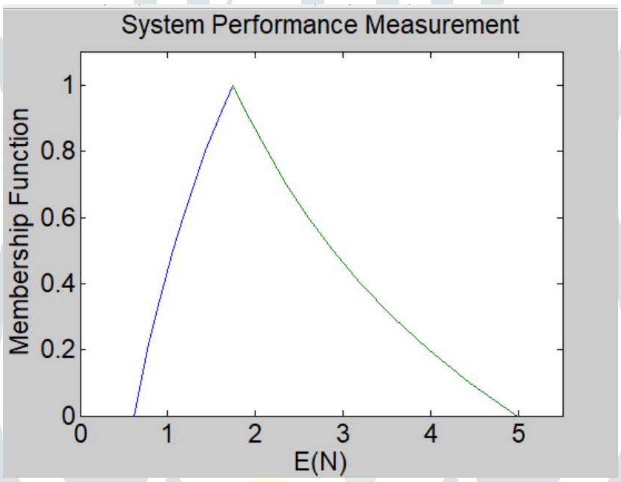
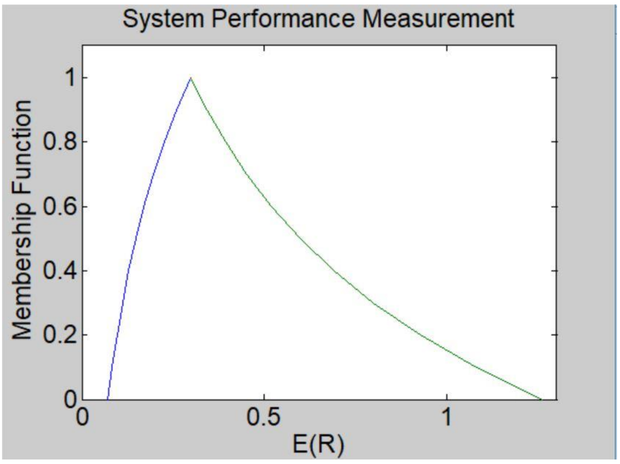


Table – 3. The α – cuts of E[R] at α values

α	$\tilde{\lambda}^L$	$\tilde{\lambda}^U$	$\tilde{\mu}^L$	$\tilde{\mu}^U$	$\tilde{\theta}^L$	$\tilde{\theta}^U$	$\tilde{\sigma}^L$	$\tilde{\sigma}^U$	$\tilde{\beta}^L$	$\tilde{\beta}^U$	$E[R]^L$	$E[R]^U$
0	3	7	25	29	14	18	36	40	46	50	0.0709	1.2591
0.1	3.2	6.8	25.2	28.8	14.2	17.8	36.2	39.8	46.2	49.8	0.0826	1.0786
0.2	3.4	6.6	25.4	28.6	14.4	17.6	36.4	39.6	46.4	49.6	0.0959	0.9272
0.3	3.6	6.4	25.6	28.4	14.6	17.4	36.6	39.4	46.6	49.4	0.1110	0.7995
0.4	3.8	6.2	25.8	28.2	14.8	17.2	36.8	39.2	46.8	49.2	0.1283	0.6911
0.5	4	6	26	28	15	17	37	39	47	49	0.1479	0.5986
0.6	4.2	5.8	26.2	27.8	15.2	16.8	37.2	38.8	47.2	48.8	0.1704	0.5194
0.7	4.4	5.6	26.4	27.6	15.4	16.6	37.4	38.6	47.4	48.6	0.1960	0.4512
0.8	4.6	5.4	26.6	27.4	15.6	16.4	37.6	38.4	47.6	48.4	0.2253	0.3924
0.9	4.8	5.2	26.8	27.2	15.8	16.2	37.8	38.2	47.8	48.2	0.2588	0.3415
1.0	5	5	27	27	16	16	38	38	48	48	0.2973	0.2973

Figure - 3. Membership Function Graph of E[R]



With the help of MATLAB software we perform α – cuts of arrival , service, retrial, failure, repair rate and fuzzy expected number of jobs in queue at eleven distinct levels of α namely 0, 0.1, 0.2, 0.3, …1. Crisp intervals for fuzzy expected number of jobs in queue E(Q) at different possibility α levels are presented in above table. Similarly the performance measures such as fuzzy expected number of jobs in system E(N) and fuzzy expected orbit size E(R) were also presented in the table. The α – cut represents the possibility that these three performance measure will lie in the associated range. The above data will be very suitable for designing a queuing system.

VII. CONCLUSION

In this paper, we have used the α – cut approach to analyze a retrial fuzzy queue model with unreliable server. The DSW algorithm greatly simplifies manipulation of the extension principle for continuous valued fuzzy variables such as fuzzy numbers defined on the real line. A numerical example was given to illustrate the effectiveness of the proposed technique. Also, the proposed method can be employed to analyze the other fuzzy queuing systems.

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