A chromatic number to the transformation Graph (G - + -)

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Abstract: *Let* $G = \{V, E\}$ *be a connected simple graph. The transformation graph* G^{-+-} of this G *is the graph with the union of vertex set and edge set in which the adjacency of two vertices a and b is defined as follows: (i) a and b in V(G) are adjacent if and only if they are non- adjacent in G (ii) a and b in E(G) are adjacent if and only if they are adjacent in G (iii)a and b in V(G) while the other is in E(G) , they are not incident in G . In this paper we established the color class and chromatic number to the transformation Cycle, Path , Star graphs.*

Keywords: Path graph, Cycle graph, Star graph, Transformation, Vertex Coloring, Chromatic number.

Introduction: 1.0

In Graph, graph coloring is one of the most important concept . The proper coloring of a graph is the coloring of the vertices and edges with minimal number of colors such that no two vertices should have the same color .A graph that permits a k-coloring is called k-colorable. The chromatic number $\chi(G)$ of a graph G, $\chi(G)$ is minimum number of colors needed for proper coloring.

Wu and Meng introduced the transformation graph G^{xyz} of G. Since the set {+,-} has eight distinct three permutations, they introduce eight types of transformation graphs. We shall investigate the transformation graph G^{-+-} of some graphs.

Definition: 1.1

A **graph** G is an ordered pair (V(G) , E(G)) consisting of a non- empty set V(G) of vertices and a set E(G) , disjoint from V(G) of edges together with an incidence function Ψ_G that associates with each edge of G is an ordered pair of vertices of G.

Definition: 1.2

walk is an alternating sequence of vertices and edges starting and ending with vertices.
A walk in which all the vertices are distinct is called a **path**. A path containing n- vertices is denoted by P_n.

A closed path is called **Cycle**. A cycle containing n-vertices is denoted by C_n, the length of a cycle is the number of edges occurring on it.

Definition: 1.3

A Star graph is a graph in which n-1 vertices have degree 1 and a single vertex have degree n-1. The n-1 vertex are connected to a single central vertex. A star graph with total n-vertex is termed as Sn.

Definition: 1.4

The **transformation graph** G^{xyz} of G is defined on the vertex set $V(G) \cup E(G)$. Two vertices (or edges) α and β of G are joined by an edge in G^{-+-} if and only if their associativity in is consistant with the corresponding term of G.

Definition:1.5

Let $G = (V(G), E(G))$ be a graph and x,y,z be three variables taking values $+$ or $-$. The **transformation graph** G^{xyz} is the graph having $V(G) \cup E(G)$ as the vertex set and for $\propto, \beta \in V(G) \cup E(G)$ $\propto$ *and* β are adjacent in G^{xyz} if and only if one of the following holds:

(i) $\propto, \beta \in V(G)$. $\propto$ *and* β are adjacent in G if $x = +$; $\propto$ *and* β are not adjacent in G if $x = -$.

(ii) $\propto, \beta \in E(G)$. $\propto$ *and* β are adjacent in G if $y = +$; $\propto$ *and* β are not adjacent in G if $y = -$.

(iii) $\propto \in V(G)$, $\beta \in E(G)$. $\propto$ *and* β are incidentt in G if $z = +$; $\propto$ *and* β are not incident in G if $z = -$.

Theorem: 2.1

Let G = P_n (n ≥ 4) be any path graph with n-vertices, then $\chi(G^{-+-}) = \left\lceil \frac{n}{2} \right\rceil + 1$

Proof:

Let G = P_n be a path graph with 'n' vertices

To prove $\chi(G^{-+-}) = \left\lceil \frac{n}{2} \right\rceil + 1$

The vertex set of G is V(G) = $\{v_i \,/ 1 \le i \le n\}$ and the edge set of G is E(G) = $\{e_i \,/ 1 \le i \le n-1\}$

The adjacency of G is,

Each $v_i \in V(P_n)$ is adjacent to,

$N(v_i) = \{v_{i-1}, v_{i+1} \,/ \, 2 \le i \le n-1\}$

$N(v_i) = v_2$ and $N(v_n) = v_{n-1}$

Each $e_i \in E(P_n)$ is adjacent to,

$N(e_i) = \{e_{i-1}, e_{i+1}\}$

$N(e_1) = e_2$ and $N(e_n) = e_{n-1}$

Each $e_i \in E(G)$ is incident with v_{i-1} and v_{i+1} where $1 \le i \le n-1$

The vertex set of G^{-+-} is $V(G^{-+-}) = V(Pn) \cup E(Pn)$

By the definition of the transformation (P_n^{-+-})

The adjacency in $V(G^{-+-})$ is as follows:

Those pair of vertices (v_i, v_i) are not adjacent in G are neighbouring vertices in G^{-+-}.

Those pair of edges (e_i, e_j) which are connected in G , are neighbouring vertices in G^{-+-}.

In similar, the pair (v_i, e_i) are not incident in G , are neighbouring vertices in G^{-+-}

Now, the vertices of $V(G^{-+-})$ can be classified as follows:

$$C_1 = \left\{e_{2j},\ 1 \le i \le \left\lceil \frac{n}{2} \right\rceil - 1\right\} \qquad \text{........ (1)}$$

$$C_i = \left\{v_{2i-3}, e_{2i-3}, v_{2i-2} \,/\, 2 \le i \le \left\lceil \frac{n}{2} \right\rceil\right\} \qquad \text{........ (2)}$$

$$C_k = \begin{cases} v_n & \text{if } n \text{ is odd} \\ v_{n-1}, e_{n-1}, v_n & \text{if } n \text{ is even} \end{cases} \Bigg\} \text{ where } k = \left\lceil \frac{n}{2} \right\rceil + 1 \qquad \text{........ (3)}$$

The elements of equation (1) , are independent in G^{-+-}.

Hence, a particular color C_1 can be given to all the vertices of equation (1)

The elements of equation (2) , are independent in G^{-+-}.

Therefore the new colors C_i , to apply all the vertices in equation (2) .

In similar, The elements of equation (3) , is independent in G^{-+-} .

Therefore, we use a different color C_k , to give the vertices of equation (3) .

Therefore, the total numbers of color class is the minimum coloring number of the graph G^{-+-}.

That is, $\chi (G^{-+-}) = 1+\left\lceil\frac{n}{2}\right\rceil-1+1$

$= \left\lceil\frac{n}{2}\right\rceil + 1$

Therefore, the chromatic number of is G^{-+-},

$\Rightarrow \chi (G^{-+-}) = \left\lceil\frac{n}{2}\right\rceil + 1$

Hence, the proof.

Theorem: 2.2

Let G = C_n (n = 1, 2, 3,.. ,n) be the cycle graph with n-vertices, If (n ≥ 3) then

$$\chi (G^{-+-}) = \left\lceil\frac{n}{2}\right\rceil + 1$$

Proof:

Let Cn be the cycle graph with 'n' vertices of G

To prove χ (G- + -) $=\left\lceil\frac{n}{2}\right\rceil + 1$

The vertex set of G is V(G) = $\{v_i /1\leq i \leq n\}$ and the edge set of G is E(G) = $\{e_i /1\leq i \leq n\}$

The adjacency of G is,

Each vertex v_i is adjacent to

$N(v_i) = \{v_{i-1,}v_{i+1} /2 \leq i \leq n-1\}$

Each e_i is adjecent to

N(e_i) = { e_{i-1}, e_{i+1}}

Each e_i is incident with v_i and v_{i+1} where 1 ≤ i≤ n-1

N(e_n) = $\{v_1, v_n\}$

The vertex set of Cn- + - is V(Cn- + -) = V(Cn) ∪E(Cn).

By the definition of the transformation (G- + -)

The adjacency in V(G- + -) is as follows:

Those pair of vertices (v_i, v_j) are not adjacent in Cn , are neighbouring vertices in G- + - .

Those pair of edges (e_i , e_j) which are adjacent in Cn , are neighbouring vertices in G- + - .

In similar, the pair (e_i, v_i) which are not incident in Cn , are neighbouring vertices in G- + - .

Now, the vertices of V(G) can be classified as follows:

C1 = $\left\{ e_{2j}, 1 \le j \le \left\lceil \frac{n}{2} \right\rceil \right\}$ ....(1)

Ci = $\{v_{2i-3}, e_{2i-3}, v_{2i-2}$ / $2 \le i \le \left\lceil \frac{n}{2} \right\rceil \}$ ....(2)

Ck= $\begin{cases} v_n, e_n & , \quad if\ n\ is\ odd \\ v_{n-1}, e_{n-1}, v_n & , \quad if\ n\ is\ even \end{cases}$ where k $= \left\lceil \frac{n}{2} \right\rceil + 1$ ....(3)

The elements of equation (1) , are independent in G- + -.

Hence, a particular color C1 can be given to all the vertices of equation (1).

The elements of equation (2) , are independent in G- + -.

Therefore the new colors Ci , to apply all the vertices in equation (2) .

In similar, The elements of equation (3) , is independent in G- + - .

Therefore, we use a different color to color Ck , to give the vertices of equation (3) .

Therefore, the total numbers of color class is the minimum coloring number of the graph G- + -.

That is , χ (G- + -) = $1 + \left\lceil \frac{n}{2} \right\rceil - 1 + 1$

$= \left\lceil \frac{n}{2} \right\rceil + 1$

Therefore, the chromatic number of is G- + - ,

χ (G- + -) = $\left\lceil \frac{n}{2} \right\rceil + 1$

Hence, the proof.

Theorem: 2.3

Let G = S_n be a star graph with 'n' vertices of G, then χ (G^{-+-}) = n.

Proof:

Given S_n be a star graph with 'n' vertices of G.

To prove χ (G^{-+-}) = n

Choose a vertex v_0 be the centre vertex which degree is n-1.

The vertex set of S_n is V(G) =$\{v_i$ /$0 \le i \le n\}$ and the edge set of S_n is E(G) = $\{e_i$ /$1 \le i \le n\}$

The adjacency of G is,

Each v_i is adjacent to , N(v_i) = $\{v_0\}$ for all i = 1,2,……..n

Each e_i is incident with v_i and v_i .

The vertex set of G is V(G^{-+-}) = V(S_n) ∪ E(S_n)

The adjacency in V(G^{-+-}) is as follows:

Since all $v_i \in$ V(S_n) , $0 \le i \le n$ are independent in G, hence $v_i \in$V(G^{-+-}), $0 \le i \le n$ form a clique (complete subgraph) of n vertices in G^{-+-} .

Hence, we need n colors to color all the vertices $v_i \in$ V(G^{-+-}). All elements $e_i \in$ E(S_n) are adjacent in G.

Therefore, $e_i \in$ V(G^{-+-}) are independent in G^{-+-} incident with the vertex $v_i \in$ V(S_n).

Therefore, the pair (v_i, e_i) ∈ V(G^{-+-}) are disjoint.

Hence, we can use the same color to color the pair of vertices $(v_i, e_i) \in V(G^{-+-})$.

$N(v_i) = \{v_i \ /1 \leq i \leq n\}$ where $v_0 \in V(S_n)$. Therefore, v_0 is an isolated vertex in $V(G^{-+-})$.

Hence, we can use any one of the color which was assigned for the

vertices $(v_i, e_i) \in V(G^{-+-})$.

Therefore, the total number of the color class is the minimum coloring number of the graph G^{-+-}.

That is, $\chi(G^{-+-}) = n$

Hence, the proof.

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