A COMPARATIVE STUDY ON A MINIMUM SPANNING TREE BASED HEURISTIC FOR THE TRAVELLING SALESMAN TOUR

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Abstract: This paper propose a comparison study on a heuristic to find Travelling Salesman Tour(TST) in a connected network. The approach first identifies a vertex and two associated edges that are desirable for inclusion in the required TST. If we let this vertex be denoted by p and two selected edges emanating from this vertex be denote (p,q) and (p,k) then we find a path joining the two vertex q and k passing through all the remaining vertex of the given network . A sum of these lengths,
That is length of the links (p,q) an (p,k) along with the length of the path that joins the vertex p an q passing through all remaining vertices will results feasible TST, hence gives an upper bound on the TST. A simple procedure is outlined to identify: (1) the vertex p, (2) the two corresponding links (p,q) and (p,k) and (3) the path joining the vertex q and k passing through all the remaining vertex. The approach is based on the minimum spanning tree; hence the complexity of the TST is reduced. Thes network in the present context has been assume to be connected network with at least two edges emanating from each vertex.

Keywords: **Connected network, Minimum Spanning Tree, Travelling Salesman Tour.**

I. INTRODUCTION

Comparison is essential to our life for selection alternative options. Comparison among proposed method in a different conjectures have their own advantages and disadvantage in vertex of three conjectures taken as: This paper compared among three conjectures, these conjectures related to optimum solution to the Travelling Salesman Tour.

Origin of the Travelling Salesman problem is as the Konisberg , a town in Prussia , had seven bridges over the river Pregel as shown in the following figure (a) and its corresponding network representation is illustrate in figure (b). The problems was to find a tour through the town that crosses each bridge exactly once. Leonhard Euler gave a formal solution led to the to this problem, and it is believed that the Konisberg situation led to the development of the graph theory.

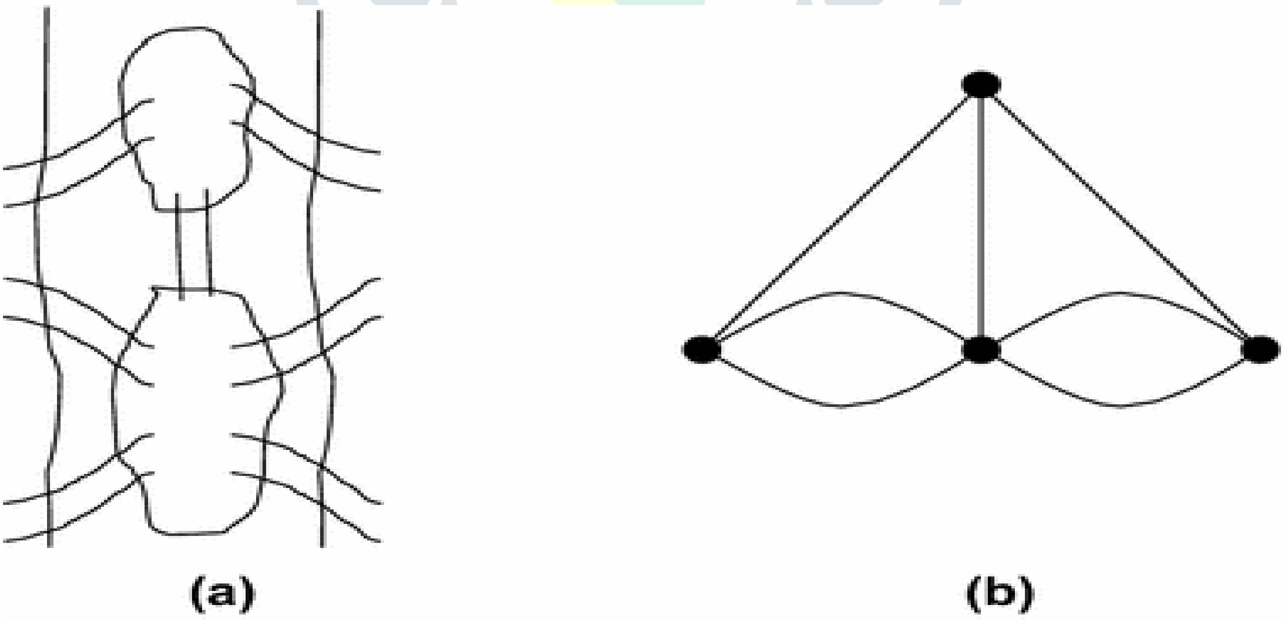

Figure 1: The Seven bridge of Konisberg (a) and corresponding graph (b)

A Minimum spanning tree (MST) can be obtained by any known greedy approach [3][4]. The index value at each vertex in the MST will be a number between 1 and (n-1) in a n-vertex completely connected network. To find a MST under a restriction that the index value at each vertex is ≤ 2 [6]. Such a MST with index restricted to ≤ 2 has been called a MST path.

This MST path will have two vertices with index value 1 and all the remaining vertices will have index value 2. A heuristic has been developed to find a Travelling Salesman Tour (TST), using the MST path; thus converting the (Non-deterministic) NP-hard problem to a relatively easier from. This paper explains a heuristic to find the Travelling Salesman Tour in a connected network. The approach first identifies a vertex and two associated edges that are desirable for inclusion in the required TST. If we let this vertex be denoted by **p** and two selected edges emanating from this vertex be denoted by (p,q) and (p,k), then we find a path joining the two vertices q and k passing through all the remaining vertices of the given network. A sum of these lengths, i.e. length of the links (p,q) and (p,k) along with the length of the path that joins the vertices q and k passing through all the remaining vertices will the result in a feasible TST, hence gives an upper bound on the TST. A simple procedure is outlined to identify: (1) the vertex p (2) the two corresponding links (p,q) and (p,k), and (3) the path joining the vertices q and k passing through all the remaining vertices . This approach is based on the Minimum Spanning Tree; hence the complexity of the TST is reduced. The network in the present context has been assumed to be a connected network with at least two edges emanating from each vertex.

This paper consist of six sections . First section introduced the graph theoretical conepts. The preliminaries are given in the second section which are very much support to the paper. A heuristic has been developed to find a travelling salesman tour (TST) ,using the MST path; the travelling salesman problem is reviewed. The proposed heuristic is present in this paper in section III . Illustrative examples have been explained in section IV. Comparison are discussed in section five. Finally the conclusions are given in section six.

II. PRELIMINARIES

This chapter describes the basic concepts of minimum spanning tree and travelling salesman tour which are very much support this paper.

Definition 2.1

Minimum Spanning tree is a subset of the edges of a connected graph. A planner graph and its minimum spanning tree each edges is with its which here is roughly proportional to its length. It is connected graph is said to **minimally connected** if removal of any one edges from it disconnected graph. It is no a circuit.A graph with n vertices, (n-1) edges and no circuit is connected.

Definition 2.2

A graph G is said to be **Connected Graph** if any vertices can be reached from any other vertices by means of path. That is the graph is said to be connected if there is at least one path between every pair of vertices in G.

Definition 2.3

Travelling Salesman Problem (TSP): Given a set of cities and distance between every pair of cities , the problem is to find the shortest possible route that visits every city exactly once and returns back to the starting point.

Note: Optimum solution = Best Possible Solution.

Notations

LB	=	Lower Bound
UB	=	Upper Bound
LEdge	=	Length of the edge (edge)
MST	=	Minimum Spanning Tree
TST	=	Travelling Salesman Tour
L^{i}MSTG'(n-1)	=	Length of the MST
LTSTG(n)	=	Length of the feasible TST
TSP	=	Travelling Salesman problem

III.MINIMUM SPANNING TREE BASED HEURISTIC FOR THE TSP

3.1 Useful Conjectures

Let a connected network of **n** vertices and **m** edges be denoted by G(n,m). After excluding a vertex **p** and all edges that emanate from the vertex **p**, the remaining network is denoted by G'(n-1,m- d_p) where d_p is the number of edges emanating from the vertex **p.** Note that the network G'(n-1,m-d_p) is an (n-1) vertex connected network. The MST of the network G'(n-1,m-d_p) will be Comprised of (n-2) edges. Let the length of the MST be denoted by LMSTG'(n-1,m-d_p).

Conjecture 1:

The MST can be converted to a MST path by repeated applications of the index-balancing theorem[18]. After i iterations of the index-balancing theorem (i=1,2……*l*) , let the length of the MST be denoted by $L^iMSTG'(n-1,m-d_p)$, where:

$$LMSTG'(n-1,m-d_p) \leq L^1MSTG'(n-1,m-d_p) \leq\ldots\ldots\leq L^iMSTG'(n-1,m-d_p) \leq\ldots\leq L^lMSTG'(n-1,m-d_p) \qquad (1)$$

i.e. each iteration of the index-balancing theorem leads to an increase in the MST length. At the l^{th} iteration, when the index at each vertex is $\leq$ 2, the MST becomes a path. The imbalance of index numbers in the MST with regard to the MST path will always be even number, that is the high and low index values will be equal and the total index value will be 2(n-2).

Conjecture 2:

After establishing a feasible Travelling Salesman Tour for a particular **q** and **k** vertex combination, the search for a better tour commencing from any other combination of two links emanating from the vertex **p** can be fathomed at i^{th} (i=0,1,2,……,*l*) index-balancing theorem when:

$$L(p,q) + L(p,k) + L^iMSTG'(n-1,m-d_p) \geq LTSTG(n,m) \qquad (2)$$

(the current upper bound)

Conjecture 3:

The number of edges in a TST in G(n,m) will be **n.** The sum of the two selected edges together with MST of $G'(n-1,m-d_p)$ will constitute a collection of edges, but a feasible Travelling Salesman problem solution will be realized only when the length of the selected two edges is added to the MST path that starts and finishes at the vertex q and k. Since we are interested in a path between the q and k, which is passing through all the other remaining vertices, we can set the link **(q,k) = ∞,** if it exists. Simply, the link (q,k) will form a loop with the links (p,q) and (p,k). Let the length of this MST path be $L^lMSTG'(n-1,m-d_p)$. This MST path with the two selected links (p,q) and (p,k) will form a feasible TST, resulting in an upper bound of the Travelling Salesman Tour.

3.2 The three components of the heuristic

3.2.1 Determination of a MST path joining vertices q and k

For the given network G (n,m), the vertex p ϵ n and two associated links {(p,q),(p,k)} ϵ m, the focus is to find the MST path joining the vertices q and k, passing through all the remaining vertices in the network $G'(n-1,m-d_p)$. Note that if the shortest path joining vertex q and k can be determine under the condition that it passes through all the vertices in the network $G'(n-1,m-d_p)$, then that path has an alternative interpretation of MST path (that is all intermediate vertices on the path have an index 2 and the vertices q and k have index 1). This MST of the network $G'(n-1,m-d_p)$ will have (n-2) links in it.

The method is described below:

Step 1: Set the link (q,k) = ∞, if it exists in the network $G'(n-1,m-d_p)$.

Step 2: Find the Minimum Spanning Tree of the network $G'(n-1,m-d_p)$, which will be comprised of (n-2) links. The sum of these edges gives the MST length, denoted by $LMSTG'(n-1,m-d_p)$.

Step 3: Using the index-balancing theorem[11] convert this spanning tree to a Minimum Spanning Tree path joining the vertex q and k passing through all the remaining vertices in $G'(n-1,m-d_p)$.

Step 4: The Travelling Spanning Tree length will be: $LTSTG(n,m) = L(p,q) + L(p,k) + L^lMSTG'(n-1,m-d_p)$. Since this is a feasible Travelling Salesman Tour, it becomes an upper bound. Note that it may not be the optimal Travelling Spanning Tree solution.

3.2.2 Identification of the vertex p

A few definitions are necessary.

1. **Basic edge:** If an edge belongs to the Travelling Salesman Tour or the Minimum Spanning Tree, it is basic, else it is non-basic.

2. **Index of a vertex** is the number of basic edges emanating from that vertex in the given Travelling Spanning Tour or the Minimum Spanning Tree.

The vertex with lowest index value in G(n,m) is selected as the vertex 'p'. If lowest index vertices are more than one, we resolve the tie arbitrarily and selected one of them as the vertex p.

Consider that the lowest index at the vertex 'p' is denoted by r, where r ≥ 2. The number of ways two links can be selected in r_{c_2} number of ways. These combinations can be arranged in increasing order and we label them as 1,2,…. r_{c_2} . The selected vertex p and the two associated links with minimum cost are denoted by (p,q) and (p,k). The complexity of the proposed approach will depend on the index value of the selected vertex p. If the lowest index value in the given network is r, where r ≥ 2, the optimum Travelling Spanning Tree will require determination of r_{c_2} number of travelling salesman tours for different combinations of two edges from the vertex p. In a completely connected network G(n,m), the maximum number of problems that will be solved is given by $(n-1)_{c_2}$.

3.2.3 Identification of the two links from the vertex p

Since all possible combinations have been arranged in an increasing order with respect to the cost, the combination l will be associated with the two minimum edges. If there is a tie, we arrange them in non-decreasing order and call them 1,2,…*l*.

3.3 The index balancing theorem

The Minimum Spanning Tree of and '(n-1)' vertex network will have (n-2) links, an a total of 2(n-2) index values. In a normal MST, the index values can be a number such that each vertex can have index between 1 ≤ n_i ≤ (n-1) and total index will be 2(n-2). An application of the index-balancing theorem can decrease the index value at a high index value vertex and increase the index value at a vertex of low index value[11]. Adding the same constant to all edges emanating from the same vertex does not change their relative merit, but can create alternatives for the Minimum Spanning Tree. Thus, additional added quantity can create alternatives to obtain new Minimum Spanning Trees which balances indexing. [7][8].

3.4 Determination of an optimal TST in the network G(n,m)

The following steps are followed.

Step A: for the given network, identify the following:

1) The vertex p.
2) The index of the vertex p, let it be r, r ≥ 2.
3) The number of combinations, two at a time, is given by r_{c_2} =1,2,…*l*.
   Let us denote a function of length , these combinations by $C_1, C_2, \ldots, C_l$ and their corresponding lengths by $L(C_1) \leq L(C_2) \leq \ldots \leq L(C_l)$.
4) Identify the vertices q and k associated with the least cost combination, C_1.
5) Set the link (q,k) = ∞ in the network $G'(n-1, m-d_p)$.
6) Find the MST of the network $G'(n-1, m-d_p)$, where the link (q,k) = ∞.
7) Find the length of the Minimum Spanning Tree, which is denoted by LMSTG'(n-1).
8) The number of index-imbalances in the MSTG'(n-1) is denoted by Number of MSTG'(n-1)=N, i=1,2,…N.
9) Apply the index-balancing until the Minimum Spanning Tree becomes the Minimum Spanning Tree path between the vertices q and k for the combination C_1.
   Find a feasible Travelling Salesman Tour and denote its length

LTSTG(n) = L(p,q) + L(p,k) + LMSTPG'(n-1) = $L(c_1)$ + LMSTPG'(n-1)

10) The UB = LTSTG(n).

11) Set k = 1.

Step B: Set k = k+1. If k+1 > l, the current Upper Bound is the required optimal Travelling Salesman Tour.

Step C: for the k^{th} combination from the vertex p, do the following

1) Identify the two associated edges with the k^{th} combination.
2) Identify the vertex that will have the index 1 in this

   k^{th} combination. Call these two vertices q and k.
3) Set the links (q,k) = ∞ in the network $G'(n-1, m-d_p)$.
4) Find the MST of the $G'(n-1, m-d_p)$, from 3 above.
5) Determine the number of index imbalances in 4 above
6) Set i = 0.
7) For the current combination, check
   $$\text{LEdge1} + \text{LEdge2} + L^i\text{MSTG}'(n-1) \leq \text{LTSTG}(n) \qquad (3)$$

If satisfied, go to step 8.
If not satisfied, go to step B.

8) Set i = i+1.
9) Apply the i^{th} index-balancing.
If the Minimum Spanning Tree is path satisfying the vertex index requirement, return to step 7.

Step D: Terminate the search for this combination, and go to **step E.**

Step E: If a feasible Travelling Spanning Tree is obtained and it is less than the current upper bound, replace the existing Upper Bound by the new value and return to **Step B.**

IV. NUMERICAL ILLUSTRATIONS

4.1. Conjecture 1

Consider a problem find the Travelling Salesman Tree for the network G(9,15) given in figure 2.

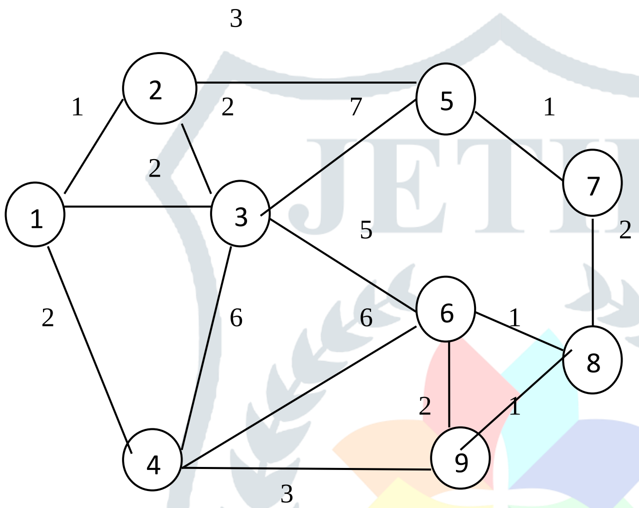

Figure 2: Network to determine the TST

. **Table 1** Index values of vertices

Vertex	1	2	3	4	5	6	7	8	9
Index	3	3	5	4	3	4	2^*	3	3

Asterisk indicates the minimum index.

For the network given in figure 2, the vertex index values are given in Table 1. From the index values in Table 1, vertex 7 is selected as the vertex l. The two links from the vertex 7 will be (7,5),(7,8). Since G'(n-1,m-d_p) then the network G'(8,13) is given in figure 3, links (5,8) does not exist. The problem is to find the Minimum Spanning Tree path in figure 3 joining the vertices 5 and 8, and passing through vertices 1, 2, 3, 4, 6 and 9. The Minimum Spanning Tree of the network in figure 3 will be as shown in figure 4. The length of this MST denoted by LMST = 13. Hence the LB = 13+1+2 = 16 which is not feasible. From the Minimum Spanning Tree network in figure 4, it is clear that vertices 1 and 8 are high degree vertices. The index at vertex 1 is 3, which should be 2 an the index at vertex 8 is 2,which should be 1 as it is a terminal vertex. The vertices 3 and 6 are low degrees vertices as their current degree is 1. Thus, the following adjustments will be required.

Adding 1 to all edges emanating from vertex 1 will change the Minimum Spanning Tree selection of edge (2,1) to edge (2,3) and balance of degree at vertices 1 and 3 will be satisfied.

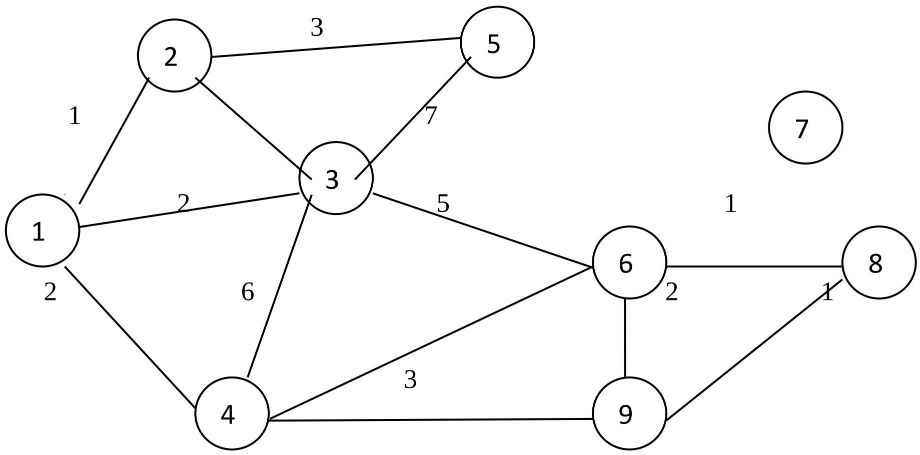

Figure 3: The network G'(8,13), excluding the vertex 7.

Similarly, adding 1 unit to all edges emanating from the vertex 8 will change the MST selection to the edge (9,6) ,replacing the edge (9,8). This change will fix up the degrees at vertices 6 and 8.

Thus , the final selection will be as shown by dark solid lines in **figure 5.**

The TST will be comprised of edges { (7,5), (7,8) and (5,2), (2,3), (3,1), (1,4), (4,9), (9,6), (6,8)}. The length of the TST will be 18, which is optimal.

4.2. Conjecture 2

Reconsider the above problem, where one more link (7,3) has been added(see figure 6).

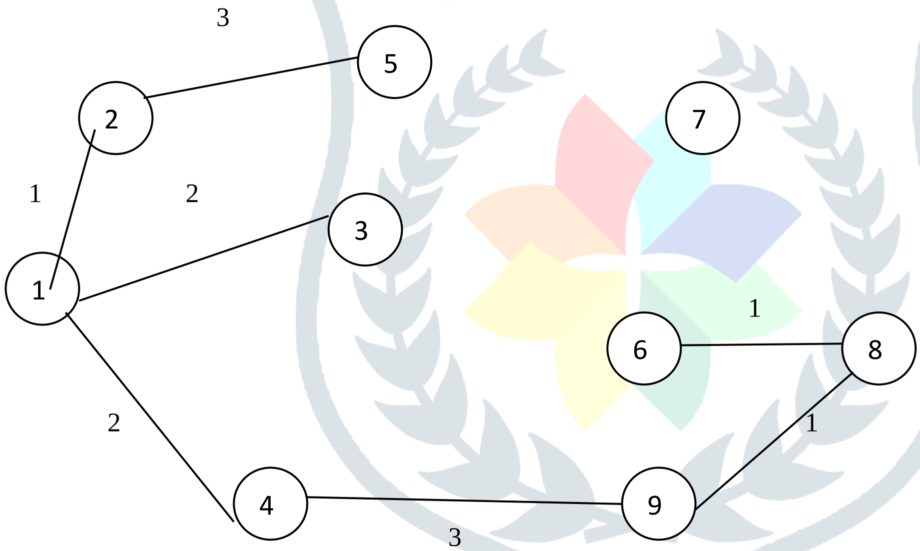

Figure 4 : The MST of G'.

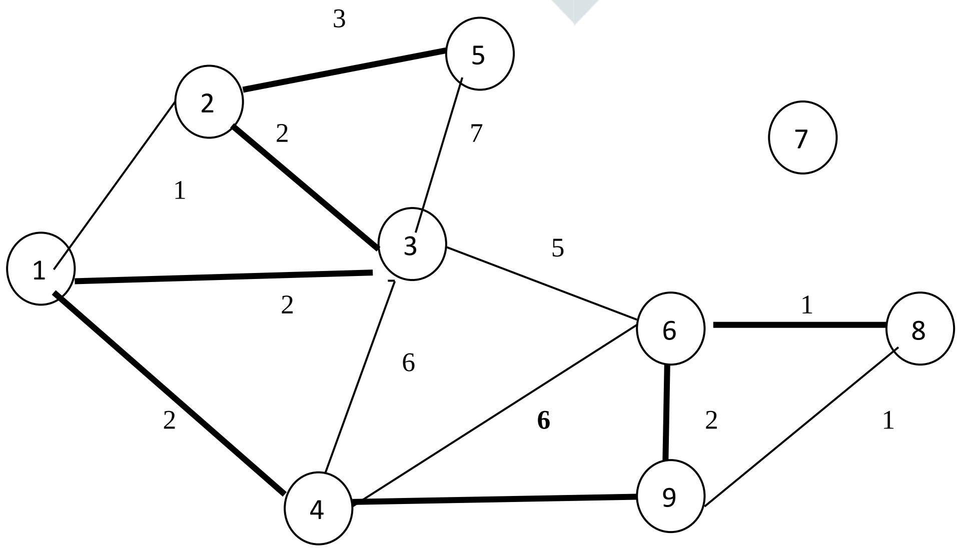

Figure 5: The modified MST path is shown in dark lines.

The vertex index will be as given in Table 2.

Thus, vertex 1,2,5,7,8 and 9 are low index vertices with value 3. Once again vertex 7 is selected arbitrarily as a starting point. There will be three possible combinations for the initial edge selection, and these are:$\{(7,5), (7,8),\}_{(1+2=)\text{cost } 3.}$ Its cost of (1+2) = 3.,$\{(7,5), (7,3)\}_{(1+7)\text{cost } 8.}$ Its cost is (1+7) = 8 and $\{(7,3), (7,8)\}_{(7+2)=\text{cost } 9}$.Its cost is (7+2) = 9. The first selection, is the same as in above problem, which will establish an upper bound LTSTG(9) = 18. Now let us consider the 2nd alternative, that is$\{(7,5), (7,3)\}_{(1+7)=\text{cost } 8}$.Its cost is (1+7) = 8.

The new lower bound will be = 1+7+13= 21,because $\{L(p,q) + L(p,k) + L^{i}MSTG'(n-1, m-d_p) \geq LTSTG(n,m)\}$.

Which is > 18 (the current upper bound). Hence no need for further investigation.

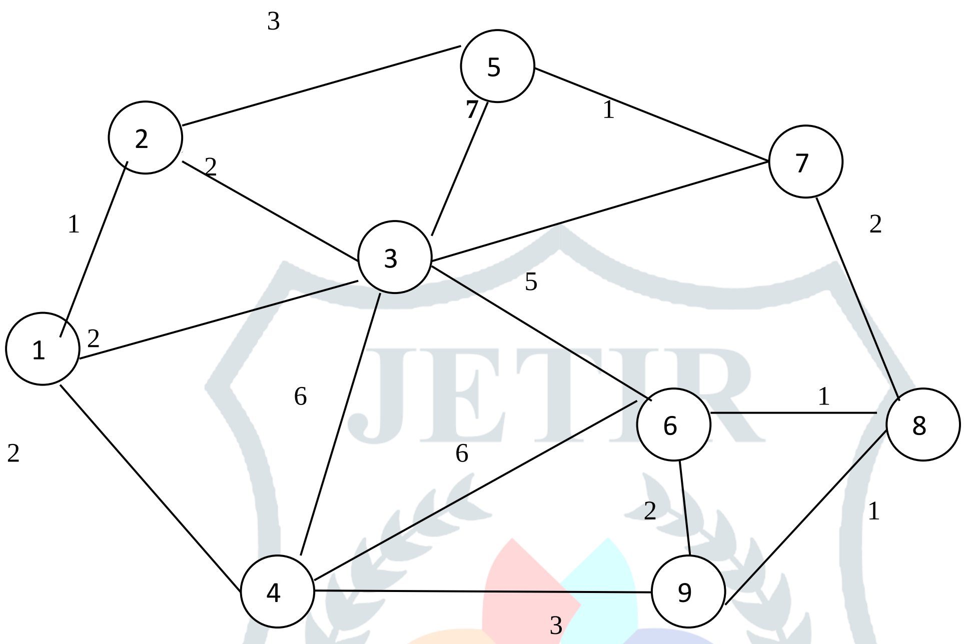

Figure 6: Network of the above problem with an extra link.

Table 2 Index values of vertices

Vertex	1	2	3	4	5	6	7	8	9
Index	3	3	6	4	3	4	3	3	3

Now we recommence with 3rd alternative $\{ (7,3), (7,8)\}_{\text{cost } 9}$. Once again the infeasible lower bound will be 22 and requires no further investigation. Hence optimal TST will be of 18, which is the same as in the above problem.

4.3. According by Cowen[2]:

Reconsider the same figure 6 and our convenience, we modified the connected network**.** The vertex index for the network in figure 7 is given in table 2. The index is lowest at vertex 7, therefore it is selected as the starting vertex p. For full investigation, the number of combinations will be 3 given by $\{ (7,5), (7,8)\}_{(1+2\ )=\ \text{cost } 3}$. Its cost is (1+2) = 3. $\{(7,5), (7,3)\}_{(7+1)\ =\ \text{cost } 8}$. Its cost is (7+1) = 8. $\{ (7,3), (7,8)\}_{(7+2)=\text{cost } 9}$. Its cost is (7+2) = 9. The network in figure 7 is obtained after removing the vertex 7 and all the edges emanating from this vertex. Since edges (7,5) and (7,8) are the two minimum edges from vertex 1, we first set the link (3,7) = ∞ and find the Minimum Spanning Tree path joining vertices 3 and 8 passing through all the remaining vertices, which are vertices 1, 2,4 and 6,9. The Minimum Spanning Tree path of the network G'(8,13) is shown in figure 4.

The MST comprised of the links $\{ (5,2)_1 , (2,3)_2 , (1,4)_3$ and $(4,9)_4, (6,8)_5,(9,8)_6\}$. The total cost is 3+2+2+3+2+1=13 Note that vertex 3 is a high index vertex as its index value is 6, which should be 2. Similarly, vertex 7 is a low index vertex with index value 2, which should be 1. If we add 1 unit to all links emanating from vertex 8, an alternatives will be created and we can select (9,6), replacing the links (9,8). Thus, the Minimum Spanning Tree will be $\{(5,2)_1 , (2,3)_2 , (1,4)_3$ and $(4,9)_4, (6,8)_5,(9,6)_6\}$.The total cost is 3+2+2+3+2+2=15 which is a Minimum Spanning Tree path leading to a feasible Travelling Salesman Tour.

Length of TST = L(p,q) + L(p,k) + LMSTG'(n-1) = L(c_1) + LMSTPG'(n-1)

Length of Travelling Salesman Tour will be = 13+1+7 = 21, resulting in the Upper Bound(UB) = LTSTG (n) = 21.

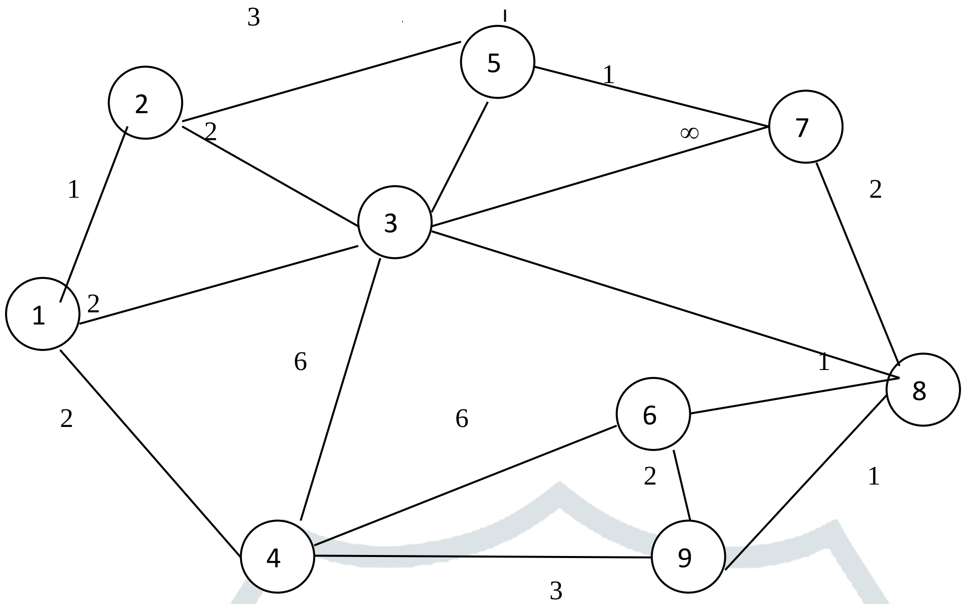

Figure 7: Modified network considered by Cowen [2]

Table 3: Vertex index for the network in figure 7.

Vertex	1	2	3	4	5	6	7	8	9
Index	3	3	4	4	4	3	3	4	3

For optimality, one has to investigate the remaining two more combinations, that is{(7,3), (7,8)}$_{(7+2)=\text{cost } 9}$ .Its cost is (7+2) = 9 .

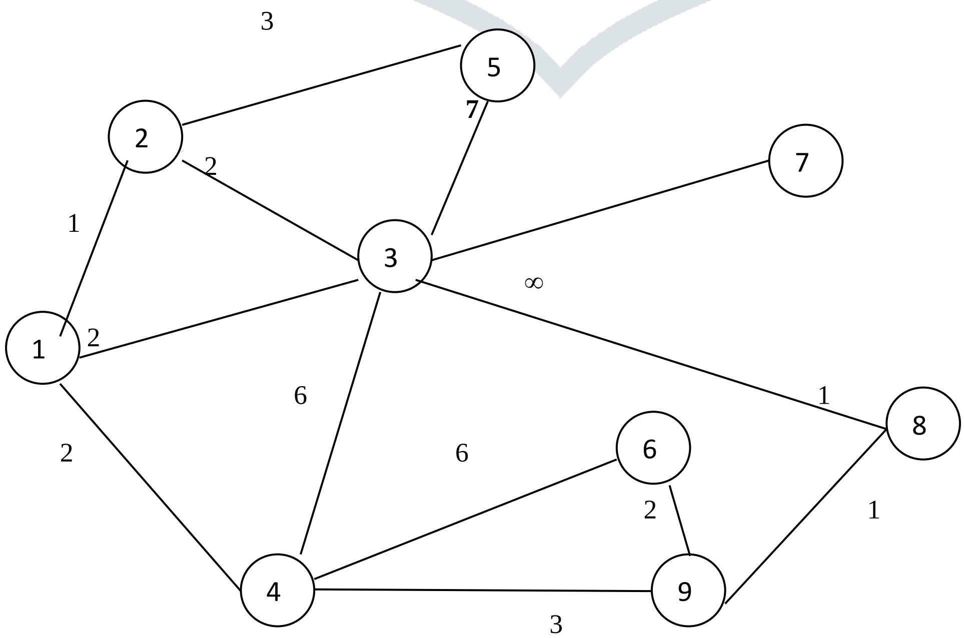

Figure 8: Modified network considered by Cowen [5]

Case Investigate { (7,5),(7,8)}

Set the link (3,8) = ∞ and find the MST, which will be formed of links (2,5),(5,6),(6,4) and (4,3) as shown in figure 5.
LEdge1 + LEdge2 + L^i MSTG'(n-1) ≤ LTSTG (n).The TST will be = L(1,3) + L(1,2) + MST path vertex 2 to 3. This will be length = {$(5,2)_1$, $(2,3)_2$, $(1,4)_3$ and $(4,9)_4$, $(6,8)_5$,$(9,8)_6$}. The total cost is 3+2+2+3+2+3+7=22. Thus, the Upper Bound is replaced by its new value of 22. This solution is as shown in figure 7 with the Travelling Salesman Problem . Its total distance is 22. The above Travelling Salesman Tour length is 21 is less than the solution obtained by Cowen [2]. Which had a TST of path is 5-2-3-1-4-9-6-8 and corresponding total length is 22.

Similarly one can investigate the last combination and find the TST length. The MST Path for this combination gives 5- 2- 3-1- 4 - 9 - 6 - 8, with TST cost = 18+1+2=22 which is > than 21. Thus, the optimal solution is 21 , as in the *case* .

V. comparison among the three conjectures

Conjecture 1:

This conjecture is minimum spanning tree converted into minimum spanning tree path repeated applications of index balancing theorem. It is based on the below inequality,

$$LMSTG'(n-1,m-d_p) \leq L^iMSTG'(n-1,m-d_p) \leq \ldots\ldots \leq L^iMSTG'(n-1,m-d_p) \leq \ldots \leq L^lMSTG'(n-1,m-d_p)$$

The high and low index values will be equal . The total index value will be 2(n-2). It is easy to understand .It is easy to find MST and it cannot find TST(travelling salesman tour). From this example, the optimal length of the path is 16 .

Conjecture 2:

In this conjecture **,**we add more than one edge with the given graph. It is similar to conjecture 1 . But it not necessary more restrictions. Because it have UB greater than or equal to the value of length of TST. It is more efficient to all other conjectures . It is very easy , fast and less computing time compare with other conjectures . It have UB > the optimal path of the length .This conjecture is followed below inequality,

$$L(p,q) + L(p,k) + L^iMSTG'(n-1,m-d_p) \geq LTSTG(n,m) .$$
(the current upper bound)

Conjecture 3:

This conjecture more difficult to compare with other conjectures. It have more computing time. It have lengthy computational process. It is not clear to understand. When at least one vertex in the given graph a low index value.

VI.CONCLUSION

The index value plays a major role in the proposed conjectures , when at least one vertex has a low index value at the lowest index value is m, the number of sub-problems solved will be given by m_{c_2} . In the completely connected n vertex network, the worst case will have $(n-1)_{c_2}$ combinations. The proposed conjectures converts the problem in the three parts to establish an upper bound. The approach discussed in this project uses link-weight modification to obtain alternative Minimum Spanning Trees, which eventually have a Travelling Salesman Tour interpretation. The worst case situation will arise in the case of a completely connected 'n' vertex network when each vertex will have an index value of (n-1), thus requiring comparisons of $(n-1)_{c_2}$ number of case to find the optimal tour . The proposed conjectures is likely to be more efficient when at least one vertex in the given network happen to have a low index value.

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