EFFICIENT LINEAR APPROXIMATE ML ESTIMATION FOR THE LOGISTIC DISTRIBUTION UNDER TYPE-II CENSORING

A. Vasudeva Rao
Professor
Department of Statistics
Acharya Nagarjuna University, Guntur, A.P., India.

Abstract:

For the logistic distribution, the maximum likelihood (ML) method does not yield explicit estimators for the location and scale parameters based on Type II censored samples. To address this, Balakrishnan(1992) derived the approximate MLEs (AMLEs) of the parameters, which are in non-linear form. In this paper, we construct some new AMLEs by making linear approximations to the intractable terms of the ML equations using least squares method, a new approach of linearization. Since, the new AMLEs are in linear form, we call them as linear AMLEs (LAMLEs). A Monte Carlo simulation study is made to investigate the performance of LAMLEs, as compared with MLEs and Balakrishnan's AMLEs; and we found that the LAMLEs are almost as efficient as MLEs and AMLEs. However, the LAMLEs are slightly biased than both MLEs and AMLEs. Further, we compare unbiased LAMLEs with the corresponding BLUEs based on the exact variances of the estimators and interestingly, these new unbiased LAMLEs are found just as efficient as the BLUEs in both complete and Type-II censored samples even in small samples. However, the construction of unbiased LAMLEs require only the means of order statistics of standard logistic distribution where as the construction of BLUEs require means as well as the variances and covariances of order statistics. Finally, we present some numerical examples to illustrate the construction of the new estimators developed here.

Index Terms- **Location and scale parameters of logistic distribution, Type-II censoring; Least squares method; Linear approximate MLEs; Unbiased linear approximate MLEs.**

I. INTRODUTION

The logistic distribution has several important biological, actuarial, industrial and engineering applications. The logistic distribution has been one of the most important statistical distributions because of its simplicity and also its historical importance as a growth curve (Erkelens, 1968). Its use in the context of bio – assay has been well demonstrated in a series of papers by Berkson (1944, 1951, 1953, 1955, 1957). The p.d.f. and c.d.f. of logistic distribution are respectively given by

$$p(x;\mu,\sigma) = \frac{e^{-(x-\mu)/\sigma}}{\sigma[1+e^{-(x-\mu)/\sigma}]^2}, \quad -\infty < x,\mu < \infty,\ \sigma>0 \tag{1.1}$$

$$P(x;\mu,\sigma) = \frac{1}{[1+e^{-(x-\mu)/\sigma}]}, \quad -\infty < x,\mu < \infty,\ \sigma>0 \tag{1.2}$$

where μ and σ are the location and scale parameters respectively. Let

$$X_{r+1} \le X_{r+2} \le \ldots \le X_{n-s}$$

be a doubly Type-II censored sample (*r* left most observations and *s* right most observations are censored from a planned sample of size n) from the above logistic population. Since, the doubly censored sample includes left censored sample (s=0), right censored sample (r=0) and complete sample (r=s=0), the following development includes all these cases. The likelihood function based on the above doubly censored sample is given by

$$L = \frac{n!}{r!s!}[\mathrm{P}(X_{r+1};\mu,\sigma)]^r[1\text{-}\mathrm{P}(X_{n-s};\mu,\sigma)]^s\prod_{i=r+1}^{n-s} p(X_i;\mu,\sigma)$$

which, by denoting

$$Z_i = (X_i - \mu)/\sigma \tag{1.3}$$

can be written as

$$L = \frac{n!}{r!s!}\sigma^{-(n-r-s)}[F(Z_{r+1})]^r[1\text{-}F(Z_{n-s})]^s\prod_{i=r+1}^{n-s} f(Z_i) \tag{1.4}$$

where $\quad f(Z) = e^{-Z}/(1+e^{-Z})^2 \quad$ and $\quad F(Z)=1/(1+e^{-Z})$ (1.5)

are respectively the p.d.f. and c.d.f. of standard logistic variate Z .
From Eq. (1.5) we may note that

$$f(Z) = F(Z)[1\text{-}F(Z)] \quad (1.6)$$

From the likelihood (1.4) the ML equations for μ and σ, after using the relation (1.6), are given by

$$\frac{\partial \log L}{\partial \mu} = -\frac{1}{\sigma}\left[r\{1-F(Z_{r+1})\} - sF(Z_{n-s}) + \sum_{i=r+1}^{n-s}\{1-2F(Z_i)\} \right] = 0 \quad (1.7)$$

$$\frac{\partial \log L}{\partial \sigma} = -\frac{1}{\sigma}\left[n-r-s+rZ_{r+1} + \sum_{i=r+1}^{n-s} Z_i - rZ_{r+1}F(Z_{r+1}) - sZ_{n-s}F(Z_{n-s}) - 2\sum_{i=r+1}^{n-s} Z_i F(Z_i) \right] = 0 \quad (1.8)$$

Since, the above ML equations do not admit explicit solutions for μ and σ, Balakrishnan (1992) derived the approximate maximum likelihood estimators (AMLEs) of μ and σ by expanding the function $F(Z_i)$ appearing in (1.7) and (1.8) in Taylor series (truncated at second and higher order derivatives) in a narrow neighborhood of the i^{th} standard logistic population quantile (defined from Eq.(1.5)),

$$\xi_i = F^{-1}(p_i) = \log\left(p_i/q_i\right), \text{ where } q_i = 1 - p_i \text{ and } p_i = i/(n+1),\ i = r+1,\dots,n-s \quad (1.9)$$

which is given by

$$F(Z_i) \approx \alpha_i + \beta_i Z_i \quad (1.10)$$

where $\beta_i = F'(\xi_i) = f(\xi_i) = p_i q_i$

and $\alpha_i = F(\xi_i) - \beta_i \xi_i = p_i - p_i q_i \log\left(p_i/q_i\right)$ (using Eqs. (1.6) & (1.9)) (1.11)

Here $F'(.) = f(.)$ is the derivative of the function $F(.)$. After discarding the details of the derivations (pl. see pp.178- 179, Balakrishnan and Cohen, 1991), the AMLEs derived by Balakrishnan (1992), denoted by $\hat{\mu}$ and $\hat{\sigma}$, are given by

$$\hat{\mu} = \text{B-}\hat{\sigma}C \quad \text{and} \quad \hat{\sigma} = \left(-D + \sqrt{D^2 + 4AE}\right)\Big/ 2A \quad (1.12)$$

where

$$\text{A}=n-r-s,\ \text{B}=\Phi^{-1}\Big[2\sum_{i=r+1}^{n-s}\beta_i X_i + r\beta_{r+1}X_{r+1} + s\beta_{n-s}X_{n-s}\Big]$$

$$C = \Phi^{-1}\Big[n\text{ - }s\text{ - }r\alpha_{r+1} - s\alpha_{n-s} - 2\sum_{i=r+1}^{n-s}\alpha_i\Big]$$

$$\text{D}=\sum_{i=r+1}^{n-s}(1-2\alpha_i)X_i + r(1-\alpha_{r+1})X_{r+1} - s\alpha_{n-s}X_{n-s} - \Phi BC$$

$$\text{E}=2\sum_{i=r+1}^{n-s}\beta_i X_i^2 + r\beta_{r+1}X_{r+1}^2 + s\beta_{n-s}X_{n-s}^2 - \Phi B^2, \quad \Phi = 2\sum_{i=r+1}^{n-s}\beta_i + r\beta_{r+1} + s\beta_{n-s}$$

Through a simulation study, Balakrishnan compared the AMLEs with the corresponding MLEs and BLUEs and recommended that the AMLEs are remarkably efficient.

Though the above AMLEs are in explicit form, their exact means and variances are not derived, because they are in complicated non-linear form. However, Balakrishnan (1992) simulated the approximations of the means, variances and covariance of the AMLEs, based on 10000 Monte Carlo runs, for various sample sizes and over various choices of censoring. Apart from the linear approximation (1.10) suggested by Balakrishnan (1992), if we make another linear approximation to the non-tractable function *G(z)=zF(z)* in the ML equation (1.8), then the resulting approximated ML equations will yield linear estimators of location and scale parameters in logistic distribution, so that we can easily obtain the exact means, biases and variances of the estimators. Moreover, the biasness of these linear estimators can easily be corrected, so that the resultant unbiased estimators can be comparable with the BLUEs.

With the above motivation, in this paper, we have derived some new AMLEs of location and scale parameters of logistic distribution based on doubly Type II censored samples by suggesting one more linear approximation, apart from the linear approximation (1.10). Since, the newly derived AMLEs are linear; we call them as linear AMLEs (LAMLEs). For making linear approximations to the non-tractable functions $F(Z)$ and $G(Z)$, we adopt two approaches of linearization namely,

1. Truncated Taylor series, as explained above and is adopted by Balakrishnan (1992) to derive the AMLEs given in Eq. (1.12).
2. Principle of least squares method, which is suggested by Vasudeva Rao *et al.* (2017) for constructing LAMLE of the scale parameter in scaled Type I GLD.

In Section 2 of the paper, we present the derivation details of the LAMLEs along with the exact means, variances and covariance of the estimators. Section 3 is devoted for relative comparison of the LAMLEs based on least squares method with the corresponding LAMLEs constructed based on Taylor series method. Based on the sampling characteristics bias and MSE, a Monte Carlo simulation study is made to examine the performance of the LAMLEs as compared the corresponding MLEs and AMLEs; the results are presented in Section 4. In Section 5, we obtain unbiased LAMLEs by making bias correction to LAMLEs and compare them with the corresponding BLUEs based on the exact variances of the estimators. The newly constructed estimators are demonstrated with two real data sets and the results are presented in Section 6.

II. LINEAR APPROXIMATE MAXIMUM LIKELIHOOD ESTIMATION

As explained in the above section, we derive some new approximate MLEs of μ and σ by suggesting one more linear approximation, which is given below, in the ML equation (1.8) apart from the linear approximation (1.10) so that the approximated ML equations yield linear estimators for μ and σ.

$$G(Z_i) = Z_i F(Z_i) \approx \gamma_i + \delta_i Z_i \qquad (i = r+1, \ldots, n-s) \tag{2.1}$$

Here δ_i and γ_i can be obtained by expanding the function $G(Z_i)$, using Taylor series, in a narrow neighborhood of ξ_i, where ξ_i is as defined in Eq. (1.9), and are given by

$$\delta_i = G'(\xi_i) = F(\xi_i) + \xi_i f(\xi_i) = p_i + p_i q_i \log\left(p_i / q_i\right)$$

$$\text{and} \quad \gamma_i = G(\xi_i) - \delta_i \xi_i = \left(p_i - \delta_i\right) \log\left(p_i / q_i\right) \quad \text{(using Eqs. (1.6) \& (1.9))} \tag{2.2}$$

Upon using the linear approximations (1.10) and (2.1), the ML equations (1.7) and (1.8) will be approximated as

$$\frac{\partial \log L}{\partial \mu} \cong \frac{\partial \log L^*}{\partial \mu} = -\frac{1}{\sigma}\left[n - s - r\alpha_{r+1} - s\alpha_{n-s} - 2\sum_{i=r+1}^{n-s} \alpha_i - r\beta_{r+1} Z_{r+1} - s\beta_{n-s} Z_{n-s} - 2\sum_{i=r+1}^{n-s} \beta_i Z_i \right] = 0 \tag{2.3}$$

$$\frac{\partial \log L}{\partial \sigma} \cong \frac{\partial \log L^*}{\partial \sigma} = -\frac{1}{\sigma}\left[n - s - r + r\gamma_{r+1} - s\gamma_{n-s} - 2\sum_{i=r+1}^{n-s} \gamma_i + r(1-\delta_{r+1}) Z_{r+1} - s\delta_{n-s} Z_{n-s} + \sum_{i=r+1}^{n-s} (1 - 2\delta_i) Z_i \right] = 0 \tag{2.4}$$

Upon using (1.3) in Eqs. (2.3) and (2.4); and solving them jointly for solutions of μ and σ, denoted by $\tilde{\mu}$ and $\tilde{\sigma}$, which are given by

$$\tilde{\mu} = \sum_{i=r+1}^{n-s} \lambda_i X_i = \boldsymbol{\lambda}'\mathbf{x} \quad \text{and} \quad \tilde{\sigma} = \sum_{i=r+1}^{n-s} \omega_i X_i = \boldsymbol{\omega}'\mathbf{x} \tag{2.5}$$

where $\mathbf{x} = (X_{r+1}, \ldots, X_{n-s})'$, $\boldsymbol{\lambda} = (\lambda_{r+1}, \ldots, \lambda_{n-s})'$, $\boldsymbol{\omega} = (\omega_{r+1}, \ldots, \omega_{n-s})'$ (2.6)

$$\lambda_i = [cm_i - dl_i]/\Delta, \quad \omega_i = [l_i \sum_{j=r+1}^{n-s} m_j - m_i \sum_{j=r+1}^{n-s} l_j]/\Delta, \quad \Delta = c\sum_{j=r+1}^{n-s} m_j - d\sum_{j=r+1}^{n-s} l_j$$

$$c = n - s - r\alpha_{r+1} - s\alpha_{n-s} - 2\sum_{i=r+1}^{n-s} \alpha_i, \quad d = n - r - s - r\gamma_{r+1} - s\gamma_{n-s} - 2\sum_{i=r+1}^{n-s} \gamma_i$$

$$l_i = \begin{cases} (2+r)\beta_i & \text{if } i = r+1 \\ 2\beta_i & \text{if } r+1 < i < n-s \\ (2+s)\beta_i & \text{if } i = n-s \end{cases} \quad \text{and} \quad m_i = \begin{cases} (2+r)\delta_i - r - 1 & \text{if } i = r+1 \\ 2\delta_i - 1 & \text{if } r+1 < i < n-s \\ (2+s)\delta_i - 1 & \text{if } i = n-s \end{cases}$$

Since, $\tilde{\mu}$ and $\tilde{\sigma}$ are in linear form, we call them as linear approximate MLEs (LAMLEs). From Eq. (2.6), we may notice that

$$\sum_{i=r+1}^{n-s} \lambda_i = 1 \quad \text{and} \quad \sum_{i=r+1}^{n-s} \omega_i = 0 \tag{2.7}$$

The means, biases, variances, covariance and standard errors of $\tilde{\mu}$ and $\tilde{\sigma}$ can be obtained easily and are given by

$$E(\tilde{\mu}) = \mu + \sigma\boldsymbol{\lambda}'\mathbf{a}, \; E(\tilde{\sigma}) = \sigma\boldsymbol{\omega}'\mathbf{a}, \; Bias(\tilde{\mu}) = \sigma\boldsymbol{\lambda}'\mathbf{a} \text{ and } Bias(\tilde{\sigma}) = \sigma(\boldsymbol{\omega}'\mathbf{a} - 1) \tag{2.8}$$

$$Var\,(\tilde{\mu}) = \sigma^2\boldsymbol{\lambda}'\boldsymbol{\Omega}\boldsymbol{\lambda}, \; Var\,(\tilde{\sigma}) = \sigma^2\boldsymbol{\omega}'\boldsymbol{\Omega}\boldsymbol{\omega} \text{ and } Cov(\tilde{\mu}, \tilde{\sigma}) = \sigma^2\boldsymbol{\lambda}'\boldsymbol{\Omega}\boldsymbol{\omega} \tag{2.9}$$

$$SE(\tilde{\mu}) = \tilde{\sigma}\sqrt{\boldsymbol{\lambda}'\boldsymbol{\Omega}\boldsymbol{\lambda}} \quad \textit{and} \quad SE(\tilde{\sigma}) = \tilde{\sigma}\sqrt{\boldsymbol{\omega}'\boldsymbol{\Omega}\boldsymbol{\omega}} \tag{2.10}$$

where $\mathbf{a} = (a_{r+1:n}, \ldots, a_{n-s:n})'$ and $\boldsymbol{\Omega} = ((b_{ij:n}))$, $r+1 \le i, j \le n-s$ are respectively the mean vector and variance-covariance matrix of the order statistics ($Z_{r+1} \le Z_{r+2} \le \ldots \le Z_{n-s}$) of standard logistic distribution. The elements of $\mathbf{a}$ and $\boldsymbol{\Omega}$ are tabulated by Balakrishnan and Malik (1995) for sample size up to 50 and these values can be used to compute the above means, biases, variances, covariance and standard errors of the LAMLEs $\tilde{\mu}$ and $\tilde{\sigma}$.

From Eq. (2.6), it may be noted that the basic requirement for the computation of the linear coefficients $\boldsymbol{\lambda}$ of $\tilde{\mu}$ and $\boldsymbol{\omega}$ of $\tilde{\sigma}$ is the evaluation of the constants (α_i, β_i) and (γ_i, δ_i) (for i=1,...,n) and those can be computed using the Eqs. (1.11) and (2.2), which are obtained by applying truncated Taylor series to the functions $F(Z_i)$ and $G(Z_i)$. This approach of linear approximation was originally suggested by Balakrishnan (1989) for deriving approximate MLE of the scale parameter of Rayleigh distribution based on censored samples. Later, this approach was extensively used by many authors while obtaining approximate MLEs of the parameters in different probability models under various types of censoring.

For evaluating the constants (α_i, β_i) and (γ_i, δ_i) (for i=1,...,n), we also adopt a new approach of linearization namely, least squares method, which is suggested recently by Vasudeva Rao *et al.* (2017) for constructing linear approximate MLE of the scale parameter in scaled Type I GLD. We briefly explained this method below with respect to logistic model.

The motivation for LS method of linear approximation is based on the fact that *a small part of any (non-linear) curve can be well approximated by a straight line, which is fitted by applying the least squares method to a set of points on the part of the curve.*

The order statistics $Z_{r+1}, Z_{r+2}, ..., Z_{n-s}$ of standard logistic model are assumed to lie in narrow neighborhoods of the corresponding standard population quantiles, which are defined by (from Eq. (1.5))

$$\xi_i = F^{-1}(p_i) = \log_e\left(p_i/q_i\right), \text{ where } p_i = \frac{i-0.3}{n+0.4}, \; i = r+1, ..., n-s \tag{2.11}$$

Here, it may be noted that, with respect to the choice of p_i, the definition of the population quantile ξ_i given in Eq. (1.9) is different from that given in the above equation, which is a modified one according to the latest formula (please see Paul and David, 1995, ch.6, pp.147).

From the above, we have $p_i - p_{i-1} = 1/(n+0.4)$, so that the probabilities $p_{r+1}, ..., p_{n-s}$ are in equi-distant. Now, consider the probability interval

$$\left[p_i - 0.5/(n+0.4),\; p_i + 0.5/(n+0.4)\right]$$

so that p_i is the midpoint of the interval. Now, for a given natural number *m*, we define the probabilities

$$p_{ij} = p_i + \frac{j-m-1}{2m(n+0.4)}, \quad j = 1, 2, ..., 2m+1$$

which are spread over uniformly in the above interval evenly and p_i is the middle value. Using the above probabilities, we define

$$z_{ij} = F^{-1}(p_{ij}) = \log_e\left(p_{ij}/q_{ij}\right), \; q_{ij} = 1 - p_{ij}, \; j = 1, 2, ..., 2m+1$$

so that z_{ij}'s lie in a neighborhood of ξ_i. The above definition of p_{ij} ensures that the values of z_{ij}'s will be distributed evenly in the entire neighborhood of ξ_i so that $z_{i,2m+1}$ is the overlapping point of the neighborhoods of ξ_i and ξ_{i+1}. Corresponding to the z_{ij}'s we compute the functional values $F(z_{ij})$'s and then the bivariate data $(z_{ij}, F(z_{ij}))$, $j = 1, 2, ..., 2m+1$, represent the points on the curve $F(z)$ in a neighborhood of ξ_i. Now, based on this bivariate data, if we apply the traditional least squares method for fitting of the linear approximation (1.10), we get

$$\beta_i = \left[\sum_{j=1}^{2m+1} z_{ij}F(z_{ij}) - (2m+1)\bar{z}_i\overline{F(z_i)}\right] \Bigg/ \left[\sum_{j=1}^{2m+1} z_{ij}^2 - (2m+1)\bar{z}_i^2\right] \text{ and } \alpha_i = \overline{F(z_i)} - \beta_i\,\bar{z}_i \tag{2.12}$$

$$\text{where } \bar{z}_i = \frac{1}{2m+1}\sum_{j=1}^{2m+1} z_{ij}, \; \overline{F(z_i)} = \frac{1}{2m+1}\sum_{j=1}^{2m+1} F(z_{ij})$$

Similarly, γ_i and δ_i of the linear approximation (2.1) are given by

$$\delta_i = \left[\sum_{j=1}^{2m+1} z_{ij}G(z_{ij}) - (2m+1)\bar{z}_i\overline{G(z_i)}\right] \Bigg/ \left[\sum_{j=1}^{2m+1} z_{ij}^2 - (2m+1)\bar{z}_i^2\right] \text{ and } \gamma_i = \overline{G(z_i)} - \delta_i\,\bar{z}_i \tag{2.13}$$

$$\text{where } \bar{z}_i = \frac{1}{2m+1}\sum_{j=1}^{2m+1} z_{ij}, \; \overline{G(z_i)} = \frac{1}{2m+1}\sum_{j=1}^{2m+1} G(z_{ij})$$

Remark: In the above method, we may take any *m*≥1. However, Vasudeva Rao *et al.* (2017) have suggested an optimal choice of *m*=5, and we have taken the same choice of *m* in this paper.

III. RELATIVE COMPARISON OF THE LAMLEs

In this section, we compare the LAMLEs constructed above using the two approaches of linearization are compared based on bias and mean square error (MSE). We denote the LAMLEs constructed based on Taylor series method and least squares method respectively by $(\tilde{\mu}_{TS}, \tilde{\sigma}_{TS})$ and $(\tilde{\mu}_{LS}, \tilde{\sigma}_{LS})$.

The bias and MSE (variance+bias^2) of LAMLEs are computed using the Eqs. (2.8) and (2.9) and are presented in *Table 1* for complete and doubly Type-II censored samples of size n=5&10 with all possible choices of *r* and *s*. As mentioned in the above section, the necessary means and variances of order statistics of standard logistic distribution are borrowed from Balakrishnan and Malik (1995). From *Table 1*, it is evident that both biases and MSEs of $(\tilde{\mu}_{LS}, \tilde{\sigma}_{LS})$ are slightly smaller than those of $(\tilde{\mu}_{TS}, \tilde{\sigma}_{TS})$ at any given n, r and s values. Thus, the LAMLEs constructed based on least squares method are slightly less biased and more efficient than those constructed based on Taylor series method in both complete and Type-II censored samples.

Thus, when compared with Taylor series method, least squares method of evaluation of the linear approximations (1.10) and (2.1) is yielding slightly better LAMLEs of μ and σ in terms of both biasness and MSE. This is due to the fact that for instance in evaluating the linear approximation given in Eq. (1.10), Taylor series method take into account only one point namely $(\xi_i, F(\xi_i))$ on the curve, because, geometrically, in this method the part of the curve around ξ_i will be approximated by the tangent line drawn to the curve at ξ_i; whereas least squares method take into account 2m+1 =11 (for m=5) points on the curve around ξ_i including $(\xi_i, F(\xi_i))$. Here, it may be noted that unlike Taylor series method, least squares method does not require the derivatives of the functions $F(.)$ and $G(.)$.

Note: Here onwards LAMLEs means that the LAMLEs constructed based on least squares method and we retain the notation $(\tilde{\mu},\tilde{\sigma})$ for LAMLEs of (μ,σ).

IV. COMPARSION OF THE LAMLES WITH MLEs AND AMLEs

Though both AMLEs and LAMLEs are in explicit form, we cannot compare them analytically, because AMLEs do not have exact means and variances (as mentioned in Section 1) like LAMLEs. Therefore, we can compare them only through a simulation study. Hence, in order to assess the performance of LAMLEs, we compare them with the corresponding AMLEs and MLEs based on a Monte Carlo simulation study. We have to compute the MLEs of (μ,σ) by solving the equations (1.7) and (1.8) using some iterative procedure and here, we use *Newton-Raphson* iterative method, which yields numerical solution of (μ,σ) using the following iterative equation.

$$\begin{pmatrix}\mu^{(i)}\\ \sigma^{(i)}\end{pmatrix}=\begin{pmatrix}\mu^{(i-1)}\\ \sigma^{(i-1)}\end{pmatrix}-\left[\begin{matrix}\dfrac{\partial^2\log L}{\partial\mu^2} & \dfrac{\partial^2\log L}{\partial\mu\partial\sigma}\\ \dfrac{\partial^2\log L}{\partial\mu\partial\sigma} & \dfrac{\partial^2\log L}{\partial\sigma^2}\end{matrix}\right]^{-1}_{\begin{pmatrix}\mu\\ \sigma\end{pmatrix}=\begin{pmatrix}\mu^{(i-1)}\\ \sigma^{(i-1)}\end{pmatrix}}\left[\begin{matrix}\dfrac{\partial\log L}{\partial\mu}\\ \dfrac{\partial\log L}{\partial\sigma}\end{matrix}\right]_{\begin{pmatrix}\mu\\ \sigma\end{pmatrix}=\begin{pmatrix}\mu^{(i-1)}\\ \sigma^{(i-1)}\end{pmatrix}}\qquad i=1,2,\ldots \tag{4.1}$$

where $\dfrac{\partial\log L}{\partial\mu}$ and $\dfrac{\partial\log L}{\partial\sigma}$ are given in Eqs. (1.7) and (1.8) respectively; their second order derivatives $\dfrac{\partial^2\log L}{\partial\mu^2}$, $\dfrac{\partial^2\log L}{\partial\sigma^2}$ and $\dfrac{\partial^2\log L}{\partial\mu\partial\sigma}$ can be obtained directly and the details of derivation are omitted here. To initiate the iterative equation (4.1), we use the LAMLEs $(\tilde{\mu},\tilde{\sigma})$ as the initial solution $(\mu^{(0)},\sigma^{(0)})$ of (μ,σ); and the MLEs of (μ,σ) thus obtained are denoted by $(\hat{\mu},\hat{\sigma})$.

We have simulated the biases and MSEs of $(\tilde{\mu},\tilde{\sigma})$, $(\hat{\mu},\hat{\sigma})$ and $(\hat{\mu},\hat{\sigma})$ based on 1000 samples of sizes n=10, 20 and 30 generated from standard logistic distribution. We tabulate these biases and MSEs in *Table 2* for specified values of n=10(10)30, r=0(1)n/4 and s=0(1)n/2 and the table reveals the following brief observations.

- Generally, with respect to bias, $\tilde{\mu}$ and $\hat{\mu}$ are almost equally biased and are slightly more biased when compared with $\hat{\mu}$. However, $\hat{\sigma}$ is less biased when compared with both $\tilde{\sigma}$ and $\hat{\sigma}$.
- With respect to MSE, all the three estimators $(\tilde{\mu},\tilde{\sigma})$, $(\hat{\mu},\hat{\sigma})$ and $(\hat{\mu},\hat{\sigma})$ are performing almost equally irrespective of sample size, scheme of censoring and magnitude of censoring.

Conclusion: *Based on MSE, the LAMLEs constructed in Section 2 are almost as efficient as both MLEs and AMLEs in all left, right and doubly Type II censored samples even in heavy censoring. However, LAMLEs are slightly more biased when compared with MLEs.*

V. CONSTRUCTION OF UNBIASED LAMLEs AND COMPARSION WITH BLUEs

Though the LAMLEs, given in Eq. (2.5), are empirically proved almost as efficient as the MLEs, they are biased than MLEs. In particular, $\tilde{\sigma}$ is much biased than $\hat{\sigma}$. However, in complete and symmetric censored samples, $\tilde{\mu}$ is an unbiased estimator. Since, unbiasedness is a desirable property of a good estimator, from Eq. (2.8), we have obtained unbiased estimators of (μ,σ), denoted by (μ^*,σ^*), are given by

$$\mu^*=\tilde{\mu}-\boldsymbol{\lambda}'\mathbf{a}(\boldsymbol{\omega}'\mathbf{a})^{-1}\tilde{\sigma}\quad\text{and}\quad\sigma^*=(\boldsymbol{\omega}'\mathbf{a})^{-1}\tilde{\sigma} \tag{5.1}$$

Since, the above estimators obtained from the LAMLEs by adjusting them for unbiasedness we call them as *unbiased* LAMLEs. For constructing $(\tilde{\mu},\tilde{\sigma})$ we do not require **a**, but for the construction of (μ^*,σ^*), we require **a**, whose components can be obtained from Balakrishnan and Malik (1995) for up to a sample size n=50. Alternatively, the components of **a** can be computed using the following recurrence relation (pl. see pages 40-41 for theorem 3.6.1 with k=1 in Balakrishnan and Cohen, 1991).

$$a_{i+1:n+1}=a_{i:n}+1/i\quad\text{for } 1\le i\le n,\text{ where } a_{1:1}=0\ \text{ and }\ a_{1:n+1}=-a_{n+1:n+1} \tag{5.2}$$

Using Eq. (5.2), we tabulate the means of order statistics in standard logistic distribution in *Table 3* for sample size n=2(1)100.

The variances, covariance and standard errors of (μ^*,σ^*) are given by

$$Var\ (\mu^*)=\sigma^2\left[\boldsymbol{\lambda}'\boldsymbol{\Omega}\boldsymbol{\lambda}+(\boldsymbol{\lambda}'\mathbf{a})^2(\boldsymbol{\omega}'\mathbf{a})^{-2}\boldsymbol{\omega}'\boldsymbol{\Omega}\boldsymbol{\omega}-2\boldsymbol{\lambda}'\mathbf{a}(\boldsymbol{\omega}'\mathbf{a})^{-1}\boldsymbol{\lambda}'\boldsymbol{\Omega}\boldsymbol{\omega}\right] \tag{5.3}$$

$$Var\ (\sigma^*)=\sigma^2(\boldsymbol{\omega}'\mathbf{a})^{-2}\boldsymbol{\omega}'\boldsymbol{\Omega}\boldsymbol{\omega}\quad\text{and}\quad Cov(\mu^*,\sigma^*)=\sigma^2\left[(\boldsymbol{\omega}'\mathbf{a})^{-1}\boldsymbol{\lambda}'\boldsymbol{\Omega}\boldsymbol{\omega}-(\boldsymbol{\lambda}'\mathbf{a})(\boldsymbol{\omega}'\mathbf{a})^{-2}\boldsymbol{\lambda}'\boldsymbol{\Omega}\boldsymbol{\lambda}\right]$$

$$SE(\mu^*)=\sigma^*\sqrt{Var\ (\mu^*)/\sigma^2}\quad\text{and}\quad SE(\sigma^*)=\sigma^*\sqrt{Var\ (\sigma^*)/\sigma^2}$$

It is well known that among the class of linear unbiased estimators best linear unbiased estimators (BLUEs) are the most efficient estimators since they have least variance. Lloyd (1952) obtained explicit formulae for the BLUEs of location and scale parameters in any location-scale family distribution based on a given Type-II censored sample. According to these formulae, the evaluation of linear coefficients of the BLUEs requires means, variances and covariances of order statistics obtained from the corresponding standardized population.

In the literature, there are many types of simplified linear estimators as alternatives to the BLUEs. The well-known Gupta's simplified linear estimators proposed by Gupta (1952) are such simplified linear estimators and are unbiased. The construction of these simplified linear estimators requires only the means of order statistics. These estimators gave good results in case of normal distribution and it is found that they are performing with almost 99% efficiency as compared with BLUEs. Some other simplified linear estimators, referred to as *unbiased nearly best linear estimators* of location and scale parameters of an arbitrary distribution have been proposed by Blom (1962). Downton (1966) has also proposed some simplified linear estimators for the location and scale parameters.

Just like the above simplified linear estimators, the construction of unbiased LAMLEs requires only the means of order statistics. Since, (μ^*, σ^*) are linear unbiased estimators, we compare them with the BLUEs of (μ, σ), denoted by (μ^{**}, σ^{**}), based on the exact variances of the estimators. We tabulate the relative efficiencies of (μ^*, σ^*) in relation to (μ^{**}, σ^{**}) in *Table 4* from complete and Type-II censored samples of size n =5(5)10 with all possible choices of *r* and *s*. *Table 4* reveals the following conclusion.

- *In both complete as well as Type-II censored samples (even censoring is heavy) (μ^*, σ^*) are performing with more than 99% efficiency as compared with the respective (μ^{**}, σ^{**}). In other words, the unbiased LAMLEs are just as efficient as the BLUEs in both complete and censored samples irrespective of its sample size (however small it may be) and magnitude of censoring.*

A Remark: *Bias corrected* AMLEs are compared with the BLUEs through a simulation study; whereas *unbiased* LAMLEs are compared with the BLUEs analytically.

Conclusion of the Study: Thus, our investigation of some alternative efficient linear estimators to MLEs of μ and σ in logistic distribution is surprisingly produces some remarkably efficient new linear unbiased estimators (*unbiased* LAMLEs), which are just as efficient as BLUEs. Hence, just like as the LAMLEs are some alternative efficient estimators to MLEs, the *unbiased* LAMLEs are some alternative efficient estimators to the BLUEs for the estimation of location and scale parameters in logistic distribution. The Construction of BLUEs requires not only the means of standardized order statistics, but also variances and covariances of standardized order statistics, where as the construction of *unbiased* LAMLEs requires only means of standardized order statistics. Another advantage is that unlike BLUEs, no readymade tables are required for the computation of the *unbiased* LAMLEs and we may compute them easily based on LAMLEs. Therefore, we recommend unbiased LAMLEs to replace the BLUEs, especially when it is difficult to compute the BLUEs.

VI. ILLUSTRATIOINS

Example 1:

Sarhan and Greenberg (1962) have presented data resulting from an experiment in which students were learning to measure strontium-90 concentrations in samples of milk. The test substance was supposed to contain 9.22 picocuries per liter. The measurements, each involving readings and calculations, were made, but, since the measurement error was known to be relatively larger at the extremes, especially the upper one, a decision was made to censor the two smallest and the three largest observations, leaving the following Type-II censored sample

8.2, 8.4, 9.1, 9.8, 9.9

This Type-II censored sample has been used by Harter and Moore (1967) to calculate the MLEs of (μ, σ). We compute the LAMLEs for the above sample using Eq. (2.5) by taking n=10, r=2 and s=3. In the following table, we present the LAMLEs along with MLEs and AMLEs.

LAMLEs	**MLEs** (Harter and Moore (1967))	**AMLEs** (Pl. see page 185 of Balakrishnan and Cohen, 1991)
$\tilde{\mu}$= 9.2683 $\tilde{\sigma} = 0.8469$ $SE(\tilde{\mu})$=0.4830 $SE(\tilde{\sigma}) = 0.2964$	$\hat{\mu} = 9.2718$ $\hat{\sigma} = 1.5678/(\pi/\sqrt{3}) = 0.8644$ $SE(\hat{\mu})$=0.4908 $SE(\hat{\sigma}) = 0.5653/(\pi/\sqrt{3}) = 0.3117$	$\hat{\mu}$=9.2683 $\hat{\sigma} = 1.5755/(\pi/\sqrt{3}) = 0.8686$ $SE(\hat{\mu})$=0.4961 $SE(\hat{\sigma}) = 0.5685/(\pi/\sqrt{3}) = 0.3134$

The standard errors of the LAMLEs, presented in the above table, are computed using Eq. (2.10) borrowing the necessary variances and covariances of order statistics of standard logistic distribution from Balakrishnan and Malik (1995). The MLEs obtained by us using Eq.(4.1) are $(\hat{\mu}, \hat{\sigma}) = (9.2718, 0.8643)$, which are agreed with those obtained by Harter and Moore (1967). From the above table, we may observe that the LAMLE and AMLE of μ are same and are slightly deviated from the corresponding MLE; whereas the LAMLE of σ is slightly deviated from the corresponding both AMLE and MLE. However, the standard errors of the LAMLEs are smaller than those of AMLEs as well as MLEs.

Similarly, in the following table we present unbiased LAMLEs, which are computed from Eq. (5.1) using the necessary means of order statistics from *Table 3* with n=10. The standard errors of the estimates are computed by using the tabulated variances of the unbiased LAMLEs in *Table 4* with n=10, r=2, s=3. For comparison purpose, we present the BLUEs and their standard errors; and from the table we may notice that unbiased LAMLEs and their standard errors are very close to the respective BLUEs and their standard errors.

Unbiased LAMLEs	**BLUEs** (Pl. see page 184 of Balakrishnan and Cohen, 1991)
μ* = 9.3029, σ*= 1.0344 SE(μ*)= $\sigma * \sqrt{0.3264}$ =0.5910 SE(σ*)= $\sigma * \sqrt{0.1834}$ = 0.4430	μ** =9.3032, σ**=1.8758 $/(\pi/\sqrt{3})$ = 1.0342 SE(μ**) =0.5908, SE(σ**)=0.8033 $/(\pi/\sqrt{3})$ =0.4428

Example 2:
Davis (1952) has given lifetimes in hours of 417 40-watt incandescent lamps taken from 42 weekly forced-life test samples. Davis has indicated that the normal distribution provides a good model for these data, although the frequency distribution of the data appears to be somewhat more peaked in the center and flatter in the flanks than the normal distribution. This is indeed how the logistic distribution compares with the normal as pointed out by Chew (1968), and so the assumption of the logistic distribution will be quite appropriate for this data. The first 20 observations from the data are given below, after having been arranged in increasing order of magnitude.

785, 855, 905, 918, 919, 920, 929, 936, 948, 950, 972, 1035, 1045, 1067, 1092, 1126, 1156, 1162, 1170, 1196.

For the sake of illustration, we have censored 50% of largest observations. Thus, here we have n=20, r=0 and s=10. For this censored sample, we compute MLEs, LAMLEs and unbiased LAMLEs of μ and σ. For comparison purpose, we have presented these estimates along with AMLEs and BLUEs. The standard errors of the estimates are given in brackets.

ESTIMATE OF μ					**ESTIMATE OF σ**				
MLE	AMLE	LAMLE	ULAMLE	BLUE	MLE	AMLE	LAMLE	ULAMLE	BLUE
953.79 (14.45)	953.64 (15.34)	953.31 (14.67)	956.40 (16.42)	956.28 (16.31)	33.28 (9.03)	34.63 (9.63)	33.18 (8.90)	36.45 (10.74)	36.21 (10.67)

The AMLEs, BLUEs and their standard errors are barrowed from pages 185-186 of Balakrishnan and Cohen (1991). However, the estimates of σ and their standard errors given above are obtained by dividing the corresponding actual values with $\pi/\sqrt{3}$.

From the above example, it is observed that LAMLEs and their standard errors are close to the respective MLEs and their standard errors even in case of heavy censoring, which ascertain our theoretical conclusion namely these LAMLEs are almost as efficient as the corresponding MLEs. Similarly, we may observe that the unbiased LAMLEs are very close to the corresponding BLUEs. The standard errors of the corresponding estimates are also very close to each other.

Thus, the above two illustrations are ascertaining our theoretical findings that LAMLES are almost as efficient as the respective MLEs and the unbiased LAMLEs are just as efficient as the corresponding BLUEs.

Table 1: Relative performance of LAMLEs $(\hat{\mu}_{TS}, \hat{\sigma}_{TS})$ and $(\hat{\mu}_{LS}, \hat{\sigma}_{LS})$ of the parameters (μ, σ) of logistic distribution based on exact bias and exact MSE from complete and Type II censored samples of size n=5&10 with all possible choices of r and s.

n	r	s	$Bias/\sigma$		MSE/σ^2		n	r	s	$Bias/\sigma$		MSE/σ^2	
			$\tilde{\mu}_{TS}$	$\tilde{\mu}_{LS}$	$\tilde{\mu}_{TS}$	$\tilde{\mu}_{LS}$				$\tilde{\sigma}_{TS}$	$\tilde{\sigma}_{LS}$	$\tilde{\sigma}_{TS}$	$\tilde{\sigma}_{LS}$
5	0	0	0.0000	0.0000	0.6336	0.6282	5	0	0	-0.1684	-0.1408	0.1461	0.1454
	0	1	-0.0888	-0.0502	0.6503	0.6445		0	1	-0.2296	-0.1952	0.1909	0.1888
	0	2	-0.2542	-0.1932	0.7807	0.7635		0	2	-0.3230	-0.2826	0.2755	0.2720
	1	0	0.0888	0.0502	0.6501	0.6444		1	0	-0.2296	-0.1952	0.1904	0.1883
	1	1	0.0000	0.0000	0.6525	0.6524		1	1	-0.3307	-0.2904	0.2700	0.2649
	1	2	-0.1987	-0.1814	0.7649	0.7615		1	2	-0.5145	-0.4766	0.4472	0.4392
	2	0	0.2542	0.1932	0.7807	0.7635		2	0	-0.3230	-0.2826	0.2747	0.2712
	2	1	0.1987	0.1814	0.7649	0.7615		2	1	-0.5145	-0.4766	0.4472	0.4392
	3	0	0.6473	0.5602	1.4262	1.3770		3	0	-0.5025	-0.4539	0.4530	0.4476
10	0	0	0.0000	0.0000	0.3091	0.3076	10	0	0	-0.0780	-0.0737	0.0714	0.0713
	0	1	-0.0193	-0.0088	0.3097	0.3087		0	1	-0.0926	-0.0857	0.0795	0.0792
	0	2	-0.0431	-0.0279	0.3144	0.3130		0	2	-0.1085	-0.1009	0.0914	0.0911
	0	3	-0.0754	-0.0565	0.3281	0.3261		0	3	-0.1281	-0.1198	0.1078	0.1074
	0	4	-0.1219	-0.0994	0.3606	0.3573		0	4	-0.1537	-0.1444	0.1309	0.1304
	0	5	-0.1934	-0.1662	0.4331	0.4277		0	5	-0.1893	-0.1784	0.1643	0.1637
	0	6	-0.3123	-0.2784	0.5988	0.5894		0	6	-0.2423	-0.2287	0.2155	0.2147
	0	7	-0.5347	-0.4899	1.0170	0.9993		0	7	-0.3301	-0.3123	0.3020	0.3010
	0	8	-1.0418	-0.9817	2.3387	2.3008		0	8	-0.5030	-0.4801	0.4762	0.4748
	1	0	0.0193	0.0088	0.3097	0.3086		1	0	-0.0926	-0.0857	0.0795	0.0793
	1	1	0.0000	0.0000	0.3098	0.3094		1	1	-0.1107	-0.1006	0.0896	0.0892
	1	2	-0.0248	-0.0200	0.3136	0.3132		1	2	-0.1314	-0.1202	0.1049	0.1043
	1	3	-0.0597	-0.0513	0.3265	0.3258		1	3	-0.1579	-0.1457	0.1268	0.1260
	1	4	-0.1125	-0.1005	0.3594	0.3579		1	4	-0.1944	-0.1810	0.1591	0.1581
	1	5	-0.1984	-0.1822	0.4384	0.4350		1	5	-0.2484	-0.2335	0.2093	0.2080
	1	6	-0.3536	-0.3325	0.6362	0.6302		1	6	-0.3373	-0.3206	0.2952	0.2934
	1	7	-0.6870	-0.6622	1.2138	1.2028		1	7	-0.5110	-0.4941	0.4703	0.4680
	2	0	0.0431	0.0279	0.3146	0.3132		2	0	-0.1085	-0.1009	0.0911	0.0908
	2	1	0.0248	0.0200	0.3138	0.3135		2	1	-0.1314	-0.1202	0.1042	0.1037
	2	2	0.0000	0.0000	0.3159	0.3158		2	2	-0.1589	-0.1465	0.1256	0.1243
	2	3	-0.0369	-0.0333	0.3265	0.3263		2	3	-0.1961	-0.1827	0.1570	0.1559
	2	4	-0.0964	-0.894	0.3575	0.3569		2	4	-0.2507	-0.2362	0.2065	0.2051
	2	5	-0.2026	-0.1921	0.4419	0.4401		2	5	-0.3400	-0.3246	0.2921	0.2902
	2	6	-0.4259	-0.4128	0.6921	0.6885		2	6	-0.5137	-0.4988	0.4678	0.4654
	3	0	0.0754	0.0565	0.3281	0.3261		3	0	-0.1281	-0.1198	0.1078	0.1074
	3	1	0.0597	0.0513	0.3265	0.3258		3	1	-0.1579	-0.1457	0.1268	0.1260
	3	2	0.0369	0.0333	0.3265	0.3263		3	2	-0.1961	-0.1827	0.1570	0.1559
	3	3	0.0000	0.0000	0.3327	0.3327		3	3	-0.2513	-0.2370	0.2057	0.2042
	3	4	-0.0661	-0.0628	0.3574	0.3572		3	4	-0.3411	-0.3261	0.2909	0.2889
	3	5	-0.2050	-0.1991	0.4433	0.4425		3	5	-0.5149	-0.5007	0.4667	0.4642
	4	0	0.1219	0.0994	0.3606	0.3573		4	0	-0.1537	-0.1444	0.1309	0.1304
	4	1	0.1125	0.1005	0.3594	0.3579		4	1	-0.1944	-0.1810	0.1591	0.1581
	4	2	0.0964	0.0894	0.3575	0.3569		4	2	-0.2507	-0.2362	0.2065	0.2051
	4	3	0.0661	0.0628	0.3574	0.3572		4	3	-0.3411	-0.3261	0.2909	0.2889
	4	4	0.0000	0.0000	0.3685	0.3685		4	4	-0.5152	-0.5012	0.4663	0.4639
	5	0	0.1934	0.1662	0.4384	0.4353		5	0	-0.1893	-0.1784	0.1643	0.1637
	5	1	0.1984	0.1822	0.4384	0.4353		5	1	-0.2484	-0.2335	0.2093	0.2080
	5	2	0.2026	0.1921	0.4419	0.4401		5	2	-0.3400	-0.3246	0.2921	0.2902
	5	3	0.2050	0.1991	0.4433	0.4425		5	3	-0.5149	-0.5007	0.4667	0.4642
	6	0	0.3123	0.2784	0.5988	0.5894		6	0	-0.2423	-0.2287	0.2155	0.2147
	6	1	0.3536	0.3325	0.6362	0.6302		6	1	-0.3373	-0.3206	0.2952	0.2934
	6	2	0.4259	0.4128	0.6921	0.6885		6	2	-0.5137	-0.4988	0.4678	0.4654
	7	0	0.5347	0.4899	1.0169	0.9992		7	0	-0.3301	-0.3123	0.3020	0.3009
	7	1	0.6870	0.6622	0.2138	1.2028		7	1	-0.5110	-0.4941	0.4762	0.4680
	8	0	1.0418	0.9817	2.3387	2.3008		8	0	-0.5030	-0.4801	0.4762	0.4748

Table 2: Comparison of MLEs $(\hat{\mu},\hat{\sigma})$, LAMLEs $(\tilde{\mu},\tilde{\sigma})$ and Balakrishnan's AMLEs $(\breve{\mu},\breve{\sigma})$ of the parameters (μ,σ) of logistic distribution based on biases and MSEs (simulated based on 1000 Monte-Carlo runs) from complete and Type II censored samples of size n=10(10)30 with r=0(1)n/4 and s=0(1)n/2.

n	r	s	$\frac{bias(\hat{\mu})}{\sigma}$	$\frac{bias(\tilde{\mu})}{\sigma}$	$\frac{bias(\breve{\mu})}{\sigma}$	$\frac{MSE(\hat{\mu})}{\sigma^2}$	$\frac{MSE(\tilde{\mu})}{\sigma^2}$	$\frac{MSE(\breve{\mu})}{\sigma^2}$	$\frac{bias(\hat{\sigma})}{\sigma}$	$\frac{bias(\tilde{\sigma})}{\sigma}$	$\frac{bias(\breve{\sigma})}{\sigma}$	$\frac{MSE(\hat{\sigma})}{\sigma^2}$	$\frac{MSE(\tilde{\sigma})}{\sigma^2}$	$\frac{MSE(\breve{\sigma})}{\sigma^2}$
10	0	0	.0021	.0013	.0000	.2994	.2993	.3003	-.0713	-.0852	-.0316	.0721	.0722	.0754
10	0	1	-.0040	-.0075	-.0165	.3007	.3009	.3015	-.0803	-.0967	-.0507	.0785	.0788	.0797
10	0	2	-.0144	-.0228	-.0319	.3018	.3019	.3023	-.0889	-.1064	-.0617	.0918	.0920	.0930
10	0	3	-.0369	-.0497	-.0563	.3160	.3168	.3170	-.1060	-.1235	-.0794	.1063	.1067	.1078
10	0	4	-.0775	-.0934	-.0956	.3502	.3521	.3516	-.1327	-.1488	-.1053	.1349	.1352	.1364
10	0	5	-.1311	-.1496	-.1446	.4298	.4329	.4315	-.1599	-.1753	-.1309	.1637	.1639	.1655
10	1	0	.0084	.0104	.0179	.3010	.3009	.3018	-.0828	-.0991	-.0535	.0819	.0822	.0847
10	1	1	.0024	.0016	.0012	.3021	.3022	.3025	-.0948	-.1136	-.0776	.0898	.0903	.0902
10	1	2	-.0091	-.0148	-.0161	.3026	.3024	.3025	-.1073	-.1273	-.0942	.1080	.1084	.1086
10	1	3	-.0344	-.0446	-.0446	.3162	.3166	.3165	-.1310	-.1510	-.1198	.1270	.1276	.1275
10	1	4	-.0812	-.0950	-.0923	.3504	.3518	.3513	-.1689	-.1875	-.1583	.1640	.1645	.1644
10	1	5	-.1487	-.1660	-.1587	.4361	.4384	.4373	-.2131	-.2313	-.2026	.2081	.2085	.2084
10	2	0	.0259	.0339	.0424	.3084	.3087	.3098	-.1033	-.1208	-.0775	.0921	.0928	.0945
10	2	1	.0207	.0259	.0270	.3089	.3095	.3099	-.1205	-.1406	-.1080	.1008	.1019	.1005
10	2	2	.0085	.0086	.0087	.3082	.3084	.3085	-.1402	-.1613	-.1324	.1244	.1256	.1242
10	2	3	-.0196	-.0242	-.0232	.3189	.3192	.3191	-.1762	-.1969	-.1709	.1525	.1539	.1523
10	2	4	-.0752	-.0834	-.0804	.3505	.3512	.3508	-.2358	-.2546	-.2315	.2090	.2107	.2088
10	2	5	-.1672	-.1785	-.1723	.4420	.4434	.4426	-.3180	-.3356	-.3143	.2917	.2933	.2914
20	0	0	-.0014	-.0011	-.0014	.1526	.1530	.1536	-.0349	-.0424	-.0127	.0361	.0363	.0369
20	0	1	-.0021	-.0031	-.0057	.1524	.1527	.1530	-.0372	-.0456	-.0192	.0370	.0372	.0372
20	0	2	-.0035	-.0062	-.0094	.1527	.1530	.1533	-.0386	-.0477	-.0224	.0390	.0392	.0390
20	0	3	-.0075	-.0121	-.0155	.1531	.1536	.1538	-.0444	-.0540	-.0291	.0418	.0421	.0418
20	0	4	-.0118	-.0183	-.0213	.1545	.1551	.1553	-.0494	-.0590	-.0347	.0447	.0450	.0445
20	0	5	-.0140	-.0218	-.0239	.1553	.1559	.1560	-.0497	-.0595	-.0351	.0484	.0487	.0484
20	0	6	-.0195	-.0285	-.0294	.1573	.1580	.1579	-.0532	-.0631	-.0385	.0513	.0515	.0513
20	0	7	-.0275	-.0378	-.0373	.1599	.1608	.1605	-.0585	-.0684	-.0436	.0557	.0558	.0559
20	0	8	-.0357	-.0469	-.0447	.1668	.1678	.1674	-.0629	-.0727	-.0476	.0626	.0627	.0628
20	0	9	-.0458	-.0580	-.0536	.1793	.1804	.1798	-.0680	-.0776	-.0521	.0694	.0694	.0697
20	0	10	-.0621	-.0752	-.0683	.1995	.2008	.2000	-.0765	-.0859	-.0599	.0788	.0789	.0791
20	1	0	-.0005	.0008	.0029	.1528	.1531	.1537	-.0369	-.0453	-.0188	.0388	.0391	.0396
20	1	1	-.0011	-.0012	-.0014	.1525	.1529	.1531	-.0394	-.0487	-.0260	.0398	.0401	.0399
20	1	2	-.0026	-.0044	-.0053	.1528	.1531	.1533	-.0411	-.0512	-.0297	.0420	.0423	.0419
20	1	3	-.0067	-.0104	-.0115	.1532	.1537	.1539	-.0475	-.0580	-.0374	.0452	.0455	.0451
20	1	4	-.0112	-.0168	-.0176	.1546	.1552	.1553	-.0531	-.0636	-.0438	.0481	.0486	.0479
20	1	5	-.0137	-.0205	-.0207	.1553	.1560	.1560	-.0537	-.0644	-.0449	.0522	.0525	.0521
20	1	6	-.0194	-.0275	-.0269	.1572	.1580	.1578	-.0579	-.0687	-.0492	.0554	.0558	.0554
20	1	7	-.0280	-.0374	-.0357	.1597	.1606	.1603	-.0641	-.0750	-.0555	.0600	.0603	.0601
20	1	8	-.0370	-.0474	-.0444	.1666	.1676	.1671	-.0696	-.0803	-.0610	.0682	.0685	.0683
20	1	9	-.0482	-.0595	-.0551	.1793	.1804	.1798	-.0759	-.0865	-.0673	.0766	.0767	.0768
20	1	10	-.0663	-.0785	-.0724	.2002	.2016	.2007	-.0863	-.0967	-.0776	.0880	.0881	.0881
20	2	0	.0012	.0043	.0072	.1529	.1530	.1534	-.0395	-.0485	-.0231	.0406	.0409	.0413
20	2	1	.0006	.0024	.0029	.1526	.1527	.1529	-.0424	-.0524	-.0310	.0419	.0421	.0419
20	2	2	-.0009	-.0008	-.0010	.1529	.1530	.1531	-.0444	-.0551	-.0353	.0444	.0448	.0444
20	2	3	-.0049	-.0069	-.0072	.1533	.1535	.1536	-.0514	-.0627	-.0438	.0481	.0484	.0480
20	2	4	-.0096	-.0134	-.0135	.1546	.1549	.1550	-.0577	-.0690	-.0512	.0517	.0522	.0515
20	2	5	-.0123	-.0174	-.0170	.1554	.1557	.1557	-.0589	-.0703	-.0529	.0565	.0569	.0564
20	2	6	-.0185	-.0247	-.0237	.1574	.1577	.1576	-.0639	-.0754	-.0583	.0606	.0609	.0606
20	2	7	-.0276	-.0352	-.0332	.1600	.1605	.1603	-.0714	-.0829	-.0659	.0658	.0661	.0658
20	2	8	-.0374	-.0460	-.0430	.1672	.1678	.1674	-.0782	-.0896	-.0729	.0755	.0758	.0755
20	2	9	-.0499	-.0594	-.0553	.1806	.1813	.1809	-.0863	-.0974	-.0812	.0859	.0861	.0860
20	2	10	-.0701	-.0805	-.0751	.2031	.2040	.2035	-.0995	-.1103	-.0944	.1005	.1007	.1005
20	3	0	.0048	.0095	.0124	.1535	.1536	.1540	-.0439	-.0534	-.0284	.0434	.0438	.0440
20	3	1	.0042	.0076	.0083	.1533	.1533	.1535	-.0472	-.0578	-.0371	.0450	.0454	.0451
20	3	2	.0027	.0044	.0045	.1535	.1535	.1536	-.0497	-.0611	-.0421	.0482	.0486	.0481
20	3	3	-.0013	-.0016	-.0017	.1538	.1539	.1540	-.0577	-.0696	-.0517	.0523	.0528	.0523
20	3	4	-.0061	-.0082	-.0081	.1550	.1552	.1553	-.0651	-.0769	-.0602	.0562	.0569	.0560
20	3	5	-.0091	-.0125	-.0120	.1557	.1559	.1559	-.0671	-.0790	-.0628	.0618	.0623	.0617
20	3	6	-.0157	-.0204	-.0193	.1576	.1579	.1578	-.0734	-.0854	-.0695	.0665	.0670	.0664
20	3	7	-.0256	-.0316	-.0296	.1601	.1605	.1603	-.0826	-.0946	-.0789	.0728	.0733	.0728
20	3	8	-.0366	-.0436	-.0407	.1671	.1676	.1673	-.0915	-.1033	-.0880	.0838	.0843	.0838
20	3	9	-.0509	-.0589	-.0550	.1807	.1813	.1809	-.1024	-.1139	-.0991	.0971	.0974	.0971
20	3	10	-.0743	-.0830	-.0781	.2043	.2050	.2046	-.1199	-.1309	-.1168	.1162	.1165	.1163

Table 2 (continued)

n	r	s	$\frac{bias(\hat{\mu})}{\sigma}$	$\frac{bias(\tilde{\mu})}{\sigma}$	$\frac{bias(\hat{\mu})}{\sigma}$	$\frac{MSE(\hat{\mu})}{\sigma^2}$	$\frac{MSE(\tilde{\mu})}{\sigma^2}$	$\frac{MSE(\hat{\mu})}{\sigma^2}$	$\frac{bias(\hat{\sigma})}{\sigma}$	$\frac{bias(\tilde{\sigma})}{\sigma}$	$\frac{bias(\hat{\sigma})}{\sigma}$	$\frac{MSE(\hat{\sigma})}{\sigma^2}$	$\frac{MSE(\tilde{\sigma})}{\sigma^2}$	$\frac{MSE(\hat{\sigma})}{\sigma^2}$
20	4	0	.0097	.0162	.0188	.1546	.1547	.1551	-.0496	-.0593	-.0347	.0480	.0484	.0485
20	4	1	.0093	.0145	.0150	.1545	.1546	.1547	-.0536	-.0643	-.0445	.0499	.0503	.0499
20	4	2	.0079	.0114	.0113	.1547	.1547	.1548	-.0568	-.0683	-.0503	.0532	.0537	.0531
20	4	3	.0040	.0055	.0053	.1550	.1550	.1551	-.0661	-.0780	-.0612	.0578	.0584	.0577
20	4	4	-.0008	-.0011	-.0012	.1561	.1562	.1562	-.0750	-.0867	-.0712	.0624	.0631	.0622
20	4	5	-.0042	-.0059	-.0055	.1566	.1568	.1568	-.0781	-.0899	-.0750	.0687	.0694	.0686
20	4	6	-.0114	-.0142	-.0134	.1583	.1584	.1584	-.0861	-.0980	-.0835	.0746	.0752	.0745
20	4	7	-.0221	-.0263	-.0247	.1604	.1607	.1606	-.0979	-.1097	-.0955	.0829	.0836	.0828
20	4	8	-.0346	-.0397	-.0373	.1672	.1675	.1673	-.1099	-.1214	-.1077	.0974	.0982	.0973
20	4	9	-.0513	-.0574	-.0541	.1806	.1811	.1808	-.1249	-.1361	-.1229	.1135	.1141	.1134
20	4	10	-.0788	-.0856	-.0814	.2051	.2058	.2053	-.1491	-.1596	-.1473	.1392	.1398	.1391
20	5	0	.0147	.0227	.0245	.1562	.1563	.1566	-.0539	-.0637	-.0391	.0521	.0525	.0527
20	5	1	.0146	.0213	.0213	.1561	.1562	.1563	-.0585	-.0694	-.0498	.0542	.0546	.0542
20	5	2	.0134	.0184	.0178	.1563	.1562	.1563	-.0625	-.0740	-.0565	.0583	.0588	.0582
20	5	3	.0099	.0129	.0122	.1564	.1565	.1565	-.0731	-.0851	-.0689	.0637	.0643	.0636
20	5	4	.0053	.0064	.0060	.1572	.1573	.1573	-.0836	-.0953	-.0806	.0697	.0704	.0696
20	5	5	.0016	.0015	.0014	.1575	.1576	.1576	-.0879	-.0996	-.0856	.0781	.0787	.0781
20	5	6	-.0058	-.0072	-.0067	.1588	.1589	.1589	-.0981	-.1097	-.0962	.0865	.0870	.0864
20	5	7	-.0172	-.0197	-.0186	.1606	.1607	.1607	-.1129	-.1243	-.1113	.0984	.0989	.0983
20	5	8	-.0308	-.0344	-.0325	.1673	.1675	.1673	-.1286	-.1397	-.1272	.1153	.1159	.1152
20	5	9	-.0498	-.0543	-.0516	.1807	.1810	.1808	-.1490	-.1597	-.1478	.1374	.1379	.1373
20	5	10	-.0817	-.0870	-.0835	.2060	.2063	.2061	-.1825	-.1926	-.1814	.1709	.1714	.1709
30	0	0	-.0042	-.0037	-.0039	.1052	.1053	.1055	-.0231	-.0281	-.0083	.0241	.0242	.0246
30	0	1	-.0043	-.0045	-.0057	.1051	.1053	.1054	-.0234	-.0288	-.0107	.0245	.0245	.0247
30	0	2	-.0050	-.0062	-.0079	.1054	.1055	.1056	-.0254	-.0313	-.0139	.0255	.0255	.0257
30	0	3	-.0060	-.0082	-.0100	.1055	.1057	.1058	-.0270	-.0332	-.0164	.0267	.0268	.0269
30	0	4	-.0068	-.0099	-.0117	.1051	.1052	.1053	-.0278	-.0342	-.0175	.0277	.0278	.0279
30	0	5	-.0079	-.0119	-.0135	.1053	.1055	.1055	-.0288	-.0354	-.0189	.0296	.0296	.0298
30	0	6	-.0108	-.0155	-.0169	.1051	.1053	.1053	-.0323	-.0389	-.0226	.0309	.0310	.0311
30	0	7	-.0126	-.0181	-.0191	.1059	.1062	.1062	-.0337	-.0404	-.0241	.0328	.0329	.0330
30	0	8	-.0143	-.0204	-.0209	.1062	.1065	.1065	-.0346	-.0413	-.0249	.0348	.0348	.0350
30	0	9	-.0184	-.0251	-.0251	.1074	.1077	.1076	-.0381	-.0448	-.0283	.0359	.0359	.0361
30	0	10	-.0225	-.0298	-.0291	.1099	.1103	.1102	-.0412	-.0478	-.0311	.0383	.0383	.0385
30	0	11	-.0217	-.0294	-.0280	.1124	.1128	.1127	-.0391	-.0457	-.0290	.0404	.0404	.0407
30	0	12	-.0250	-.0333	-.0310	.1156	.1160	.1158	-.0408	-.0474	-.0304	.0426	.0426	.0429
30	0	13	-.0300	-.0387	-.0354	.1217	.1221	.1219	-.0435	-.0500	-.0328	.0466	.0466	.0469
30	0	14	-.0346	-.0439	-.0394	.1274	.1278	.1276	-.0456	-.0521	-.0346	.0504	.0503	.0507
30	0	15	-.0448	-.0544	-.0488	.1356	.1363	.1358	-.0514	-.0577	-.0400	.0534	.0534	.0536
30	1	0	-.0038	-.0029	-.0019	.1053	.1053	.1054	-.0244	-.0298	-.0117	.0245	.0246	.0249
30	1	1	-.0039	-.0036	-.0037	.1052	.1052	.1053	-.0247	-.0306	-.0142	.0248	.0249	.0249
30	1	2	-.0046	-.0054	-.0059	.1054	.1054	.1056	-.0268	-.0332	-.0177	.0259	.0260	.0260
30	1	3	-.0056	-.0073	-.0080	.1055	.1056	.1057	-.0286	-.0353	-.0204	.0271	.0272	.0272
30	1	4	-.0065	-.0091	-.0097	.1051	.1052	.1052	-.0295	-.0364	-.0218	.0282	.0283	.0283
30	1	5	-.0077	-.0111	-.0116	.1053	.1054	.1054	-.0306	-.0377	-.0234	.0302	.0302	.0303
30	1	6	-.0106	-.0149	-.0152	.1052	.1053	.1053	-.0344	-.0415	-.0274	.0315	.0316	.0315
30	1	7	-.0125	-.0175	-.0176	.1060	.1062	.1062	-.0360	-.0432	-.0292	.0335	.0336	.0336
30	1	8	-.0143	-.0199	-.0195	.1063	.1065	.1064	-.0370	-.0443	-.0303	.0355	.0355	.0356
30	1	9	-.0185	-.0248	-.0240	.1075	.1077	.1076	-.0409	-.0481	-.0342	.0370	.0371	.0371
30	1	10	-.0228	-.0297	-.0284	.1100	.1104	.1103	-.0443	-.0514	-.0376	.0394	.0395	.0396
30	1	11	-.0222	-.0295	-.0276	.1126	.1129	.1128	-.0423	-.0495	-.0356	.0419	.0420	.0421
30	1	12	-.0258	-.0337	-.0311	.1159	.1163	.1161	-.0442	-.0514	-.0375	.0444	.0445	.0446
30	1	13	-.0311	-.0395	-.0361	.1223	.1226	.1225	-.0474	-.0545	-.0406	.0489	.0490	.0491
30	1	14	-.0362	-.0451	-.0409	.1283	.1286	.1285	-.0499	-.0571	-.0430	.0530	.0530	.0533
30	1	15	-.0471	-.0563	-.0513	.1369	.1375	.1371	-.0565	-.0634	-.0495	.0566	.0567	.0567
30	2	0	-.0032	-.0013	.0002	.1056	.1056	.1057	-.0259	-.0318	-.0145	.0254	.0254	.0257
30	2	1	-.0033	-.0020	-.0017	.1055	.1055	.1056	-.0263	-.0327	-.0173	.0258	.0258	.0259
30	2	2	-.0040	-.0038	-.0038	.1057	.1057	.1058	-.0285	-.0354	-.0210	.0269	.0270	.0270
30	2	3	-.0050	-.0058	-.0060	.1058	.1059	.1060	-.0305	-.0377	-.0239	.0282	.0283	.0283
30	2	4	-.0059	-.0075	-.0078	.1054	.1054	.1055	-.0315	-.0389	-.0255	.0294	.0295	.0294
30	2	5	-.0071	-.0096	-.0097	.1055	.1056	.1057	-.0328	-.0404	-.0273	.0314	.0315	.0314
30	2	6	-.0101	-.0134	-.0134	.1054	.1054	.1054	-.0368	-.0445	-.0317	.0329	.0330	.0329
30	2	7	-.0121	-.0161	-.0159	.1061	.1063	.1063	-.0387	-.0464	-.0337	.0350	.0351	.0350
30	2	8	-.0140	-.0186	-.0181	.1064	.1065	.1065	-.0399	-.0477	-.0352	.0372	.0373	.0372
30	2	9	-.0184	-.0237	-.0228	.1075	.1077	.1076	-.0441	-.0518	-.0395	.0388	.0389	.0387
30	2	10	-.0229	-.0288	-.0274	.1100	.1103	.1102	-.0480	-.0556	-.0433	.0416	.0417	.0416
30	2	11	-.0225	-.0289	-.0270	.1125	.1127	.1126	-.0461	-.0538	-.0416	.0441	.0442	.0441
30	2	12	-.0265	-.0333	-.0309	.1157	.1160	.1158	-.0485	-.0561	-.0440	.0469	.0470	.0469
30	2	13	-.0322	-.0395	-.0364	.1220	.1223	.1222	-.0522	-.0597	-.0477	.0520	.0521	.0520
30	2	14	-.0378	-.0457	-.0419	.1279	.1282	.1280	-.0553	-.0629	-.0507	.0563	.0563	.0563
30	2	15	-.0495	-.0577	-.0532	.1366	.1372	.1368	-.0628	-.0702	-.0582	.0604	.0606	.0604
30	3	0	-.0020	.0008	.0024	.1057	.1057	.1058	-.0280	-.0343	-.0173	.0265	.0266	.0268
30	3	1	-.0021	.0001	.0006	.1056	.1056	.1057	-.0285	-.0352	-.0203	.0269	.0270	.0270
30	3	2	-.0028	-.0016	-.0015	.1058	.1058	.1059	-.0309	-.0382	-.0243	.0281	.0282	.0281
30	3	3	-.0038	-.0036	-.0037	.1059	.1060	.1060	-.0331	-.0407	-.0274	.0294	.0295	.0294
30	3	4	-.0047	-.0054	-.0055	.1055	.1055	.1056	-.0342	-.0421	-.0292	.0306	.0307	.0306
30	3	5	-.0060	-.0076	-.0075	.1057	.1057	.1058	-.0358	-.0438	-.0313	.0326	.0328	.0326

Table 2 (continued)

n	r	s	$\frac{bias(\hat{\mu})}{\sigma}$	$\frac{bias(\tilde{\mu})}{\sigma}$	$\frac{bias(\hat{\mu})}{\sigma}$	$\frac{MSE(\hat{\mu})}{\sigma^2}$	$\frac{MSE(\tilde{\mu})}{\sigma^2}$	$\frac{MSE(\hat{\mu})}{\sigma^2}$	$\frac{bias(\hat{\sigma})}{\sigma}$	$\frac{bias(\tilde{\sigma})}{\sigma}$	$\frac{bias(\hat{\sigma})}{\sigma}$	$\frac{MSE(\hat{\sigma})}{\sigma^2}$	$\frac{MSE(\tilde{\sigma})}{\sigma^2}$	$\frac{MSE(\hat{\sigma})}{\sigma^2}$
30	3	6	-.0090	-.0114	-.0113	.1055	.1055	.1055	-.0402	-.0482	-.0361	.0344	.0346	.0343
30	3	7	-.0112	-.0143	-.0139	.1062	.1063	.1063	-.0423	-.0504	-.0385	.0369	.0371	.0369
30	3	8	-.0132	-.0169	-.0162	.1064	.1065	.1065	-.0439	-.0520	-.0402	.0395	.0396	.0394
30	3	9	-.0178	-.0221	-.0211	.1075	.1077	.1076	-.0486	-.0566	-.0451	.0412	.0413	.0411
30	3	10	-.0225	-.0275	-.0261	.1100	.1102	.1101	-.0529	-.0609	-.0495	.0445	.0447	.0445
30	3	11	-.0225	-.0279	-.0260	.1125	.1127	.1126	-.0513	-.0593	-.0481	.0473	.0475	.0473
30	3	12	-.0268	-.0327	-.0304	.1158	.1160	.1159	-.0543	-.0622	-.0511	.0507	.0509	.0507
30	3	13	-.0331	-.0394	-.0366	.1223	.1225	.1224	-.0587	-.0665	-.0555	.0565	.0567	.0565
30	3	14	-.0394	-.0463	-.0428	.1284	.1286	.1285	-.0626	-.0704	-.0595	.0616	.0617	.0616
30	3	15	-.0520	-.0593	-.0552	.1375	.1379	.1376	-.0714	-.0790	-.0683	.0665	.0667	.0664
30	4	0	-.0007	.0029	.0045	.1059	.1059	.1060	-.0296	-.0361	-.0193	.0274	.0275	.0277
30	4	1	-.0008	.0022	.0028	.1058	.1058	.1059	-.0301	-.0372	-.0225	.0278	.0279	.0279
30	4	2	-.0014	.0005	.0007	.1060	.1060	.1061	-.0328	-.0403	-.0267	.0291	.0292	.0291
30	4	3	-.0024	-.0014	-.0014	.1061	.1061	.1062	-.0351	-.0430	-.0301	.0305	.0307	.0305
30	4	4	-.0034	-.0033	-.0033	.1056	.1057	.1057	-.0365	-.0446	-.0321	.0319	.0321	.0319
30	4	5	-.0047	-.0054	-.0054	.1058	.1059	.1059	-.0382	-.0464	-.0344	.0339	.0341	.0339
30	4	6	-.0077	-.0093	-.0091	.1056	.1056	.1056	-.0430	-.0512	-.0396	.0359	.0361	.0359
30	4	7	-.0099	-.0122	-.0118	.1063	.1064	.1064	-.0454	-.0537	-.0423	.0387	.0389	.0386
30	4	8	-.0120	-.0149	-.0143	.1065	.1066	.1065	-.0472	-.0555	-.0444	.0415	.0417	.0414
30	4	9	-.0167	-.0203	-.0193	.1075	.1077	.1076	-.0524	-.0607	-.0497	.0434	.0436	.0434
30	4	10	-.0217	-.0258	-.0245	.1099	.1102	.1101	-.0573	-.0654	-.0548	.0473	.0476	.0473
30	4	11	-.0219	-.0265	-.0247	.1125	.1127	.1126	-.0560	-.0641	-.0535	.0506	.0508	.0505
30	4	12	-.0266	-.0317	-.0295	.1158	.1160	.1159	-.0595	-.0675	-.0571	.0546	.0548	.0545
30	4	13	-.0333	-.0388	-.0362	.1224	.1226	.1225	-.0647	-.0725	-.0624	.0615	.0617	.0614
30	4	14	-.0402	-.0463	-.0430	.1287	.1289	.1288	-.0695	-.0773	-.0672	.0675	.0677	.0675
30	4	15	-.0537	-.0602	-.0564	.1379	.1383	.1380	-.0796	-.0872	-.0774	.0725	.0728	.0724
30	4	16	-.0689	-.0758	-.0715	.1519	.1524	.1520	-.0903	-.0978	-.0881	.0809	.0813	.0808
30	4	17	-.0843	-.0915	-.0866	.1665	.1671	.1667	-.1003	-.1076	-.0981	.0914	.0917	.0913
30	4	18	-.1059	-.1136	-.1079	.1867	.1875	.1869	-.1140	-.1213	-.1118	.1001	.1005	.1000
30	4	19	-.1342	-.1423	-.1359	.2247	.2258	.2249	-.1314	-.1385	-.1293	.1170	.1174	.1168
30	4	20	-.1738	-.1822	-.1751	.2666	.2679	.2668	-.1551	-.1619	-.1530	.1402	.1407	.1401
30	4	21	-.2393	-.2479	-.2402	.3498	.3514	.3500	-.1936	-.2001	-.1917	.1693	.1699	.1692
30	4	22	-.3200	-.3289	-.3205	.4802	.4818	.4803	-.2389	-.2452	-.2371	.2160	.2164	.2159
30	4	23	-.4801	-.4891	-.4802	.7384	.7409	.7385	-.3276	-.3337	-.3261	.2919	.2925	.2917
30	5	0	.0011	.0054	.0068	.1058	.1058	.1059	-.0313	-.0381	-.0212	.0281	.0282	.0283
30	5	1	.0009	.0047	.0051	.1057	.1057	.1058	-.0320	-.0393	-.0246	.0287	.0288	.0287
30	5	2	.0004	.0030	.0031	.1058	.1059	.1059	-.0348	-.0426	-.0291	.0301	.0303	.0301
30	5	3	-.0006	.0011	.0010	.1059	.1060	.1060	-.0374	-.0455	-.0328	.0317	.0318	.0317
30	5	4	-.0016	-.0007	-.0008	.1056	.1056	.1056	-.0389	-.0473	-.0350	.0331	.0333	.0331
30	5	5	-.0029	-.0029	-.0029	.1057	.1058	.1058	-.0409	-.0494	-.0375	.0353	.0355	.0353
30	5	6	-.0059	-.0068	-.0066	.1055	.1055	.1055	-.0461	-.0546	-.0432	.0374	.0377	.0374
30	5	7	-.0082	-.0097	-.0094	.1062	.1062	.1062	-.0488	-.0574	-.0462	.0404	.0407	.0404
30	5	8	-.0104	-.0126	-.0120	.1064	.1064	.1064	-.0510	-.0596	-.0486	.0434	.0437	.0434
30	5	9	-.0152	-.0181	-.0172	.1074	.1075	.1074	-.0568	-.0653	-.0546	.0458	.0461	.0457
30	5	10	-.0203	-.0238	-.0225	.1098	.1100	.1099	-.0623	-.0707	-.0603	.0502	.0505	.0502
30	5	11	-.0208	-.0247	-.0231	.1124	.1125	.1124	-.0613	-.0696	-.0593	.0539	.0541	.0539
30	5	12	-.0259	-.0303	-.0282	.1158	.1159	.1158	-.0655	-.0737	-.0637	.0590	.0592	.0589
30	5	13	-.0331	-.0379	-.0354	.1226	.1228	.1227	-.0716	-.0796	-.0699	.0670	.0672	.0670
30	5	14	-.0408	-.0461	-.0431	.1295	.1295	.1295	-.0775	-.0854	-.0759	.0750	.0751	.0750
30	5	15	-.0553	-.0611	-.0575	.1390	.1392	.1391	-.0895	-.0972	-.0878	.0797	.0800	.0797
30	5	16	-.0722	-.0783	-.0743	.1544	.1546	.1545	-.1025	-.1101	-.1009	.0910	.0913	.0910
30	5	17	-.0899	-.0964	-.0918	.1715	.1718	.1716	-.1153	-.1226	-.1138	.1056	.1058	.1056
30	5	18	-.1153	-.1221	-.1170	.1968	.1973	.1969	-.1332	-.1403	-.1318	.1205	.1209	.1205
30	5	19	-.1496	-.1567	-.1510	.2439	.2445	.2440	-.1567	-.1635	-.1553	.1460	.1463	.1460
30	5	20	-.1995	-.2068	-.2006	.2988	.2996	.2989	-.1900	-.1965	-.1887	.1804	.1807	.1804
30	5	21	-.2857	-.2931	-.2865	.4111	.4122	.4113	-.2466	-.2527	-.2454	.2288	.2293	.2287
30	5	22	-.4073	-.4150	-.4078	.6061	.6073	.6062	-.3240	-.3299	-.3229	.3131	.3134	.3130
30	6	0	.0029	.0080	.0091	.1067	.1068	.1070	-.0332	-.0400	-.0232	.0295	.0297	.0297
30	6	1	.0028	.0074	.0076	.1067	.1068	.1069	-.0340	-.0413	-.0268	.0303	.0304	.0303
30	6	2	.0023	.0058	.0056	.1068	.1069	.1070	-.0371	-.0449	-.0317	.0318	.0320	.0318
30	6	3	.0014	.0039	.0037	.1069	.1070	.1070	-.0399	-.0481	-.0357	.0335	.0337	.0335
30	6	4	.0005	.0021	.0018	.1065	.1065	.1066	-.0416	-.0501	-.0381	.0352	.0354	.0352
30	6	5	-.0008	-.0001	-.0002	.1066	.1067	.1067	-.0439	-.0524	-.0410	.0377	.0379	.0377
30	6	6	-.0038	-.0039	-.0039	.1063	.1064	.1064	-.0496	-.0581	-.0471	.0402	.0404	.0401
30	6	7	-.0061	-.0069	-.0067	.1069	.1070	.1070	-.0527	-.0612	-.0506	.0435	.0438	.0435
30	6	8	-.0084	-.0098	-.0094	.1071	.1071	.1071	-.0553	-.0638	-.0534	.0469	.0470	.0468
30	6	9	-.0133	-.0154	-.0147	.1080	.1082	.1081	-.0617	-.0702	-.0600	.0489	.0492	.0489
30	6	10	-.0186	-.0213	-.0202	.1102	.1104	.1103	-.0680	-.0763	-.0665	.0540	.0543	.0540
30	6	11	-.0194	-.0226	-.0211	.1126	.1128	.1127	-.0673	-.0755	-.0658	.0579	.0581	.0579
30	6	12	-.0248	-.0285	-.0266	.1158	.1160	.1159	-.0724	-.0805	-.0710	.0635	.0638	.0635
30	6	13	-.0326	-.0367	-.0344	.1224	.1226	.1225	-.0797	-.0875	-.0784	.0727	.0730	.0727
30	6	14	-.0411	-.0456	-.0429	.1291	.1293	.1292	-.0870	-.0947	-.0858	.0824	.0825	.0823
30	6	15	-.0569	-.0618	-.0587	.1387	.1390	.1388	-.1013	-.1088	-.1001	.0890	.0893	.0890
30	7	0	.0056	.0114	.0122	.1061	.1062	.1063	-.0359	-.0428	-.0258	.0311	.0312	.0313
30	7	1	.0056	.0109	.0108	.1061	.1062	.1062	-.0368	-.0442	-.0298	.0319	.0321	.0319
30	7	2	.0052	.0094	.0090	.1062	.1062	.1063	-.0402	-.0482	-.0350	.0339	.0341	.0338
30	7	3	.0044	.0076	.0071	.1063	.1063	.1063	-.0433	-.0516	-.0394	.0359	.0361	.0358
30	7	4	.0034	.0058	.0053	.1059	.1059	.1060	-.0454	-.0539	-.0422	.0378	.0380	.0377

Table 2 (continued)

n	r	s	$\frac{bias(\hat{\mu})}{\sigma}$	$\frac{bias(\tilde{\mu})}{\sigma}$	$\frac{bias(\hat{\mu})}{\sigma}$	$\frac{MSE(\hat{\mu})}{\sigma^2}$	$\frac{MSE(\tilde{\mu})}{\sigma^2}$	$\frac{MSE(\hat{\mu})}{\sigma^2}$	$\frac{bias(\hat{\sigma})}{\sigma}$	$\frac{bias(\tilde{\sigma})}{\sigma}$	$\frac{bias(\hat{\sigma})}{\sigma}$	$\frac{MSE(\hat{\sigma})}{\sigma^2}$	$\frac{MSE(\tilde{\sigma})}{\sigma^2}$	$\frac{MSE(\hat{\sigma})}{\sigma^2}$
30	7	5	.0021	.0036	.0033	.1060	.1060	.1061	-.0480	-.0566	-.0454	.0406	.0408	.0405
30	7	6	-.0008	-.0001	-.0003	.1058	.1058	.1058	-.0543	-.0628	-.0522	.0432	.0435	.0431
30	7	7	-.0031	-.0032	-.0032	.1064	.1064	.1064	-.0580	-.0665	-.0562	.0472	.0475	.0472
30	7	8	-.0055	-.0062	-.0060	.1065	.1065	.1065	-.0611	-.0696	-.0595	.0511	.0513	.0511
30	7	9	-.0105	-.0119	-.0113	.1074	.1075	.1074	-.0685	-.0769	-.0671	.0541	.0545	.0541
30	7	10	-.0160	-.0179	-.0171	.1096	.1097	.1096	-.0759	-.0840	-.0747	.0606	.0609	.0606
30	7	11	-.0173	-.0196	-.0185	.1121	.1122	.1121	-.0757	-.0838	-.0747	.0655	.0657	.0654
30	7	12	-.0232	-.0261	-.0245	.1155	.1156	.1155	-.0820	-.0899	-.0810	.0721	.0724	.0720
30	7	13	-.0317	-.0350	-.0331	.1224	.1225	.1224	-.0910	-.0986	-.0901	.0834	.0838	.0834
30	7	14	-.0413	-.0449	-.0426	.1297	.1297	.1297	-.1004	-.1078	-.0995	.0957	.0959	.0957
30	7	15	-.0589	-.0629	-.0602	.1404	.1406	.1404	-.1182	-.1253	-.1174	.1056	.1058	.1055

Table 3: Means of order statistics from standard logistic distribution for sample size n=2(1)100

n	a_1 / a_{11}	a_2 / a_{12}	a_3 / a_{13}	a_4 / a_{14}	a_5 / a_{15}	a_6 / a_{16}	a_7 / a_{17}	a_8 / a_{18}	a_9 / a_{19}	a_{10} / a_{20}
2	-1.0000									
3	-1.5000	0.0000								
4	-1.8333	-0.5000								
5	-2.0833	-0.8333	.0000							
6	-2.2833	-1.0833	-0.3333							
7	-2.4500	-1.2833	-0.5833	.0000						
8	-2.5929	-1.4500	-0.7833	-0.2500						
9	-2.7179	-1.5929	-0.9500	-0.4500	.0000					
10	-2.8290	-1.7179	-1.0929	-0.6167	-.2000					
11	-2.9290	-1.8290	-1.2179	-.7595	-.3667	.0000				
12	-3.0199	-1.9290	-1.3290	-.8845	-.5095	-.1667				
13	-3.1032	-2.0199	-1.4290	-.9956	-.6345	-.3095	.0000			
14	-3.1801	-2.1032	-1.5199	-1.0956	-.7456	-.4345	-.1429			
15	-3.2516	-2.1801	-1.6032	-1.1865	-.8456	-.5456	-.2679	.0000		
16	-3.3182	-2.2516	-1.6801	-1.2699	-.9365	-.6456	-.3790	-.1250		
17	-3.3807	-2.3182	-1.7516	-1.3468	-1.0199	-.7365	-.4790	-.2361	.0000	
18	-3.4396	-2.3807	-1.8182	-1.4182	-1.0968	-.8199	-.5699	-.3361	-.1111	
19	-3.4951	-2.4396	-1.8807	-1.4849	-1.1682	-.8968	-.6532	-.4270	-.2111	.0000
20	-3.5477	-2.4951	-1.9396	-1.5474	-1.2349	-.9682	-.7301	-.5104	-.3020	-.1000
21	-3.5977	-2.5477	-1.9951	-1.6062	-1.2974	-1.0349	-.8016	-.5873	-.3854	-.1909
	.0000									
22	-3.6454	-2.5977	-2.0477	-1.6618	-1.3562	-1.0974	-.8682	-.6587	-.4623	-.2742
	-.0909									
23	-3.6908	-2.6454	-2.0977	-1.7144	-1.4118	-1.1562	-.9307	-.7254	-.5337	-.3512
	-.1742	.0000								
24	-3.7343	-2.6908	-2.1454	-1.7644	-1.4644	-1.2118	-.9896	-.7879	-.6004	-.4226
	-.2512	-.0833								
25	-3.7760	-2.7343	-2.1908	-1.8120	-1.5144	-1.2644	-1.0451	-.8467	-.6629	-.4893
	-.3226	-.1603	.0000							
26	-3.8160	-2.7760	-2.2343	-1.8575	-1.5620	-1.3144	-1.0977	-.9023	-.7217	-.5518
	-.3893	-.2317	-.0769							
27	-3.8544	-2.8160	-2.2760	-1.9010	-1.6075	-1.3620	-1.1477	-.9549	-.7773	-.6106
	-.4518	-.2984	-.1484	.0000						
28	-3.8915	-2.8544	-2.3160	-1.9426	-1.6510	-1.4075	-1.1954	-1.0049	-.8299	-.6661
	-.5106	-.3609	-.2150	-.0714						
29	-3.9272	-2.8915	-2.3544	-1.9826	-1.6926	-1.4510	-1.2408	-1.0525	-.8799	-.7188
	-.5661	-.4197	-.2775	-.1381	.0000					
30	-3.9617	-2.9272	-2.3915	-2.0211	-1.7326	-1.4926	-1.2843	-1.0980	-.9275	-.7688
	-.6188	-.4752	-.3363	-.2006	-.0667					

Note: For a given sample size n, only the first (n+1)/2 means of order statistics are recorded since the remaining means of order statistics can be obtained using the relation $a_{n-i+1} = -a_i$ for i=1,…,(n+1)/2

Table 3 (Continued)

n	a_1 a_{11} a_{21}	a_2 a_{12} a_{22}	a_3 a_{13} a_{23}	a_4 a_{14} a_{24}	a_5 a_{15} a_{25}	a_6 a_{16} a_{26}	a_7 a_{17} a_{27}	a_8 a_{18} a_{28}	a_9 a_{19} a_{29}	a_{10} a_{20} a_{30}
31	-3.9950	-2.9617	-2.4272	-2.0581	-1.7711	-1.5326	-1.3260	-1.1414	-.9730	-.8164
	-.6688	-.5279	-.3919	-.2594	-.1292	.0000				
32	-4.0272	-2.9950	-2.4617	-2.0938	-1.8081	-1.5711	-1.3660	-1.1831	-1.0164	-.8618
	-.7164	-.5779	-.4445	-.3150	-.1880	-.0625				
33	-4.0585	-3.0272	-2.4950	-2.1283	-1.8438	-1.6081	-1.4044	-1.2231	-1.0581	-.9053
	-.7618	-.6255	-.4945	-.3676	-.2435	-.1213	.0000			
34	-4.0888	-3.0585	-2.5272	-2.1617	-1.8783	-1.6438	-1.4415	-1.2616	-1.0981	-.9470
	-.8053	-.6709	-.5421	-.4176	-.2962	-.1769	-.0588			
35	-4.1182	-3.0888	-2.5585	-2.1939	-1.9117	-1.6783	-1.4772	-1.2986	-1.1366	-.9870
	-.8470	-.7144	-.5876	-.4652	-.3462	-.2295	-.1144	.0000		
36	-4.1468	-3.1182	-2.5888	-2.2252	-1.9439	-1.7117	-1.5117	-1.3343	-1.1736	-1.0255
	-.8870	-.7561	-.6311	-.5107	-.3938	-.2795	-.1670	-.0556		
37	-4.1746	-3.1468	-2.6182	-2.2555	-1.9752	-1.7439	-1.5450	-1.3688	-1.2093	-1.0625
	-.9255	-.7961	-.6727	-.5542	-.4393	-.3271	-.2170	-.1082	.0000	
38	-4.2016	-3.1746	-2.6468	-2.2849	-2.0055	-1.7752	-1.5772	-1.4021	-1.2438	-1.0982
	-.9625	-.8345	-.7127	-.5958	-.4827	-.3726	-.2646	-.1582	-.0526	
39	-4.2279	-3.2016	-2.6746	-2.3134	-2.0349	-1.8055	-1.6085	-1.4344	-1.2771	-1.1327
	-.9982	-.8716	-.7512	-.6358	-.5244	-.4161	-.3101	-.2058	-.1026	.0000
40	-4.2535	-3.2279	-2.7016	-2.3412	-2.0634	-1.8349	-1.6388	-1.4656	-1.3094	-1.1660
	-1.0327	-.9073	-.7882	-.6743	-.5644	-.4577	-.3536	-.2513	-.1503	-.0500
41	-4.2785	-3.2535	-2.7279	-2.3683	-2.0912	-1.8634	-1.6682	-1.4959	-1.3406	-1.1983
	-1.0660	-.9418	-.8240	-.7113	-.6029	-.4977	-.3952	-.2947	-.1957	-.0976
	.0000									
42	-4.3029	-3.2785	-2.7535	-2.3946	-2.1183	-1.8912	-1.6968	-1.5254	-1.3709	-1.2295
	-1.0983	-.9751	-.8584	-.7470	-.6399	-.5362	-.4352	-.3364	-.2392	-.1431
	-.0476									
43	-4.3267	-3.3029	-2.7785	-2.4202	-2.1446	-1.9183	-1.7246	-1.5539	-1.4004	-1.2598
	-1.1295	-1.0074	-.8918	-.7815	-.6756	-.5732	-.4737	-.3764	-.2809	-.1866
	-.0931	.0000								
44	-4.3500	-3.3267	-2.8029	-2.4452	-2.1702	-1.9446	-1.7516	-1.5817	-1.4289	-1.2892
	-1.1598	-1.0386	-.9240	-.8149	-.7101	-.6089	-.5107	-.4149	-.3209	-.2282
	-.1366	-.0455								
45	-4.3727	-3.3500	-2.8267	-2.4696	-2.1952	-1.9702	-1.7779	-1.6087	-1.4567	-1.3178
	-1.1892	-1.0689	-.9553	-.8471	-.7434	-.6434	-.5464	-.4519	-.3593	-.2682
	-.1782	-.0889	.0000							
46	-4.3949	-3.3727	-2.8500	-2.4934	-2.2196	-1.9952	-1.8035	-1.6350	-1.4837	-1.3456
	-1.2178	-1.0983	-.9856	-.8784	-.7757	-.6768	-.5809	-.4876	-.3963	-.3067
	-.2182	-.1306	-.0435							
47	-4.4167	-3.3949	-2.8727	-2.5167	-2.2434	-2.0196	-1.8285	-1.6607	-1.5100	-1.3726
	-1.2456	-1.1269	-1.0150	-.9087	-.8069	-.7090	-.6143	-.5221	-.4321	-.3437
	-.2567	-.1706	-.0851	.0000						
48	-4.4380	-3.4167	-2.8949	-2.5394	-2.2667	-2.0434	-1.8529	-1.6857	-1.5357	-1.3989
	-1.2726	-1.1547	-1.0436	-.9381	-.8372	-.7403	-.6465	-.5554	-.4665	-.3794
	-.2937	-.2091	-.1251	-.0417						
49	-4.4588	-3.4380	-2.9167	-2.5616	-2.2894	-2.0667	-1.8767	-1.7101	-1.5607	-1.4246
	-1.2989	-1.1817	-1.0713	-.9666	-.8666	-.7706	-.6778	-.5877	-.4999	-.4139
	-.3294	-.2461	-.1636	-.0817	.0000					
50	-4.4792	-3.4588	-2.9380	-2.5834	-2.3116	-2.0894	-1.9000	-1.7339	-1.5851	-1.4496
	-1.3246	-1.2080	-1.0984	-.9944	-.8952	-.8000	-.7081	-.6189	-.5321	-.4472
	-.3639	-.2818	-.2006	-.1201	-.0400					
51	-4.4992	-3.4792	-2.9588	-2.6046	-2.3334	-2.1116	-1.9227	-1.7571	-1.6089	-1.4740
	-1.3496	-1.2337	-1.1247	-1.0215	-.9230	-.8286	-.7375	-.6492	-.5634	-.4795
	-.3972	-.3163	-.2364	-.1572	-.0785	.0000				
52	-4.5188	-3.4992	-2.9792	-2.6255	-2.3546	-2.1334	-1.9449	-1.7799	-1.6321	-1.4978
	-1.3740	-1.2587	-1.1503	-1.0478	-.9500	-.8563	-.7661	-.6787	-.5937	-.5108
	-.4295	-.3496	-.2708	-.1929	-.1155	-.0385				
53	-4.5380	-3.5188	-2.9992	-2.6459	-2.3755	-2.1546	-1.9667	-1.8021	-1.6549	-1.5210
	-1.3978	-1.2831	-1.1753	-1.0734	-.9763	-.8834	-.7938	-.7072	-.6231	-.5411
	-.4608	-.3819	-.3042	-.2274	-.1512	-.0755	.0000			

Table 3 (Continued)

n	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}
	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	a_{17}	a_{18}	a_{19}	a_{20}
	a_{21}	a_{22}	a_{23}	a_{24}	a_{25}	a_{26}	a_{27}	a_{28}	a_{29}	a_{30}
	a_{31}	a_{32}	a_{33}	a_{34}	a_{35}	a_{36}	a_{37}	a_{38}	a_{39}	a_{40}
54	-4.5569	-3.5380	-3.0188	-2.6659	-2.3959	-2.1755	-1.9880	-1.8238	-1.6771	-1.5438
	-1.4210	-1.3069	-1.1997	-1.0984	-1.0020	-.9097	-.8209	-.7350	-.6517	-.5705
	-.4911	-.4131	-.3364	-.2607	-.1857	-.1112	-.0370			
55	-4.5754	-3.5569	-3.0380	-2.6855	-2.4159	-2.1959	-2.0088	-1.8451	-1.6988	-1.5660
	-1.4438	-1.3301	-1.2235	-1.1228	-1.0270	-.9353	-.8472	-.7620	-.6795	-.5990
	-.5205	-.4434	-.3677	-.2930	-.2190	-.1457	-.0728	.0000		
56	-4.5936	-3.5754	-3.0569	-2.7047	-2.4355	-2.2159	-2.0292	-1.8659	-1.7201	-1.5877
	-1.4660	-1.3528	-1.2468	-1.1466	-1.0514	-.9603	-.8728	-.7883	-.7065	-.6268
	-.5490	-.4729	-.3980	-.3242	-.2513	-.1790	-.1072	-.0357		
57	-4.6115	-3.5936	-3.0754	-2.7236	-2.4547	-2.2355	-2.0492	-1.8863	-1.7409	-1.6090
	-1.4877	-1.3751	-1.2695	-1.1699	-1.0752	-.9847	-.8978	-.8140	-.7328	-.6538
	-.5768	-.5014	-.4274	-.3545	-.2825	-.2113	-.1406	-.0702	.0000	
58	-4.6290	-3.6115	-3.0936	-2.7421	-2.4736	-2.2547	-2.0688	-1.9063	-1.7613	-1.6298
	-1.5090	-1.3968	-1.2917	-1.1926	-1.0984	-1.0085	-.9222	-.8390	-.7584	-.6802
	-.6038	-.5292	-.4560	-.3839	-.3128	-.2425	-.1728	-.1035	-.0345	
59	-4.6463	-3.6290	-3.1115	-2.7603	-2.4921	-2.2736	-2.0880	-1.9260	-1.7813	-1.6502
	-1.5298	-1.4181	-1.3135	-1.2148	-1.1212	-1.0318	-.9460	-.8634	-.7834	-.7058
	-.6302	-.5562	-.4837	-.4125	-.3423	-.2728	-.2041	-.1358	-.0678	.0000
60	-4.6632	-3.6463	-3.1290	-2.7781	-2.5103	-2.2921	-2.1069	-1.9452	-1.8010	-1.6702
	-1.5502	-1.4389	-1.3348	-1.2366	-1.1434	-1.0545	-.9693	-.8872	-.8078	-.7308
	-.6558	-.5825	-.5108	-.4403	-.3708	-.3023	-.2344	-.1670	-.1001	-.0333
61	-4.6799	-3.6632	-3.1463	-2.7957	-2.5281	-2.3103	-2.1254	-1.9641	-1.8202	-1.6898
	-1.5702	-1.4593	-1.3556	-1.2578	-1.1651	-1.0767	-.9920	-.9104	-.8316	-.7552
	-.6808	-.6082	-.5371	-.4673	-.3986	-.3308	-.2638	-.1973	-.1313	-.0656
	.0000									
62	-4.6963	-3.6799	-3.1632	-2.8129	-2.5457	-2.3281	-2.1436	-1.9826	-1.8391	-1.7091
	-1.5898	-1.4793	-1.3760	-1.2787	-1.1864	-1.0985	-1.0142	-.9332	-.8549	-.7790
	-.7052	-.6332	-.5627	-.4936	-.4256	-.3586	-.2924	-.2268	-.1616	-.0968
	-.0323									
63	-4.7124	-3.6963	-3.1799	-2.8299	-2.5629	-2.3457	-2.1615	-2.0008	-1.8576	-1.7279
	-1.6091	-1.4989	-1.3960	-1.2991	-1.2072	-1.1197	-1.0360	-.9554	-.8776	-.8023
	-.7290	-.6576	-.5877	-.5193	-.4519	-.3856	-.3201	-.2553	-.1910	-.1271
	-.0635	.0000								
64	-4.7283	-3.7124	-3.1963	-2.8465	-2.5799	-2.3629	-2.1790	-2.0186	-1.8758	-1.7465
	-1.6279	-1.5182	-1.4156	-1.3191	-1.2276	-1.1406	-1.0572	-.9771	-.8998	-.8250
	-.7523	-.6814	-.6121	-.5443	-.4776	-.4119	-.3472	-.2831	-.2196	-.1566
	-.0938	-.0312								
65	-4.7439	-3.7283	-3.2124	-2.8629	-2.5965	-2.3799	-2.1963	-2.0362	-1.8936	-1.7646
	-1.6465	-1.5370	-1.4348	-1.3387	-1.2476	-1.1610	-1.0781	-.9984	-.9216	-.8472
	-.7750	-.7046	-.6359	-.5686	-.5026	-.4376	-.3735	-.3101	-.2474	-.1851
	-.1232	-.0616	.0000							
66	-4.7593	-3.7439	-3.2283	-2.8791	-2.6129	-2.3965	-2.2132	-2.0534	-1.9112	-1.7825
	-1.6646	-1.5556	-1.4537	-1.3579	-1.2673	-1.1810	-1.0985	-1.0192	-.9429	-.8689
	-.7972	-.7274	-.6592	-.5925	-.5270	-.4626	-.3991	-.3364	-.2744	-.2129
	-.1518	-.0910	-.0303							
67	-4.7744	-3.7593	-3.2439	-2.8949	-2.6291	-2.4129	-2.2299	-2.0703	-1.9284	-1.8000
	-1.6825	-1.5737	-1.4722	-1.3768	-1.2865	-1.2006	-1.1185	-1.0397	-.9637	-.8902
	-.8189	-.7496	-.6819	-.6157	-.5508	-.4870	-.4241	-.3621	-.3007	-.2399
	-.1796	-.1195	-.0597	.0000						
68	-4.7894	-3.7744	-3.2593	-2.9106	-2.6449	-2.4291	-2.2463	-2.0870	-1.9453	-1.8173
	-1.7000	-1.5916	-1.4904	-1.3953	-1.3053	-1.2198	-1.1381	-1.0597	-.9841	-.9111
	-.8402	-.7713	-.7041	-.6384	-.5740	-.5108	-.4485	-.3871	-.3264	-.2662
	-.2066	-.1473	-.0883	-.0294						
69	-4.8041	-3.7894	-3.2744	-2.9259	-2.6606	-2.4449	-2.2624	-2.1034	-1.9620	-1.8342
	-1.7173	-1.6091	-1.5083	-1.4135	-1.3239	-1.2387	-1.1573	-1.0793	-1.0041	-.9315
	-.8611	-.7926	-.7259	-.6607	-.5968	-.5340	-.4723	-.4115	-.3514	-.2919
	-.2329	-.1743	-.1161	-.0580	.0000					

Table 3 (Continued)

n	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}
	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	a_{17}	a_{18}	a_{19}	a_{20}
	a_{21}	a_{22}	a_{23}	a_{24}	a_{25}	a_{26}	a_{27}	a_{28}	a_{29}	a_{30}
	a_{31}	a_{32}	a_{33}	a_{34}	a_{35}	a_{36}	a_{37}	a_{38}	a_{39}	a_{40}
70	-4.8186	-3.8041	-3.2894	-2.9411	-2.6759	-2.4606	-2.2783	-2.1195	-1.9784	-1.8509
	-1.7342	-1.6264	-1.5258	-1.4313	-1.3420	-1.2572	-1.1762	-1.0985	-1.0237	-.9515
	-.8815	-.8134	-.7472	-.6824	-.6190	-.5568	-.4956	-.4353	-.3758	-.3169
	-.2586	-.2007	-.1431	-.0858	-.0286					
71	-4.8328	-3.8186	-3.3041	-2.9560	-2.6911	-2.4759	-2.2939	-2.1354	-1.9945	-1.8673
	-1.7509	-1.6433	-1.5430	-1.4489	-1.3599	-1.2754	-1.1947	-1.1174	-1.0429	-.9711
	-.9015	-.8338	-.7680	-.7037	-.6407	-.5790	-.5183	-.4585	-.3996	-.3413
	-.2836	-.2263	-.1694	-.1128	-.0563	.0000				
72	-4.8469	-3.8328	-3.3186	-2.9707	-2.7060	-2.4911	-2.3093	-2.1510	-2.0104	-1.8834
	-1.7673	-1.6600	-1.5600	-1.4661	-1.3775	-1.2932	-1.2129	-1.1359	-1.0618	-.9903
	-.9211	-.8538	-.7884	-.7245	-.6620	-.6007	-.5405	-.4813	-.4228	-.3651
	-.3079	-.2513	-.1950	-.1391	-.0834	-.0278				
73	-4.8608	-3.8469	-3.3328	-2.9852	-2.7207	-2.5060	-2.3244	-2.1664	-2.0260	-1.8993
	-1.7834	-1.6764	-1.5767	-1.4831	-1.3947	-1.3108	-1.2307	-1.1541	-1.0803	-1.0092
	-.9403	-.8735	-.8084	-.7449	-.6828	-.6220	-.5623	-.5035	-.4456	-.3883
	-.3318	-.2757	-.2200	-.1647	-.1097	-.0548	.0000			
74	-4.8745	-3.8608	-3.3469	-2.9995	-2.7352	-2.5207	-2.3394	-2.1816	-2.0414	-1.9149
	-1.7993	-1.6925	-1.5931	-1.4997	-1.4116	-1.3280	-1.2483	-1.1719	-1.0985	-1.0277
	-.9592	-.8927	-.8280	-.7649	-.7032	-.6428	-.5835	-.5252	-.4678	-.4111
	-.3550	-.2995	-.2444	-.1897	-.1353	-.0811	-.0270			
75	-4.8880	-3.8745	-3.3608	-3.0136	-2.7495	-2.5352	-2.3541	-2.1965	-2.0566	-1.9303
	-1.8149	-1.7084	-1.6092	-1.5161	-1.4283	-1.3450	-1.2655	-1.1895	-1.1164	-1.0459
	-.9777	-.9116	-.8472	-.7845	-.7232	-.6632	-.6044	-.5465	-.4895	-.4333
	-.3777	-.3228	-.2682	-.2141	-.1603	-.1068	-.0533	.0000		
76	-4.9014	-3.8880	-3.3745	-3.0275	-2.7636	-2.5495	-2.3686	-2.2112	-2.0715	-1.9455
	-1.8303	-1.7240	-1.6251	-1.5323	-1.4447	-1.3616	-1.2825	-1.2067	-1.1339	-1.0637
	-.9959	-.9301	-.8661	-.8038	-.7429	-.6832	-.6248	-.5673	-.5108	-.4550
	-.4000	-.3455	-.2915	-.2379	-.1847	-.1318	-.0790	-.0263		
77	-4.9145	-3.9014	-3.3880	-3.0412	-2.7775	-2.5636	-2.3828	-2.2257	-2.0862	-1.9604
	-1.8455	-1.7394	-1.6407	-1.5481	-1.4608	-1.3780	-1.2991	-1.2237	-1.1511	-1.0813
	-1.0137	-.9483	-.8846	-.8226	-.7621	-.7029	-.6448	-.5877	-.5316	-.4763
	-.4217	-.3677	-.3142	-.2612	-.2085	-.1562	-.1040	-.0520	.0000	
78	-4.9275	-3.9145	-3.4014	-3.0547	-2.7912	-2.5775	-2.3969	-2.2400	-2.1007	-1.9751
	-1.8604	-1.7545	-1.6561	-1.5638	-1.4767	-1.3942	-1.3155	-1.2403	-1.1681	-1.0985
	-1.0313	-.9661	-.9028	-.8411	-.7810	-.7221	-.6644	-.6077	-.5520	-.4971
	-.4430	-.3894	-.3365	-.2839	-.2318	-.1800	-.1284	-.0770	-.0256	
79	-4.9403	-3.9275	-3.4145	-3.0680	-2.8047	-2.5912	-2.4108	-2.2541	-2.1150	-1.9896
	-1.8751	-1.7695	-1.6712	-1.5791	-1.4923	-1.4100	-1.3317	-1.2567	-1.1848	-1.1155
	-1.0485	-.9837	-.9207	-.8593	-.7995	-.7410	-.6836	-.6274	-.5720	-.5176
	-.4638	-.4107	-.3582	-.3061	-.2545	-.2032	-.1522	-.1013	-.0506	.0000
80	-4.9530	-3.9403	-3.4275	-3.0812	-2.8180	-2.6047	-2.4245	-2.2680	-2.1291	-2.0039
	-1.8896	-1.7842	-1.6861	-1.5943	-1.5077	-1.4257	-1.3475	-1.2728	-1.2012	-1.1321
	-1.0655	-1.0009	-.9382	-.8772	-.8177	-.7595	-.7025	-.6466	-.5916	-.5376
	-.4842	-.4316	-.3795	-.3279	-.2767	-.2259	-.1754	-.1252	-.0750	-.0250
81	-4.9655	-3.9530	-3.4403	-3.0942	-2.8312	-2.6180	-2.4380	-2.2817	-2.1430	-2.0180
	-1.9039	-1.7987	-1.7008	-1.6092	-1.5229	-1.4410	-1.3632	-1.2887	-1.2173	-1.1485
	-1.0821	-1.0178	-.9554	-.8947	-.8355	-.7777	-.7210	-.6655	-.6109	-.5572
	-.5042	-.4520	-.4003	-.3492	-.2985	-.2482	-.1982	-.1484	-.0988	-.0494
	.0000									
82	-4.9778	-3.9655	-3.4530	-3.1070	-2.8442	-2.6312	-2.4514	-2.2952	-2.1567	-2.0318
	-1.9180	-1.8130	-1.7153	-1.6239	-1.5378	-1.4562	-1.3785	-1.3043	-1.2332	-1.1647
	-1.0985	-1.0345	-.9724	-.9120	-.8531	-.7955	-.7392	-.6840	-.6297	-.5764
	-.5238	-.4720	-.4207	-.3700	-.3198	-.2699	-.2204	-.1711	-.1221	-.0732
	-.0244									
83	-4.9900	-3.9778	-3.4655	-3.1196	-2.8570	-2.6442	-2.4645	-2.3085	-2.1702	-2.0455
	-1.9318	-1.8270	-1.7296	-1.6384	-1.5525	-1.4711	-1.3937	-1.3197	-1.2488	-1.1805
	-1.1147	-1.0509	-.9891	-.9289	-.8703	-.8131	-.7570	-.7022	-.6483	-.5953
	-.5431	-.4916	-.4407	-.3904	-.3406	-.2912	-.2421	-.1934	-.1448	-.0965
	-.0482	.0000								

Table 3 (Continued)

n	a_1 a_{11} a_{21} a_{31} a_{41}	a_2 a_{12} a_{22} a_{32} a_{42}	a_3 a_{13} a_{23} a_{33} a_{43}	a_4 a_{14} a_{24} a_{34} a_{44}	a_5 a_{15} a_{25} a_{35} a_{45}	a_6 a_{16} a_{26} a_{36} a_{46}	a_7 a_{17} a_{27} a_{37} a_{47}	a_8 a_{18} a_{28} a_{38} a_{48}	a_9 a_{19} a_{29} a_{39} a_{49}	a_{10} a_{20} a_{30} a_{40} a_{50}
84	-5.0021	-3.9900	-3.4778	-3.1321	-2.8696	-2.6570	-2.4775	-2.3217	-2.1835	-2.0591
	-1.9455	-1.8409	-1.7437	-1.6527	-1.5670	-1.4858	-1.4086	-1.3349	-1.2642	-1.1962
	-1.1305	-1.0670	-1.0055	-.9456	-.8872	-.8303	-.7746	-.7200	-.6664	-.6138
	-.5619	-.5108	-.4603	-.4104	-.3610	-.3120	-.2634	-.2151	-.1670	-.1192
	-.0715	-.0238								
85	-5.0140	-4.0021	-3.4900	-3.1445	-2.8821	-2.6696	-2.4903	-2.3346	-2.1967	-2.0724
	-1.9591	-1.8546	-1.7576	-1.6668	-1.5813	-1.5003	-1.4233	-1.3498	-1.2793	-1.2115
	-1.1462	-1.0829	-1.0216	-.9620	-.9039	-.8472	-.7918	-.7376	-.6843	-.6320
	-.5804	-.5297	-.4795	-.4300	-.3810	-.3324	-.2842	-.2364	-.1888	-.1414
	-.0942	-.0471	.0000							
86	-5.0257	-4.0140	-3.5021	-3.1567	-2.8945	-2.6821	-2.5030	-2.3475	-2.2096	-2.0855
	-1.9724	-1.8681	-1.7713	-1.6807	-1.5954	-1.5146	-1.4378	-1.3645	-1.2942	-1.2267
	-1.1615	-1.0985	-1.0375	-.9781	-.9203	-.8639	-.8088	-.7548	-.7018	-.6498
	-.5986	-.5482	-.4984	-.4492	-.4006	-.3524	-.3046	-.2572	-.2101	-.1631
	-.1164	-.0698	-.0233							
87	-5.0374	-4.0257	-3.5140	-3.1687	-2.9067	-2.6945	-2.5155	-2.3601	-2.2225	-2.0985
	-1.9855	-1.8815	-1.7848	-1.6944	-1.6092	-1.5287	-1.4521	-1.3790	-1.3090	-1.2416
	-1.1767	-1.1139	-1.0531	-.9940	-.9364	-.8803	-.8255	-.7717	-.7191	-.6674
	-.6165	-.5664	-.5169	-.4681	-.4198	-.3720	-.3246	-.2776	-.2309	-.1844
	-.1381	-.0920	-.0460	.0000						
88	-5.0489	-4.0374	-3.5257	-3.1806	-2.9187	-2.7067	-2.5278	-2.3726	-2.2351	-2.1114
	-1.9985	-1.8946	-1.7981	-1.7079	-1.6229	-1.5426	-1.4662	-1.3933	-1.3234	-1.2563
	-1.1916	-1.1291	-1.0685	-1.0096	-.9523	-.8964	-.8418	-.7884	-.7360	-.6846
	-.6340	-.5842	-.5351	-.4866	-.4387	-.3913	-.3443	-.2976	-.2513	-.2053
	-.1594	-.1138	-.0682	-.0227						
89	-5.0602	-4.0489	-3.5374	-3.1924	-2.9306	-2.7187	-2.5400	-2.3850	-2.2476	-2.1240
	-2.0114	-1.9076	-1.8113	-1.7212	-1.6365	-1.5563	-1.4801	-1.4074	-1.3377	-1.2708
	-1.2063	-1.1440	-1.0836	-1.0250	-.9679	-.9123	-.8580	-.8048	-.7527	-.7015
	-.6513	-.6018	-.5530	-.5048	-.4572	-.4101	-.3635	-.3172	-.2713	-.2257
	-.1803	-.1350	-.0899	-.0449	.0000					
90	-5.0715	-4.0602	-3.5489	-3.2040	-2.9424	-2.7306	-2.5521	-2.3972	-2.2600	-2.1365
	-2.0240	-1.9204	-1.8243	-1.7344	-1.6498	-1.5698	-1.4938	-1.4213	-1.3518	-1.2851
	-1.2208	-1.1587	-1.0985	-1.0401	-.9833	-.9279	-.8738	-.8209	-.7691	-.7182
	-.6682	-.6190	-.5705	-.5227	-.4754	-.4286	-.3824	-.3365	-.2909	-.2457
	-.2007	-.1559	-.1112	-.0667	-.0222					
91	-5.0826	-4.0715	-3.5602	-3.2155	-2.9540	-2.7424	-2.5640	-2.4092	-2.2722	-2.1489
	-2.0365	-1.9331	-1.8371	-1.7474	-1.6630	-1.5831	-1.5073	-1.4350	-1.3657	-1.2992
	-1.2351	-1.1732	-1.1132	-1.0551	-.9985	-.9433	-.8895	-.8368	-.7852	-.7346
	-.6849	-.6360	-.5878	-.5402	-.4933	-.4468	-.4009	-.3553	-.3101	-.2653
	-.2207	-.1763	-.1321	-.0880	-.0440	.0000				
92	-5.0936	-4.0826	-3.5715	-3.2269	-2.9655	-2.7540	-2.5757	-2.4211	-2.2842	-2.1611
	-2.0489	-1.9456	-1.8498	-1.7602	-1.6759	-1.5963	-1.5206	-1.4485	-1.3794	-1.3131
	-1.2492	-1.1875	-1.1277	-1.0698	-1.0134	-.9585	-.9049	-.8524	-.8011	-.7507
	-.7013	-.6526	-.6047	-.5575	-.5108	-.4647	-.4191	-.3738	-.3290	-.2845
	-.2403	-.1963	-.1525	-.1088	-.0652	-.0217				
93	-5.1044	-4.0936	-3.5826	-3.2381	-2.9769	-2.7655	-2.5874	-2.4329	-2.2961	-2.1731
	-2.0611	-1.9579	-1.8623	-1.7728	-1.6888	-1.6093	-1.5338	-1.4618	-1.3929	-1.3268
	-1.2631	-1.2016	-1.1420	-1.0843	-1.0281	-.9734	-.9200	-.8678	-.8167	-.7666
	-.7174	-.6690	-.6214	-.5744	-.5280	-.4822	-.4369	-.3920	-.3475	-.3034
	-.2595	-.2159	-.1725	-.1292	-.0861	-.0430	.0000			
94	-5.1152	-4.1044	-3.5936	-3.2492	-2.9881	-2.7769	-2.5989	-2.4445	-2.3079	-2.1850
	-2.0731	-1.9701	-1.8746	-1.7853	-1.7014	-1.6221	-1.5468	-1.4750	-1.4062	-1.3403
	-1.2768	-1.2155	-1.1561	-1.0985	-1.0426	-.9881	-.9349	-.8830	-.8321	-.7822
	-.7333	-.6851	-.6378	-.5911	-.5450	-.4995	-.4545	-.4099	-.3657	-.3219
	-.2784	-.2351	-.1921	-.1492	-.1065	-.0638	-.0213			

Table 3 (Continued)

n	a_1 a_{11} a_{21} a_{31} a_{41}	a_2 a_{12} a_{22} a_{32} a_{42}	a_3 a_{13} a_{23} a_{33} a_{43}	a_4 a_{14} a_{24} a_{34} a_{44}	a_5 a_{15} a_{25} a_{35} a_{45}	a_6 a_{16} a_{26} a_{36} a_{46}	a_7 a_{17} a_{27} a_{37} a_{47}	a_8 a_{18} a_{28} a_{38} a_{48}	a_9 a_{19} a_{29} a_{39} a_{49}	a_{10} a_{20} a_{30} a_{40} a_{50}
95	-5.1258	-4.1152	-3.6044	-3.2602	-2.9992	-2.7881	-2.6102	-2.4560	-2.3195	-2.1968
	-2.0850	-1.9822	-1.8868	-1.7977	-1.7139	-1.6348	-1.5596	-1.4879	-1.4194	-1.3536
	-1.2903	-1.2292	-1.1700	-1.1126	-1.0569	-1.0026	-.9496	-.8979	-.8473	-.7976
	-.7489	-.7010	-.6539	-.6075	-.5617	-.5164	-.4717	-.4274	-.3836	-.3401
	-.2969	-.2540	-.2113	-.1688	-.1265	-.0843	-.0421	.0000		
96	-5.1363	-4.1258	-3.6152	-3.2711	-3.0102	-2.7992	-2.6215	-2.4674	-2.3310	-2.2084
	-2.0968	-1.9941	-1.8989	-1.8099	-1.7263	-1.6473	-1.5723	-1.5008	-1.4324	-1.3668
	-1.3036	-1.2427	-1.1837	-1.1265	-1.0710	-1.0169	-.9641	-.9126	-.8622	-.8128
	-.7643	-.7166	-.6698	-.6236	-.5781	-.5331	-.4886	-.4447	-.4011	-.3579
	-.3151	-.2725	-.2302	-.1880	-.1461	-.1043	-.0625	-.0208		
97	-5.1468	-4.1363	-3.6258	-3.2818	-3.0211	-2.8102	-2.6326	-2.4786	-2.3424	-2.2199
	-2.1084	-2.0059	-1.9108	-1.8219	-1.7385	-1.6596	-1.5848	-1.5134	-1.4452	-1.3798
	-1.3168	-1.2560	-1.1972	-1.1402	-1.0849	-1.0310	-.9784	-.9271	-.8769	-.8277
	-.7794	-.7320	-.6854	-.6395	-.5942	-.5495	-.5053	-.4616	-.4184	-.3755
	-.3329	-.2907	-.2487	-.2069	-.1653	-.1239	-.0825	-.0412	.0000	
98	-5.1571	-4.1468	-3.6363	-3.2925	-3.0318	-2.8211	-2.6436	-2.4897	-2.3536	-2.2313
	-2.1199	-2.0175	-1.9225	-1.8338	-1.7505	-1.6718	-1.5971	-1.5259	-1.4579	-1.3926
	-1.3298	-1.2692	-1.2105	-1.1537	-1.0986	-1.0449	-.9925	-.9414	-.8914	-.8424
	-.7944	-.7472	-.7008	-.6551	-.6101	-.5656	-.5217	-.4783	-.4353	-.3927
	-.3505	-.3085	-.2669	-.2254	-.1842	-.1431	-.1021	-.0612	-.0204	
99	-5.1673	-4.1571	-3.6468	-3.3030	-3.0425	-2.8318	-2.6544	-2.5007	-2.3647	-2.2425
	-2.1313	-2.0290	-1.9342	-1.8456	-1.7624	-1.6838	-1.6093	-1.5383	-1.4704	-1.4052
	-1.3426	-1.2821	-1.2237	-1.1671	-1.1121	-1.0586	-1.0064	-.9555	-.9057	-.8569
	-.8091	-.7621	-.7159	-.6705	-.6257	-.5815	-.5378	-.4947	-.4520	-.4097
	-.3677	-.3261	-.2847	-.2436	-.2027	-.1620	-.1214	-.0808	-.0404	.0000
100	-5.1774	-4.1673	-3.6571	-3.3134	-3.0530	-2.8425	-2.6652	-2.5116	-2.3757	-2.2536
	-2.1425	-2.0403	-1.9456	-1.8572	-1.7742	-1.6957	-1.6213	-1.5505	-1.4827	-1.4177
	-1.3552	-1.2950	-1.2367	-1.1802	-1.1254	-1.0721	-1.0201	-.9694	-.9198	-.8712
	-.8236	-.7768	-.7309	-.6856	-.6411	-.5971	-.5537	-.5108	-.4684	-.4263
	-.3847	-.3433	-.3023	-.2615	-.2209	-.1805	-.1402	-.1001	-.0600	-.0200

Table 4 : Performance of unbiased LAMLEs (μ^*, σ^*) of the parameters (μ, σ) of logistic distribution compared with the corresponding BLUEs (μ^{**}, σ^{**}) based on the exact variances of the estimators from complete and Type II censored samples of size n=5 and 10 with all possible choices of *r* and *s*.

n	r	s	$\frac{V(\mu^*)}{\sigma^2}$	$\frac{V(\mu^{**})}{\sigma^2}$	$Eff(\mu^*)$	$\frac{V(\sigma^*)}{\sigma^2}$	$\frac{V(\sigma^{**})}{\sigma^2}$	$Eff(\sigma^*)$
5	0	0	.6282	.6278	99.94	.1702	.1701	99.97
		1	.6445	.6442	99.94	.2326	.2324	99.91
		2	.7774	.7774	100.00	.3734	.3727	99.81
		3	1.7212	1.7212	100.00	.8165	.8165	100.00
	1	0	.6445	.6441	99.95	.2319	.2317	99.93
		1	.6524	.6521	99.96	.3586	.3586	100.00
		2	.7899	.7899	100.00	.7741	.7741	100.00
	2	0	.7774	.7774	100.00	.3717	.3711	99.85
		1	.7899	.7899	100.00	.7741	.7741	100.00
	3	0	1.7170	1.7170	100.00	.8102	.8102	100.00
10	0	0	.3076	.3075	99.96	.0768	.0767	99.96
		1	.3086	.3084	99.93	.0860	.0860	99.96
		2	.3128	.3126	99.96	.1001	.1000	99.95
		3	.3255	.3255	99.99	.1202	.1201	99.92
		4	.3574	.3574	100.00	.1497	.1495	99.90
		5	.4333	.4333	99.99	.1954	.1952	99.89
		6	.6231	.6229	99.96	.2730	.2727	99.89
		7	1.1784	1.1778	99.95	.4302	.4298	99.92
		8	3.6272	3.6272	100.00	.9037	.9037	100.00
	1	0	.3086	.3084	99.93	.0861	.0860	99.97
		1	.3094	.3091	99.90	.0978	.0978	99.99
		2	.3131	.3129	99.94	.1161	.1161	99.99
		3	.3256	.3255	99.97	.1436	.1436	99.98
		4	.3590	.3590	99.99	.1869	.1868	99.97
		5	.4466	.4466	100.00	.2612	.2612	99.97
		6	.7019	.7018	100.00	.4130	.4130	99.98
		7	1.7605	1.7605	100.00	.8748	.8748	100.00
10	2	0	.3129	.3128	99.95	.0997	.0997	99.94
		1	.3133	.3131	99.92	.1153	.1153	99.98
		2	.3158	.3157	99.96	.1418	.1418	100.00
		3	.3264	.3264	99.98	.1834	.1834	100.00
		4	.3593	.3593	100.00	.2559	.2559	99.99
		5	.4630	.4630	100.00	.4053	.4052	100.00
		6	.9010	.9010	100.00	.8622	.8622	100.00
	3	0	.3255	.3255	99.99	.1201	.1200	99.92
		1	.3256	.3255	99.97	.1436	.1436	99.98
		2	.3264	.3264	99.98	.1834	.1834	100.00
		3	.3327	.3326	99.99	.2543	.2543	100.00
		4	.3596	.3596	100.00	.4021	.4021	100.00
		5	.4916	.4916	100.00	.8565	.8565	100.00
	4	0	.3574	.3574	100.00	.1497	.1495	99.90
		1	.3590	.3590	99.99	.1869	.1868	99.97
		2	.3593	.3593	100.00	.2559	.2559	99.99
		3	.3596	.3596	100.00	.4021	.4021	100.00
		4	.3685	.3685	100.00	.8549	.8549	100.00
	5	0	.4333	.4333	99.99	.1954	.1952	99.89
		1	.4466	.4466	100.00	.2612	.2612	99.97
		2	.4630	.4630	100.00	.4053	.4052	100.00
		3	.4916	.4916	100.00	.8565	.8565	100.00
	6	0	.6231	.6229	99.96	.2730	.2728	99.89
		1	.7019	.7019	100.00	.4130	.4130	99.98
		2	.9010	.9010	100.00	.8622	.8622	100.00
	7	0	1.1781	1.1775	99.95	.4301	.4297	99.92
		1	1.7605	1.7605	100.00	.8748	.8748	100.00
	8	0	3.6272	3.6272	100.00	.9037	.9037	100.00

REFERENCES

[1] Balakrishnan, N. (1989). Approximate MLE of scale parameter of the Rayleigh distribution with censoring, *I E E E Trans. On Reliab.*, 38, 355 – 357.

[2] Balakrishnan, N., (Ed.) (1992). *Handbook of the Logistic distribution*. New York: Marcel Dekker.

[3] Balakrishnan, N. and Cohen, A.C. (1991). *Order statistics and Inference*: Estimation Methods. Academic Press, Boston, San Diego, New York.

[4] Balakrishnan, N. and Malik, H.J. (1995). Means, variances and covariances of logistic order statistics for samples up to fifty. *Selected tables in Mathematical Statistics.*

[5] Berkson, J. (1944). Application of the logistic function to bio-assay, *J. Amer. Statist. Assoc.*, 39, 357 – 365.

[6] Berkson, J. (1951). Why I prefer logistic to probits, *Biometrics*, 7, 327 – 339.

[7] Berkson, J. (1953). A statistically precise and relatively simple method of estimating the bio-assay and quantal response based on the logistic function. *J. Amer. Statist. Assoc.*, 48, 565 – 599.

[7] Berkson, J. (1955). Maximum likelihood and minimum χ^2 estimates of the logistic function. *J. Amer. Statist. Assoc.*, 50, 130 – 162.

[8] Berkson, J. (1957). Tables for the maximum likelihood estimate of the logistic function. *Biometrics*, 13, 28 – 34.

[9] Blom, G. (1962). Nearly best linear estimates of location and scale parameters, in Contributions to Order Statistics (A.E. Sarhan and B.G. Greenberg, eds.), pp.34-46, John Wiley & Sons, Newyork.

[10] Chew, V. (1968). Some useful alternatives to the normal distribution, *Amer. Statist.*, 22, *22-24*.

[11] Davis, D.J. (1952). An analysis of some failure data, *J. Amer. Statist. Assoc.*, 47, 113-150.

[12] Downton, F. (1966). Linear estimates with polynomial coefficients, Biometrika, 53, 129-141.

[13] Erkelens, J. (1968). A method of calculation for the logistic curve. *Statistica Neerlandica*, 22, 213 – 217.

[14] Gupta, A.K. (1952). Estimation of the mean and standard deviation of a normal population from a censored sample, *Biometrika*, 39, 260-273.

[15] Harter, H.L., and Moore, A.M. (1967). Maximum likelihood estimation, from censored samples of the parameters of a logistic distribution. *J.Amer. Statist. Assoc.*, 62, 675 – 684.

[16] Lloyd, E.H. (1952). Least squares estimation of location and scale parameters using order statistics. *Biometrika*, 39, 88-95.

[17] Paul A.T. and David C.T. (1995). *Applied Reliability*, Second Edition. Van Nostrand Reinhold, New York.

[18] Sarhan, A.E. and Greenberg, B.G. (eds.) (1962). *Contributions to Order Statistics*, John Wiley & Sons, New York.

[19] Vaudeva Rao, A., Sitaramacharyulu, P. and Chenchu Ramaiah, M. (2017). Linear approximate ML estimation in scaled Type I generalized logistic distribution based on Type-II censored samples. *Commun. Statist. – Simul. Comput.*, 46 (3), 1682 – 1702.