# A STUDY ON FUZZY SOFT MAXIMAL CLOSED SET AND FUZZY SOFT MINIMAL CLOSED SET

[1]Shakila.K, [2]R.Selvi
[1]Research Scholar, [2]Assistant Professor,
[1]Department of Mathematics,
[1]Sri Parasakthi College for Women, Courtallam, India
Affiliated to Manonmaniam Sundaranar University, Tirunelveli.

*Abstract:* In this paper we introduce fuzzy soft minimal closed set, fuzzy soft maximal closed set in fuzzy soft topological space. Some of their basic properties are obtained. And also some properties of fuzzy soft minimal closed set, fuzzy soft maximal closed set, fuzzy soft minimal open set, fuzzy soft maximal open set are studied and investigated.

*Index Terms* - **Fuzzy soft topology, Fuzzy Soft Maximal closed set, Fuzzy Soft Minimal closed set, Fuzzy Soft open set and Fuzzy Soft closed set.**

## I. INTRODUCTION

Fuzzy set was introduced by L.A.Zadeh[1] in 1965 and its topological properties were first studied by C.L.Chang[2] in the year 1968. Later many authors contributed for the development of fuzzy topology. To overcome the difficulties in fuzzy set D.Molodtsov[3] proposed a new theory called soft set for modeling uncertainties and their topological properties were initiated by Shabir and Naz[4] in the year 2011.

Maji et al[5] initiated the concept of fuzzy soft set which is a combination of fuzzy set and soft set where soft set is defined over fuzzy set. Its topological properties were first studied by Tanay et al[8] in 2011. The fuzzy soft set act as an excellent tool in solving the uncertainties faced in the different fields such as medical science, engineering, economics, environment etc.

In the year 2001 and 2003, F.Nakaoka and N.Oda[9,10,11] introduced and studied the subclasses of open and closed sets such as minimal open (resp. minimal closed) sets and maximal open (resp. maximal closed set) sets. S.S.Benchalli, Basavaraj M. Ittanagi, R.S. Wali[14] introduced minimal open sets and maps in topological spaces.

## II. PRELIMINARIES

**Definition 2.1[1]:** A fuzzy set $A$ of a non-empty set $X$ is characterized by a membership function $\mu_A: X \to [0,1]$ whose value $\mu_A(x)$ represents the "degree of membership" of $x$ in $A$ for $x \in X$.

**Definition 2.2[3]:** Let $A$ be a subset of $E$. A pair $(F, A)$ is called a soft set over $X$ where $F: A \to P(X)$ defined by $F(e) \to P(X) \forall\, e \in A$. In other words, $F(e)$ may be considered as the set of $e$-approximate element of the soft set $(F, A)$.

**Definition 2.3[5]:** Let $A \subset E$ and $\mathcal{F}(X)$ be the set of all fuzzy sets in $X$. Then a pair $(f, A)$ is called a fuzzy soft set over $X$, denoted by $f_A$, where $f: A \to \mathcal{F}(X)$ is a function.

From the definition, it is clear that $f(a)$ is a fuzzy set in $U$, for each $a \in A$, and we will denote the membership function of $f(a)$ by $f_a: X \to [0,1]$.

**Definition 2.4[5]:** For two fuzzy soft sets $(f, A)$ and $(g, B)$ over a common universe $X$, we say that $(f, A)$ is a fuzzy soft subset of $(g, B)$ if

i) $A \subset B$, and
ii) For each $a \in A$, $f_a \leq g_a$, that is $f_a$ is a fuzzy subset of $g_a$.

This relationship is denoted by $(f, A) \tilde{\subseteq} (g, B)$. Similarly, $(f, A)$ is said to be a fuzzy soft superset of $(g, B)$, if $(g, B)$ is a fuzzy soft subset of $(f, A)$. This relationship is denoted by $(f, A) \tilde{\supseteq} (g, B)$.

**Definition 2.5[5]:** Two fuzzy soft sets $(f, A)$ and $(g, B)$ over a common universe $X$ are said to be fuzzy soft equal if $(f, A)$ is a fuzzy soft subset of $(g, B)$ and $(g, B)$ is a fuzzy soft subset of $(f, A)$.

**Definition 2.6[5]:** The union of two fuzzy soft sets $(f, A)$ and $(g, B)$ over a common universe $X$ is the fuzzy soft set $(h, C)$, where $C = A \cup B$ and $\forall c \in C$,

$$h(c) = \begin{cases} f_c, & if\ c \in A - B \\ g_c, & if\ c \in B - A \\ f_c \vee g_c, & if\ c \in A \cap B. \end{cases}$$

This relationship is denoted by $(f, A) \tilde{\cup} (g, B) = (h, C)$.

**Definition 2.7[5]:** The intersection of two fuzzy soft sets $(f, A)$ and $(g, B)$ over a common universe $X$ is the fuzzy soft set $(h, C)$, where $C = A \cap B$ and $\forall c \in C$, $h_c = f_c \wedge g_c$. This is denoted by $(f, A) \tilde{\cap} (g, B) = (h, C)$.

**Definition 2.8[5]:** A fuzzy soft set $(f, A)$ over $X$ is said to be a null fuzzy soft set if and only if for each $e \in A, f_e = \tilde{0}$, where $\tilde{0}$ is the membership function of null fuzzy set over $X$, which takes value 0 for all $x$ in $X$.
**Definition 2.9[5]:** A fuzzy soft set $(f, A)$ over $X$ is said to be an absolute fuzzy soft set if and only if for each $e \in A, f_e = \tilde{1}$, where $\tilde{1}$ is the membership function of absolute fuzzy set over $X$, which takes value 1 for all $x$ in $X$.
**Definition 2.10[5]:** The complement of a fuzzy soft set $(f, A)$ is the fuzzy soft set $(f^c, A)$, which is denoted by $(f, A)^c$ and where $f^c: A \to F(X)$ is a fuzzy set valued function i.e., for each $a \in A, f'(a)$ is a fuzzy set in $X$, whose membership function $f'_a(x) = 1 - f_a(x)$ for all $x \in X$. Here $f'_a$ is the membership function of $f'(a)$.
**Definition 2.11[8]:** Let $\tau$ be a collection of fuzzy soft sets over a universe $X$ with a fixed parameter set $E$, then $(f_E, \tau)$ is called fuzzy soft topology if

i) $\tilde{0}_E, \tilde{1}_E \in \tau$
ii) Union of any members of $\tau$ is a member of $\tau$.
iii) Intersection of any two members of $\tau$ is a member of $\tau$.

Each member of $\tau$ is called fuzzy soft open set i.e. A fuzzy soft set $f_A$ over $X$ is fuzzy soft open if and only if $f_A \in \tau$. A fuzzy soft set $f_A$ over $X$ is called fuzzy soft closed set if the complement of $f_A$ is fuzzy soft open set.
**Definition 2.12[8]:** A fuzzy soft set $g_B$ in a fuzzy soft topological pace $(f_E, \tau)$ is said to be a fuzzy soft neighborhood of a fuzzy soft point $e_{g_C}$ if there exists a fuzzy soft open set $h_A$ such that $e_{g_C} \in h_A \subseteq g_B$.
**Definition 2.13[10]:** A proper nonempty open subset $M$ of $X$ is said to be a maximal open set if any open set which contains $M$ is $X$ or $M$.
**Definition 2.14[11]:** A proper nonempty closed subset $N$ of a topological space $X$ is said to be maximal closed set if any closed set which contains $N$ is $X$ or $N$.
**Definition 2.15[9]:** A proper non empty open subset $M$ of $X$ is said to be a minimal open set if any open set which is contained in $M$ is $\phi$ or $M$.
**Definition 2.16[11]:** A proper non empty closed subset $N$ of $X$ is said to be a minimal closed set if any closed set which is contained in $N$ is $\phi$ or N.
**Definition 2.17[15]:** A proper nonempty fuzzy soft open subset $f_E$ of $X$ is said to be a fuzzy soft maximal open set if any fuzzy soft open set which contains $f_E$ is $X$ or $f_E$. The family of all fuzzy soft maximal open sets in a fuzzy soft topological space $(f_E, \tau)$ is denoted by $FS\ Ma\ O(f_E)$.
**Definition 2.18[15]:** A proper nonempty fuzzy soft open subset $f_E$ of $X$ is said to be a fuzzy soft minimal open set if any fuzzy soft open set which is contained in $f_E$ is either $\phi$ or $f_E$. The family of all fuzzy soft minimal open sets in a fuzzy soft topological space $(f_E, \tau)$ is denoted by $FS\ Mi\ O(f_E)$.

Throughout this paper, we represent the difference of index sets by $\Omega \backslash \Delta (= \Omega - \Delta)$ and $|\Omega|$ represent the cardinality of the set Ω.

Let $X$ be an initial universe, $E$ be the set of parameters, $P(X)$ be the set of all subsets of $X$, $F(X)$ be the set of all fuzzy sets in $X$ and $(f_E, \tau)$ be a fuzzy soft topological space with parameters in $E$.

## III. Fuzzy soft maximal closed set

**Definition 3.1:** A proper nonempty fuzzy soft closed subset $f_E$ of $X$ is said to be a fuzzy soft maximal closed set if any fuzzy soft closed set which contains $f_E$ is either $X$ or $f_E$. The family of all fuzzy soft maximal closed sets in a fuzzy soft topological space $(f_E, \tau)$ is denoted by $FS\ Ma\ C(f_E)$.
**Example 3.2:** Let $X$ be the universal set, E be the set of parameters and $(f_E, \tau)$ be a fuzzy soft topological space. $X = \{h^1, h^2, h^3\}, E = \{e_1, e_2, \}, \tau = \{\phi, 1, [\{e_1, (p_{0.5}, q_{0.3}, r_{0.2})\}, \{e_2, (p_{0.3}, q_{0.5}, r_{0.2})\}], [\{e_1, (p_1, q_0, r_{0.5})\}, \{e_2, (p_{0.5}, q_{0.3}, r_1)\}],$ $[\{e_1, (p_{0.5}, q_0, r_{0.2})\}, \{e_2, (p_{0.3}, q_{0.3}, r_{0.2})\}], [\{e_1, (p_1, q_{0.3}, r_{0.5})\}, \{e_2, (p_{0.5}, q_{0.5}, r_1)\}]\}$.
Then $FS\ Ma\ C(X) = [\{e_1, (p_{0.5}, q_1, r_{0.8})\}, \{e_2, (p_{0.7}, q_{0.7}, r_{0.8})\}]$

**Lemma 3.3:** Let $(f_E, \tau)$ be a fuzzy soft topological spaces

i) If $f_L$ is a fuzzy soft maximal closed set and $g_M$ is a fuzzy soft closed set. Then, $f_L \cup g_M = X$ or $g_M \subset f_L$.
ii) If both $f_L$ and $h_N$ are fuzzy soft maximal closed sets, then $f_L \cup h_N = X$ or $f_L = h_N$.

**Proof:**

i) Consider $f_L$ is a fuzzy soft maximal closed set. If $f_L \cup g_M = X$, the proof is obvious. Consider $g_M$ is a fuzzy soft closed set such that $f_L \cup g_M \neq X$. Since $f_L \subset f_L \cup g_M$, we have $f_L \cup g_M = f_L$ or $f_L \cup g_M = X$ which implies $f_L \cup g_M = f_L$. Hence, $g_M \subset f_L$.
ii) From the above result, $f_L \subset h_N$ and $h_N \subset f_L$. Hence, $f_L = h_N$.

**Theorem 3.4:** Let $f_E$ and $f_{E_i}$ be fuzzy soft maximal closed sets for any $i \in \Omega$.

i) If $f_E \neq f_{E_i}$ for any element $i \in \Omega$, then $\left(\bigcap_{i \in \Omega} f_{E_i}\right) \cup f_E = X$.

ii) If $\bigcap_{i\in\Omega} f_{E_i} \subset f_E$, then there exists an element $i \in \Omega$ such that $f_E = f_{E_i}$.

**Proof:**

i) If $f_E \neq f_{E_i}$ for any element $i \in \Omega$, then by lemma 3.3(ii) the proof is obvious.

ii) Let $\bigcap_{i\in\Omega} f_{E_i} \subset f_E$. Then $f_E = f_E \cup (\bigcap_{i\in\Omega} f_{E_i}) = \bigcap_{i\in\Omega}(f_E \cup f_{E_i})$. $f_E$ and $f_{E_i}$ are fuzzy soft maximal closed sets therefore by lemma 3.3(ii) $f_E \cup f_{E_i} = X$ or $f_E = f_{E_i}$. Suppose if $f_E \cup f_{E_i} = X$ then $X = \bigcap_{i\in\Omega}(f_E \cup f_{E_i}) = f_E$ which is a contradiction to $f_E$ is a fuzzy soft maximal closed set. Therefore, there exists an element $i \in \Omega$ such that $f_E \cup f_{E_i} \neq X$, which implies $f_E = f_{E_i}$.

**Corollary 3.5:** Let $f_{E_i}$ and $f_{E_j}$ be a fuzzy soft maximal closed set for any element $i \in \Omega$ and $j \in \Delta$. If $\bigcap_{j\in\Delta} f_{E_j} \subset \bigcap_{i\in\Omega} f_{E_i}$, then there exist an element $j \in \Delta$ such that $f_{E_i} = f_{E_j}$, for any element $i \in \Omega$.
**Proof:** By lemma 3.3(ii), we have the result.

**Theorem 3.6:** Assume that $|\Omega| \geq 2$ and if $f_{E_i}$ is a fuzzy soft maximal closed set for any element $i \in \Omega$ and $f_{E_i} \neq f_{E_j}$ for any $i, j \in \Omega$ & $i \neq j$. Then,

i) $X - \bigcap_{i\in\Omega\setminus\{j\}} f_{E_i} \subset f_{E_j}$, for any element $j \in \Omega$.

ii) $\bigcap_{i\in\Omega\setminus\{j\}} f_{E_i} \neq \phi$, for any element $j \in \Omega$.

**Proof:** i) Consider $j \in \Omega$ and $f_{E_i} \neq f_{E_j}$ for any $i, j \in \Omega$ & $i \neq j$. Then by lemma 3.3(ii) we have, $X - f_{E_j} \subset f_{E_i}$ for any $i, j \in \Omega$ with $i \neq j$. This implies $X - f_{E_j} \subset \bigcap_{i\in\Omega\setminus\{j\}} f_{E_i}$.
Therefore, $X - \bigcap_{i\in\Omega\setminus\{j\}} f_{E_i} \subset f_{E_j}$, for any $j \in \Omega$.

ii) Let $j \in \Omega$ such that $\bigcap_{i\in\Omega\setminus\{j\}} f_{E_i} = \phi$. That implies $f_{E_i} = \phi$, which contradicts the fact that $f_{E_i}$ is a fuzzy soft maximal closed set. Hence, $\bigcap_{i\in\Omega\setminus\{j\}} f_{E_i} \neq \phi$, for any element $j \in \Omega$.

**Theorem 3.7:** If $f_{E_i}$ is a fuzzy soft maximal closed set for any $i \in \Omega$ and $f_{E_i} \neq f_{E_j}$ for any $i, j \in \Omega$ with $i \neq j$. And $\Delta$ is a proper nonempty subset of $\Omega$, then

$$\bigcap_{i\in\Omega\setminus\Delta} f_{E_i} \not\subset \bigcap_{k\in\Delta} f_{E_k} \not\subset \bigcap_{i\in\Omega\setminus\Delta} f_{E_i}.$$

**Proof:** Let $k$ be any element of $\Delta$ such that $\bigcap_{i\in\Omega\setminus\Delta} f_{E_i} \subset f_{E_k}$. Then by theorem 3.6(i), $X = (X - \bigcap_{i\in\Omega\setminus\Delta} f_{E_i}) \cup (\bigcap_{i\in\Omega\setminus\Delta} f_{E_i}) \subset \bigcap_{k\in\Delta} f_{E_k} \Rightarrow X \subset \bigcap_{k\in\Delta} f_{E_k}$, which is a contradiction. Hence, $\bigcap_{i\in\Omega\setminus\Delta} f_{E_i} \not\subset \bigcap_{k\in\Delta} f_{E_k}$. Again consider $f_{E_k} \subset \bigcap_{i\in\Omega\setminus\Delta} f_{E_i}$ then for some element $i \in \Omega$, we have $f_{E_k} \subset f_{E_i}$ implies $f_{E_k} = f_{E_i}$, which contradicts the assumption. Hence, $\bigcap_{k\in\Delta} f_{E_k} \not\subset \bigcap_{i\in\Omega\setminus\Delta} f_{E_i}$. Therefore, $\bigcap_{i\in\Omega\setminus\Delta} f_{E_i} \not\subset \bigcap_{k\in\Delta} f_{E_k} \not\subset \bigcap_{i\in\Omega\setminus\Delta} f_{E_i}$.

**Theorem 3.8:** Assume that $|\Omega| \geq 2$. Let $f_{E_i}$ be a fuzzy soft maximal closed set for any element $i \in \Omega$ and $f_{E_i} \neq f_{E_j}$ for any $i, j \in \Omega$ & $i \neq j$, then for any element $f_{E_j}$ of $\Omega$ we have

$$f_{E_j} = (\bigcap_{i\in\Omega} f_{E_i}) \cup (X - \bigcap_{i\in\Omega\setminus\{j\}} f_{E_i})$$

**Proof:** Let $j$ be an element of $\Omega$, then by theorem 3.6(i) we have

$$\begin{aligned}(\bigcap_{i\in\Omega} f_{E_i}) \cup (X - \bigcap_{i\in\Omega\setminus\{j\}} f_{E_i}) &= ((\bigcap_{i\in\Omega\setminus\{j\}} f_{E_i}) \cap f_{E_j}) \cup (X - \bigcap_{i\in\Omega\setminus\{j\}} f_{E_i}) \\ &= \left((\bigcap_{i\in\Omega\setminus\{j\}} f_{E_i}) \cup (X - \bigcap_{i\in\Omega\setminus\{j\}} f_{E_i})\right) \cap \left(f_{E_j} \cup (X - \bigcap_{i\in\Omega\setminus\{j\}} f_{E_i})\right) \\ &= X \cap f_{E_j} \\ &= f_{E_j}\end{aligned}$$

**Theorem 3.9:** If $f_{E_i}$ is a fuzzy soft maximal closed set for any $i \in \Omega$ and $f_{E_i} \neq f_{E_j}$ for any $i, j \in \Omega$ with $i \neq j$. If $\Delta$ is a proper nonempty subset of $\Omega$, then $\bigcap_{i\in\Omega} f_{E_i} \subset \bigcap_{k\in\Delta} f_{E_k}$.
**Proof:** Let $\Delta \neq \phi$ and $\Delta \subset \Omega$, then there exists an elements $l$ & $j$ of $\Omega$ such that $l \notin \Delta$ and $j \in \Delta$. $\bigcap_{i\in\Omega} f_{E_i} \subset f_{E_j}$ when $\Delta$ contains only one element.
If $\bigcap_{i\in\Omega} f_{E_i} = f_{E_j}$, then $f_{E_j} \subset f_{E_i}$ for any $i \in \Omega$. Since, $f_{E_i}$ is a fuzzy soft maximal closed set for any $i \in \Omega$, $f_{E_j} = f_{E_i}$ which is a contradiction. Hence, $\bigcap_{i\in\Omega} f_{E_i} \subset f_{E_j}$.
If $|\Delta| \geq 2$ then by theorem 3.8, we have

$$f_{E_l} = (\bigcap_{i\in\Omega} f_{E_i}) \cup (X - \bigcap_{i\in\Omega\setminus\{l\}} f_{E_i}) \text{ and } f_{E_j} = (\bigcap_{k\in\Delta} f_{E_k}) \cup (X - \bigcap_{k\in\Delta\setminus\{j\}} f_{E_k}).$$

If $\bigcap_{i\in\Omega} f_{E_i} = \bigcap_{k\in\Delta} f_{E_k}$ then $\bigcap_{k\in\Delta} f_{E_k} = \bigcap_{i\in\Omega} f_{E_i} \subset \bigcap_{i\in\Omega\setminus\{l\}} f_{E_i} \subset \bigcap_{k\in\Delta} f_{E_k}$. Hence, $\bigcap_{i\in\Omega\setminus\{l\}} f_{E_i} = \bigcap_{k\in\Delta} f_{E_k} \subset \bigcap_{k\in\Delta\setminus\{j\}} f_{E_k}$, which implies $f_{E_l} \supset f_{E_j}$. Thus, $f_{E_l} = f_{E_j}$ with $l \neq j$ which is a contradiction. Hence, $\bigcap_{i\in\Omega} f_{E_i} \subset \bigcap_{k\in\Delta} f_{E_k}$.

**Theorem 3.10:** Let $f_{E_i}$ be a fuzzy soft maximal closed set for any $i$ of $\Omega$ and $f_{E_i} \neq f_{E_j}$ for any elements $i$ & $j$ of $\Omega$ with $i \neq j$. If $\bigcap_{i\in\Omega} f_{E_i}$ is a fuzzy soft open sets, then $f_{E_i}$ is a fuzzy soft open set for any $i \in \Omega$.
**Proof:** Let $j$ be any element of $\Omega$ then by theorem 3.8

$$f_{E_j} = (\bigcap_{i\in\Omega} f_{E_i}) \cup (X - \bigcap_{i\in\Omega\setminus\{j\}} f_{E_i}).$$
$$= (\bigcap_{i\in\Omega} f_{E_i}) \cup (\bigcup_{i\in\Omega\setminus\{j\}} (X - f_{E_i}))$$

Since $\bigcup_{i\in\Omega\setminus\{j\}} (X - f_{E_i})$ is a fuzzy soft open set, $f_{E_i}$ is a fuzzy soft open set.
**Theorem 3.11:** Assume that $|\Omega| \geq 2$ and if $f_{E_i}$ is a fuzzy soft maximal closed set for any element $i \in \Omega$ and $f_{E_i} \neq f_{E_j}$ for any $i$ & $j \in \Omega$, $i \neq j$. If $\bigcap_{i\in\Omega} f_{E_i} = \phi$, then $\{f_{E_i} / i \in \Omega\}$ is the set of all fuzzy soft maximal closed sets of a fuzzy soft topological space.
**Proof:** Consider that there exists a fuzzy soft maximal closed set $f_{E_k}$ of a fuzzy soft topological space which is not equal to $f_{E_i}$ for any $i \in \Omega$. Then by theorem 3.6, $X = \bigcap_{i\in\Omega} f_{E_i} = \bigcap_{i\in(\Omega\cup k)\setminus\{k\}} f_{E_i} \neq \phi$, which is a contradiction. Hence, $\{f_{E_i} / i \in \Omega\}$ is the set of all fuzzy soft maximal closed sets of a fuzzy soft topological space.

## IV. FUZZY SOFT MINIMAL CLOSED SET:

**Definition 4.1:** A proper nonempty fuzzy soft closed subset $f_E$ of $X$ is said to be a fuzzy soft minimal closed set if any fuzzy soft closed set which is contained in $f_E$ is either $\phi$ or $f_E$. The family of all fuzzy soft minimal closed sets in a fuzzy soft topological space $(f_E, \tau)$ is denoted by $FS\ Mi\ C(f_E)$.
**Example 4.2:** Let $X$ be the universal set, E be the set of parameters and $(f_E, \tau)$ be a fuzzy soft topological space. $X = \{h^1, h^2, h^3\}, E = \{e_1, e_2, \}, \tau = \{\phi, 1, [\{e_1, (p_{0.5}, q_{0.3}, r_{0.2})\}, \{e_2, (p_{0.3}, q_{0.5}, r_{0.2})\}], [\{e_1, (p_1, q_0, r_{0.5})\}, \{e_2, (p_{0.5}, q_{0.3}, r_1)\}],$ $[\{e_1, (p_{0.5}, q_0, r_{0.2})\}, \{e_2, (p_{0.3}, q_{0.3}, r_{0.2})\}], [\{e_1, (p_1, q_{0.3}, r_{0.5})\}, \{e_2, (p_{0.5}, q_{0.5}, r_1)\}]\}$.
Then $FS\ Mi\ C(X) = [\{e_1, (p_0, q_{0.7}, r_{0.5})\}, \{e_2, (p_{0.5}, q_{0.5}, r_0)\}]$.

**Lemma 4.3:** Let $(f_E, \tau)$ be a fuzzy soft topological spaces

i) If $f_L$ is a fuzzy soft minimal closed set and $g_M$ is a fuzzy soft closed set. Then, $f_L \cap g_M = \phi$ or $f_L \subset g_M$.
ii) If both $f_L$ and $h_N$ are fuzzy soft minimal closed sets, then $f_L \cap h_N = \phi$ or $f_L = h_N$.

**Theorem 4.4:** Let $f_E$ and $f_{E_i}$ are fuzzy soft minimal closed sets for any $i \in \Omega$. If $f_E \subset \bigcup_{i\in\Omega} f_{E_i}$, then there exists an element $j \in \Omega$ such that $f_E = f_{E_j}$.
**Proof:** Let, $f_E \subset \bigcup_{i\in\Omega} f_{E_i}$, then $f_E = f_E \cap (\bigcup_{i\in\Omega} f_{E_i}) = \bigcup_{i\in\Omega} (f_E \cap f_{E_i})$. Given, $f_E$ and $f_{E_i}$ are fuzzy soft minimal closed sets therefore by lemma 4.3(ii), $f_E \cap f_{E_i} = \phi$ or $f_E = f_{E_i}$. Suppose if $f_E \cap f_{E_i} = \phi$ then $\bigcup_{i\in\Omega} (f_E \cap f_{E_i}) = \phi = f_E$ which is a contradiction. Hence, $f_E = f_{E_j}$.

**Theorem 4.5:** Let $f_E$ and $f_{E_i}$ are fuzzy soft minimal closed sets for any $i \in \Omega$, if $f_E \neq f_{E_i}$ for any element $i \in \Omega$, then $(\bigcup_{i\in\Omega} f_{E_i}) \cap f_E = \phi$.
**Proof:** Consider, $(\bigcup_{i\in\Omega} f_{E_i}) \cap f_E \neq \phi$, then there exists an element $i \in \Omega$ such that $f_{E_i} \cap f_E \neq \phi$. But by lemma 4.3(ii) $f_E = f_{E_i}$, which is a contradiction. Hence, $(\bigcup_{i\in\Omega} f_{E_i}) \cap f_{E_i} = \phi$.

**Theorem 4.6:** Assume that $|\Omega| \geq 2$, if $f_{E_i}$ is a fuzzy soft minimal closed set for any element $i$ of $\Omega$ and $f_{E_i} \neq f_{E_j}$ for any elements $i, j \in \Omega$ where $i \neq j$. Then $(\bigcup_{i\in(\Omega\setminus\{j\})} f_{E_i}) \cap f_{E_j} = \phi$ for any element $j \in \Omega$.
**Proof:** Consider $(\bigcup_{i\in(\Omega\setminus\{j\})} f_{E_i}) \cap f_{E_j} \neq \phi$. Then $\bigcup_{i\in(\Omega\setminus\{j\})} (f_{E_i} \cap f_{E_j}) \neq \phi \Rightarrow f_{E_i} \cap f_{E_j} \neq \phi$, which is a contradiction by lemma 4.3(ii). Hence, $(\bigcup_{i\in(\Omega\setminus\{j\})} f_{E_i}) \cap f_{E_j} = \phi$.

**Theorem 4.7:** Let $f_{E_i}$ be a fuzzy soft minimal closed set for any element $i \in \Omega$ and $f_{E_i} \neq f_{E_j}$ for any elements $i, j \in \Omega$ where $i \neq j$. If $\Delta$ is a proper nonempty subset of $\Omega$, then $(\bigcup_{i\in(\Omega\setminus\Delta)} f_{E_i}) \cap (\bigcup_{k\in\Delta} f_{E_k}) = \phi$.
**Proof:** Suppose $(\bigcup_{i\in(\Omega\setminus\Delta)} f_{E_i}) \cap (\bigcup_{k\in\Delta} f_{E_k}) \neq \phi \Rightarrow \bigcup (f_{E_i} \cap f_{E_k}) \neq \phi, i \in \Omega\setminus\Delta$ and $k \in \Delta \Rightarrow f_{E_i} \cap f_{E_k} \neq \phi$ for some $i \in \Omega$. Thus, by lemma 4.3(ii) we have, $f_{E_i} = f_{E_k}$, which contradicts the fact that $f_{E_i} \neq f_{E_k}$. Hence, $(\bigcup_{i\in(\Omega\setminus\Delta)} f_{E_i}) \cap (\bigcup_{k\in\Delta} f_{E_k}) = \phi$.

**Theorem 4.8:** Let $f_{E_i}$ be a fuzzy soft minimal closed set for any element $i \in \Omega$ and $f_{E_j}$ be a fuzzy soft minimal closed set for any element $j \in \Delta$. If there exist an element $l \in \Delta$ such that $f_{E_i} \neq f_{E_l}$ for any $i \in \Omega$, then $\bigcup_{l\in\Delta} f_{E_l} \not\subset \bigcup_{i\in\Omega} f_{E_i}$.
**Proof:** Assume that there exists an element $l \in \Delta$ satisfying $f_{E_i} \neq f_{E_l}$ for any $i \in \Omega$, such that $\bigcup_{l\in\Delta} f_{E_l} \subset \bigcup_{i\in\Omega} f_{E_i}$, this implies that $f_{E_l} \subset \bigcup_{i\in\Omega} f_{E_i}$ for some element $l \in \Delta$. Thus, $f_{E_l} = f_{E_k}$ for any $k \in \Omega$, by theorem 4.4, which is a contradiction. Hence, $\bigcup_{l\in\Delta} f_{E_l} \not\subset \bigcup_{i\in\Omega} f_{E_i}$.

**Theorem 4.9:** Let $f_{E_i}$ be a fuzzy soft minimal closed set for any $i \in \Omega$ and $f_{E_i} \neq f_{E_k}$ for any $i, k \in \Omega$ with $i \neq k$. Then, $\bigcup_{m\in\Delta} f_{E_m} \subset \bigcup_{i\in\Omega} f_{E_i}$ if $\Delta$ is a proper nonempty subset of $\Omega$.

**Proof:** Let $j$ be any element of $\Omega\backslash\Delta$, then $f_{E_j}$ is a fuzzy soft minimal closed set of the family $\{f_{E_j}: j \in \Omega\backslash\Delta\}$. Then, $f_{E_j} \cap (\cup_{m\in\Delta} f_{E_m}) = \cup_{m\in\Delta}(f_{E_j} \cap f_{E_m}) = \Phi$ and $f_{E_j} \cap (\cup_{i\in\Omega} f_{E_i}) = \cup_{i\in\Omega}(f_{E_j} \cap f_{E_i}) = f_{E_j}$. If $\cup_{m\in\Delta} f_{E_m} = \cup_{i\in\Omega} f_{E_i}$, then $\Phi = f_{E_j}$ which is a contradiction to the fact $f_{E_j}$ is a fuzzy soft minimal closed set. Hence, $\cup_{m\in\Delta} f_{E_m} \neq \cup_{i\in\Omega} f_{E_i}$ and $\cup_{m\in\Delta} f_{E_m} \subset \cup_{i\in\Omega} f_{E_i}$.

**Theorem 4.10:** Assume that $|\Omega| \geq 2$ and if $f_{E_i}$ is a fuzzy soft minimal closed set for any element $i \in \Omega$ and $f_{E_i} \neq f_{E_j}$ for any $i, j \in \Omega$ & $i \neq j$. Then,

i) $f_{E_j} \subset X - \cup_{i\in\Omega\backslash\{j\}} f_{E_i}$, for some $j \in \Omega$.

ii) $\cup_{i\in\Omega\backslash\{j\}} f_{E_i} \neq X$, for any element $j \in \Omega$.

**Proof:** i) Consider $j \in \Omega$ and $f_{E_i} \neq f_{E_j}$ for any $i, j \in \Omega$ & $i \neq j$. Then by theorem 4.5, $(\cup_{i\in\Omega} f_{E_i}) \cap f_{E_j} = \phi$, for some $i, j \in \Omega$

$\Rightarrow (\cup_{i\in\Omega} f_{E_i}) \cap f_{E_j} = \phi$

$\Rightarrow f_{E_i} \cap f_{E_j} = \phi$ by lemma 4.3(ii)

$\Rightarrow f_{E_i} \subset X - f_{E_j}$.

$\Rightarrow \cup_{i\in\Omega\backslash\{j\}} f_{E_i} \subset X - f_{E_j}$.

Therefore, $f_{E_j} \subset X - \cup_{i\in\Omega\backslash\{j\}} f_{E_i}$, for some $j \in \Omega$.

ii) Let $j \in \Omega$ such that $\cup_{i\in\Omega\backslash\{j\}} f_{E_i} = X$. That implies $f_{E_i} = \phi$, which contradicts the fact that $f_{E_i}$ is a fuzzy soft minimal closed set. Hence, $\cup_{i\in\Omega\backslash\{j\}} f_{E_i} \neq X$, for any element $j \in \Omega$.

**Theorem 4.11:** Assume that $|\Omega| \geq 2$. Let $f_{E_i}$ be a fuzzy soft minimal closed set for any element $i \in \Omega$ and $f_{E_i} \neq f_{E_j}$ for any $i, j \in \Omega$ & $i \neq j$, then for any element $f_{E_j}$ of $\Omega$ we have

$$f_{E_j} = (\cup_{i\in\Omega} f_{E_i}) \cap (X - \cup_{i\in\Omega\backslash\{j\}} f_{E_i})$$

**Proof:** Let $j$ be any element of $\Omega$ then by theorem 4.10

$$(\cup_{i\in\Omega} f_{E_i}) \cap (X - \cup_{i\in\Omega\backslash\{j\}} f_{E_i}) = \left((\cup_{i\in\Omega\backslash\{j\}} f_{E_i}) \cup f_{E_j}\right) \cap (X - \cup_{i\in\Omega\backslash\{j\}} f_{E_i}).$$

$$= \left((\cup_{i\in\Omega\backslash\{j\}} f_{E_i}) \cap (X - \cup_{i\in\Omega\backslash\{j\}} f_{E_i})\right) \cup \left((f_{E_j}) \cap (X - \cup_{i\in\Omega\backslash\{j\}} f_{E_i})\right)$$

$$= \phi \cup f_{E_j}$$

$$= f_{E_j}$$

**Theorem 4.12:** Let $f_{E_i}$ be a fuzzy soft minimal closed set for any $i$ of $\Omega$ and $f_{E_i} \neq f_{E_j}$ for any elements $i$ & $j$ of $\Omega$ with $i \neq j$. If $\cup_{i\in\Omega} f_{E_i}$ is a fuzzy soft open sets, then $f_{E_i}$ is a fuzzy soft open set for any $i \in \Omega$.

**Proof:** Let $j$ be any element of $\Omega$ then by theorem 4.11

$$f_{E_j} = (\cup_{i\in\Omega} f_{E_i}) \cap (X - \cup_{i\in\Omega\backslash\{j\}} f_{E_i}).$$

$$= (\cup_{i\in\Omega} f_{E_i}) \cap (\cap_{i\in\Omega\backslash\{j\}} (X - f_{E_i})).$$

But by our assumption $\cap_{i\in\Omega\backslash\{j\}} (X - f_{E_i})$ is a fuzzy soft open set, therefore $f_{E_i}$ is a fuzzy soft open set.

**Theorem 4.13:** Assume that $|\Omega| \geq 2$ and if $f_{E_i}$ is a fuzzy soft minimal closed set for any element $i \in \Omega$ and $f_{E_i} \neq f_{E_j}$ for any $i$ & $j \in \Omega$, $i \neq j$. If $\cup_{i\in\Omega} f_{E_i} = X$, then $\{f_{E_i} / i \in \Omega\}$ is the set of aall fuzzy soft minimal closed sets of a fuzzy soft topological space.

**Proof:** Consider that there exists a fuzzy soft minimal closed set $f_{E_k}$ of a fuzzy soft topological space which is not equal to $f_{E_i}$ for any $i \in \Omega$. Then by theorem 4.10, $X = \cup_{i\in\Omega} f_{E_i} = \cup_{i\in(\Omega\cup k)\backslash\{k\}} f_{E_i} \neq X$, which is a contradiction. Hence, $\{f_{E_i} / i \in \Omega\}$ is the set of all fuzzy soft minimal closed sets of a fuzzy soft topological space.

## V. Relations between Fuzzy Soft Minimal Open set, Fuzzy Soft Maximal Open set, Fuzzy Soft Minimal Closed set and Fuzzy Soft Maximal Closed set

**Theorem 5.1:** Let$f_E$ be a nonzero fuzzy soft subset of a fuzzy soft topological space $(f_E, \tau)$. Then the following holds

i) $f_E$ is a fuzzy soft minimal closed set if and only if $X - f_E$ i.e, $(f_E)^c$ is a fuzzy soft maximal open set.

ii) $f_E$ is a fuzzy soft maximal closed set if and only if $X - f_E$ i.e, $(f_E)^c$ is a fuzzy soft minimal open set.

**Proof:** It is obvious from definitions

**Example 5.2:** Let $X$ be the universal set, E be the set of parameters and $(f_E, \tau)$ be a fuzzy soft topological space. $X = \{h^1, h^2, h^3\}, E = \{e_1, e_2,\}, \tau = \{\phi, 1, [\{e_1, (p_{0.5}, q_{0.3}, r_{0.2})\}, \{e_2, (p_{0.3}, q_{0.5}, r_{0.2})\}], [\{e_1, (p_1, q_0, r_{0.5})\}, \{e_2, (p_{0.5}, q_{0.3}, r_1)\}], [\{e_1, (p_{0.5}, q_0, r_{0.2})\}, \{e_2, (p_{0.3}, q_{0.3}, r_{0.2})\}], [\{e_1, (p_1, q_{0.3}, r_{0.5})\}, \{e_2, (p_{0.5}, q_{0.5}, r_1)\}]\}$.

Then, $FS\ Mi\ C(X) = [\{e_1, (p_0, q_{0.7}, r_{0.5})\}, \{e_2, (p_{0.5}, q_{0.5}, r_0)\}]$.

$FS\ Ma\ O(X) = [\{e_1, (p_1, q_{0.3}, r_{0.5})\}, \{e_2, (p_{0.5}, q_{0.5}, r_1)\}]$.
$FS\ Ma\ C(X) = [\{e_1, (p_{0.5}, q_1, r_{0.8})\}, \{e_2, (p_{0.7}, q_{0.7}, r_{0.8})\}]$.
$FS\ Mi\ O(X) = [\{e_1, (p_{0.5}, q_0, r_{0.2})\}, \{e_2, (p_{0.3}, q_{0.3}, r_{0.2})\}]$.

**Proposition 5.3:** Let $f_A$ and $f_B$ be a fuzzy soft subsets of $X$. If $f_A \cup f_B = X$ and $f_A \cap f_B$ is a fuzzy soft closed set and $f_A$ is a fuzzy soft open set then $f_B$ is a fuzzy soft closed set.

**Proposition 5.4:** If $f_{E_i}$ is a fuzzy soft open set for any $i \in \Omega$ and $f_{E_i} \cup f_{E_j} = X$ for any $i, j \in \Omega$ with $i \neq j$. $\cap_{i\in\Omega\backslash\{j\}} f_{E_i}$ is a fuzzy soft closed set for any element $i \in \Omega$, if $\cap_{i\in\Omega} f_{E_i}$ is a fuzzy soft closed set.
**Proof:** Consider, $j$ be any element of $\Omega$. Since $f_{E_i} \cup f_{E_j} = X$ for any $i, j \in \Omega$ with $i \neq j$ for any $i, j \in \Omega$ with $i \neq j$ implies $f_{E_j} \cup \left(\cap_{i\in\Omega\backslash\{j\}} f_{E_i}\right) = \cap_{i\in\Omega\backslash\{j\}} \left(f_{E_j} \cup f_{E_i}\right) = X$. But by our assumption we have $f_{E_j} \cup \left(\cap_{i\in\Omega\backslash\{j\}} f_{E_i}\right) = \cap_{i\in\Omega} f_{E_i}$ is a fuzzy soft closed set. Hence, $\cap_{i\in\Omega\backslash\{j\}} f_{E_i}$ is a fuzzy soft closed set for any element $i \in \Omega$.

**Theorem 5.5:** Let $f_{E_i}$ be a fuzzy soft maximal open set for any $i$ of $\Omega$ and $f_{E_i} \neq f_{E_j}$ for any elements $i$ & $j$ of $\Omega$ with $i \neq j$. $\cap_{i\in\Omega\backslash\{j\}} f_{E_i}$ is a fuzzy soft closed set for any element $j \in \Omega$, if $\cap_{i\in\Omega} f_{E_i}$ is a fuzzy soft closed set.
**Proof:** Consider $f_{E_i} \neq f_{E_j}$ any elements $i$ & $j$ of $\Omega$ with $i \neq j$ then by lemma 3.3(ii) we have, $f_{E_i} \cup f_{E_j} = X$ and using proposition 5.4, $\cap_{i\in\Omega\backslash\{j\}} f_{E_i}$ is a fuzzy soft closed set for any element $i \in \Omega$.

## REFERENCES


**[1]** L.A.Zadeh. Fuzzy sets. Information and Control, 8(3):338-345, 1965.
**[2]** C.L.Chang. Fuzzy topological spaces. Journal of Mathematical Analysis and Applications, 24(1):182-190, 1968.
**[3]** D.Molodtsov, Soft set theory-first results, Comput. Math. Appl. 37(1999) 19-31.
**[4]** M.Shabir and M.Naz, On soft topological spaces, Comput. Math. Appl., 61(2011) 1786-1799.
**[5]** P.K.Maji, R.Biswas, A.R.Roy, Fuzzy Soft Sets, J. Fuzzy Math. 9, 2001, 589-602.
**[6]** A.Aygunoglu, H.Aygun, Introduction to fuzzy soft groups, Comput. Math. Appl. 58 (2009) 1279-1286.
**[7]** B.Ahmad, A.Kharal, On fuzzy soft sets, Adv. Fuzzy Syst. 2009 (2009).
**[8]** B.Tanay and M.B.Kandemir, Topological structure of fuzzy soft sets, Computer and Math. with appl., 61(2011) 2952-2957
**[9]** Fumie Nakaoka and Nobuyuki Oda, Some applications of Minimal open sets, Int. J. Mat. Sci. 27(8), 2001, 471-476.
**[10]** Fumie Nakaoka and Nobuyuki Oda, Some Properties of Maximal open sets, Int. J. Mat. Sci. 21, 2003, 1331-1340.
**[11]** Fumie Nakaoka and Nobuyuki Oda, Minimal closed sets and maximal closed sets, Int. J. Mat. Sci. 2006, 1-8.
**[12]** S Roy, T K Samantha , A note on fuzzy soft topological space, Ann Fuzzy Math. Inform 3(2) Pp305 – 311, 2011.
**[13]** T.J.Neog, D.K.Sut, G.C.Hazarika, Fuzzy soft topological space, Int. J. Latest Trend in Math Vol.2 No.1 March 2012.
**[14]** S. S. Benchalli, Basavaraj M. Ittanagi and R. S. Wali, On Minimal Open Sets and Maps in Topological Spaces, J. Comp. & Math. Sci. Vol.2 (2), 208-220 (2011).
[**15**] Shakila.K and Selvi.R, A Note on Fuzzy Soft Maximal open set and Fuzzy Soft Minimal open set, Int. J. Scientific Research and Reviews, Vol.7, Issue 7, 472-477, 2018.