REGULAR POINT SET DOMINATION IN GRAPHS

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Abstract : Let the graph $G(V_G,E_G)$ be any connected graph. A subset $D \subseteq V_G$ is a regular point set dominating set if for each set $S \subseteq V\backslash D$ there exists a vertex u of D such that the graph $<S\cup \{u\}>$ is regular. The regular points set dominating number is least cardinality of a regular point set dominating set, is denoted by $\gamma_{rp}(G)$. The lower and upper bound of $\gamma_{rp}(G)$ are determined for specific graphs.

***Key*words : Domination, Point Set Domination, Regular Point Set Domination.**

1. Introduction

All graphs taken here are finite, undirected with neither loops nor multiple edges. Any undefined term in this paper may be found in Haynes T., Hedetniemi S., Slater P. [4].

Let $G = (V_G, E_G)$ be a graph with 'n' vertices and 'm' edges, thus $|V_G| = n$ and $|E_G| = m$. The graph $\overline{G}$ is the complement of graph G having n vertices of G and two vertices are linked if these are not linked in G. The open neighborhood of a vertex v of G defined as the set $N(v) = \{ u \in V_G ; uv \in E_G\}$.

The path with n vertices is denoted as P_n and the cycle with n vertices is denoted by C_n. The wheel on n vertices is a graph which formed by connecting a vertex to remained vertices of a cycle C_{n-1} and is denoted as W_n, $n\geq 4$. We denote a complete bipartite graph with two partite sets of cardinality m and n by $K_{m,n}$. The graph $K_{1,n-1}$ represents a star [4].The graph is a regular graph if each vertex have same degree[2,3].

A subset $D \subseteq V_G$ is a dominating set of G if for each vertex of V–D has a neighbor in D. The minimum cardinality of a dominating set is the domination number γ (G) of G. For deep analysis of domination in graph, see [4,5].

Let $G = (V_G, E_G)$ be a connected graph then the set $D\subseteq V_G$ in G is a point set dominating set if for every set V_1 of V\D there exists a vertex u of D such that the induced subgraph $<V_1\cup\{u\}>$ is connected and the minimum cardinality of a point set dominating set of G is the point set domination number, denoted as γ_p(G) [7].

Notation 1.1[4] $\Delta(G)$: The maximum degree of a vertex in G
n_0 : Set of all isolates in G.
$\alpha(G)$: vertex covering number
$\alpha'(G)$: Edge covering number
$\beta(G)$: edge independence number
diam(G): the maximum eccentricity of any vertex in graph G.

Theorem 1.2[1] Let Gbe any graph then

$$\frac{n}{1+\Delta(G)} \leq \gamma(G).$$

Also, The bound is attained iff the following conditions are satisfied for each γ (G)-set S of G.

1. S is an independent.
2. Every vertex in S is of maximum degree;
3. Each vertex in V\S is linked to only one vertex in S.

We present an extension of point set domination named as the regular point set domination (RPSD-domination). We start the analysis of that type of parameter. The minimum cardinality of this set is RPSD –dominating number and is denoted by γ_{rp} (G).

2. Main Title

In this section, we study the regular point set domination for various graphs i.e., complete graph, cyclic graph, wheel graph, complete bipartite graph. We initiate with the following straightforward observations.

Definition 2.1 Let G = (V_G,E_G) be any graph. A subset D $\subseteq$ V_G is a regular point set dominating set if for each set S$\subseteq$ V\D there exists a vertex u of D such that the graph <S$\cup${u}> is regular. The regular points set dominating number is least cardinality of a regular point set dominating set, is denoted by $\gamma_{rp}(G)$.

Example: Let G be any graph with vertex set V = $\{u_1, u_2, v_1, v_2\}$ such that the vertex set D = $\{u_1, u_2\}$ is regular point dominating set. Here, for each subset S of $\{v_1, v_2\}$ there is| a vertex u_1 such that the induced subgraph <S $\cup$ $\{u_1\}$> is regular as shown in figure 1.1

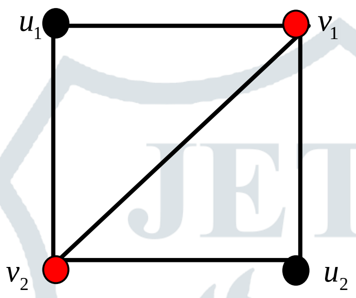

Figure 1.1 The graph G with four vertices

Observation 2.2 For any connected graph G,

$$\gamma(G) \le \gamma_p(G) \le \gamma_{rp}(G)$$

$$\gamma(G) \longrightarrow \gamma_p(G) \longrightarrow \gamma_{rp}(G)$$

Figure 1.2. Relationship between various parameters

Observation 2.3 Let G be the complete graph of order n then

$$\gamma_{rp}(K_n) = 1$$

Observation 2.4 Let G be a cyclic graph of order n then

$$\gamma_{rp}(C_n) = n-1, \ n \ge 4$$

Observation 2.5 Let the graph G is wheel graph of order n,

$$\gamma_{rp}(W_n) = n-2, \quad n \ge 4$$

Observation 2.6 If G is a complete bipartite graph G = $K_{m,n}$, m$\le$n

$$\gamma_{rp}(K_{m,n}) = n+m-1 .$$

Theorem 2.7 Let G be any graph of order n then

$$\frac{n}{1+\Delta(G)}.$$

Also, the bound is attained if the graph G = K_n or $\overline{K_n}$.

Proof: Since the lower bound for $\gamma(G)$ is $\frac{n}{1+\Delta(G)}$, by observation 2.2,

$\gamma(G)=\frac{n}{1+\Delta(G)}$.

Here, we show the next part of this theorem.

Suppose the set S is a minimal regular point set dominating set for graph G. Consider $\gamma(G)=\frac{n}{1+\Delta(G)}$ then the set S is also minimal dominating set. Therefore, by theorem(1.2), the set S is independent, every vertex in S is of maximum degree and each vertex of V\S is adjacent to only one vertex in S. Assume that the set V\S has only one vertex (say v) and the set S = $\{u_1,u_2,\ldots,u_r\}$. If a vertex v $\in N(u_i)$ for $1\le i\le r$, then $v\notin N(u_j)$ for $i\neq j$. The set S is a minimal regular point set dominating set therefore there exists set I $\subseteq$S such that < I $\cup$ {v}> is regular. Hence, the graph G is either K_2 or $\overline{K_n}$ .If any two vertices in V\S are adjacent with any two vertex of S, then $\gamma_{rp}(G)$ =1. This implies that G = K_n by observation (2.3).

Theorem 2.8: Let G be any graph of order n,

$\gamma_{rp}(G)\le \alpha(G) + n_0 +1$.

The bound is attained if G′ is a tree of order greater than 2.

Proof: Let us assume that the maximum independent set D of V_G in G. Also, let $D_1 \subseteq D$ contains all isolates n_0.Thus the set $(V\backslash D)\cup D_1\cup\{v\}$ is regular set dominating set for a vertex v of $D\backslash D_1$. Hence, we have $\gamma_{rp}(G)\le \alpha(G) + n_0 +1$.

Assume that G′ is a tree with n > 2 vertices. Let the set S be the minimal regular point set dominating set in G′. For the set S$\subseteq$ V\D there exists a vertex u $\in$ D such that <S$\cup${u} > is a null graph. Therefore the set D has more vertices than its vertex covering number or its edge covering number $\alpha'(G')$.

Since $\alpha'(G')\ge \beta'(G') = \alpha(G')$ the bound is attained.

Theorem 2.9: For the regular graph G with diam(G)<4.Then every minimal regular point set dominating set is point set dominating set in G.

Proof: Let the graph G is regular. Obviously, it is connected. Let $u_1,u_2 \in$ V\D are two vertices such that $u_1u_2\notin E$. But there exists at least one vertex of V\D such that u_1w, $wu_2 \in E$ for some w $\in$V\D. Therefore, there is no vertex u in D such that <{u}$\cup$D>is regular for the set D_1 = (V\D)-{u_2}. It is a contradiction. Hence, the set <{u_1,u_2}$\cup${u}> is connected for a vertex u of D. Thus, the set D is also a point set dominating set in G.

Conclusion

We have defined a new variant of the parameter of domination namely regular point set domination number of various graphs and we discussed the lower and upper bound of the RPSD-domination parameter. Also, we observed that the regular point set domination number is always less than of its order.

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