MATHEMATICAL MODELLING OF DRYING OF *CLARIAS BATRACHUS*

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ABSTRACT: In this paper number of mathematical models describing fish drying are developed and discussed. In this perspective, drying experiments were performed to evaluate the drying characteristic of Magur fish (*Clarius Batrachus*). An incubator was used for this experiment at three different temperatures viz. 50°C, 60°C, 70°C and at constant air velocity of 3.6 m/s. The changes in the mass of fish and principle drying parameters were recorded continuously every one hour interval from morning to evening in each test day. The test on these models was investigated by comparing the coefficient of determination (R^2), reduced chi-square ($\chi 2$) and root mean square error (RMSE) between the observed and predicted moisture ratio. On the basis of highest value of (R^2) and lowest value of reduced chi-square ($\chi 2$) and root mean square error (RMSE) the Midli et al. drying model was found to be satisfactorily describing drying curves of fish.

***Index Terms* - Mathematical modeling, Drying, Magur fish**

I. INTRODUCTION

Drying is the process of heat and mass transfer followed by removal of water and other solvent by evaporation (Haghi et al, 2008). Worldwide, huge amount of food products are dried to improve product quality, reduce packaging costs and their storage for long duration (Bala and Mondol, 2001). In addition, the main goal of modern drying is to improve drying processes by reducing energy consumption and providing high quality with minimal increase in economic input (Raghavan et al, 2005). Mathematical modeling of any drying process develops a better understanding for the controlling parameters of this complex process. Mathematical modeling is also necessary for anticipating the drying time and product moisture content, developing new products, designing the appropriate equipment and optimization of the process, new designing or for improvement in existing drying systems or even for the control of the drying process (Erbay et al, 2010).

Fish is an important source of high quality protein necessary in human diet. Although, fish have a very short span of shelf life is highly perishable food and (Jain, 2006). Jain et al, (2007) reported in their study that drying is very important as well as necessary preservation technique to maintain the fish quality and preserve fish from spoilage. Drying of fish inactivates enzymes and removes the moisture which causes bacteria and mold growth (Bellagha et al, 2002, Bala et al, 2001, Daun, 2004).

The present study is proposed to establish a suitable mathematical model of drying of Magur fish (*Clarius Batrachus*) on the basis of some experiments.

II. MATERIALS AND METHOD:

2.1 Methodology

The fresh *Clarius Batrachus* fish samples were used in this study and obtained from local market. The selection of fish was based upon the availability of species in local area. The choosen samples of fish were cleaned with tap water .Surface water was removed by blotting with absorbent paper. Drying experiments are carried out at temperatures 50°C, 60°C and 70°C in an Incubator having temperature controller (Thermotech PID-93D) and at constant air velocity of 3.6m/s. The air velocity of Incubator was measured by Anemometer (AM-4201) having least count 0.1 m/s. At suitable time intervals the weight loss is measured by means of a digital balance (ATOM-A122) having an accuracy 0.1 gm. Before starting the experiments, the system was run for at least one hour to obtain steady state conditions.

2.2 Mathematical modeling of drying curves

The drying curves obtained were processed to find the most convenient one among twelve different expressions defining drying rates, as given in Table 1, by several investigators. The initial moisture content of *Clarius Batrachus* was found to be 80.67% and was reduced to the final moisture content of 0.28%. In the literature, there are several statistical test methods used to evaluate statistically the performance of the drying models. Among these, the correlation coefficient (R^2), the mean bias error (MBE), the root mean square error (RMSE) and the reduced chi-square ($\chi 2$) are the most widely used ones (Togrul et al, 2002, El-Sebaii et al, 2002, Akpinar et al, 2003, Lui et al, 1997).

Table 1: Mathematical models used for obtaining drying curves of *Clarius Batrachus*

Model No.	Model name	Model equation	References
1	Lewis	M_R= exp (-kt)	(Lui et al,1997, O OCallaghan et al, 1971)
2	Page	M_R= exp (-kt^n)	(Agarwal et al, 1997, Zhang et al, 1991)
3	Modified page	M_R= exp[(-kt)^n]	(Overhults et al, 1973, White et al., 1981)
4	Henderson and Pabis	M_R= aexp (-kt)	(Hendreson and Pabis, 1961) Chhinnan, 1984, Westerman et al, 1973)
5	Yagcioglu et al.	M_R= aexp (-kt) + c	(Yagcioglu et al, 1999)
6	Two-term	M_R= aexp (-k_ot) + bexp(-k_1t)	(Henderson, 1974, Rahman et al, 1998)
7	Two-term exponential	M_R= aexp (-kt) + (1-a)exp(-kat)	(Sharaf-Elden et al, 1980)
8	Wang and Singh	M_R =1+at+bt^2	(Wang and Singh, 1978)
9	Diffusion approach	M_R= aexp (-kt) + (1-a)exp(-kbt)	(Kassem, 1998)
10	Verma et al.	M_R= aexp (-kt) + (1-a)exp(-gt)	(Verma et al, 1985)
11	Modified Henderson and Pabis	M_R= aexp (-kt) + bexp(-gt) +cexp(-ht)	(Karathanos, 1999)
12	Midili and Kucuk	M_R= exp (-kt^n) + bt	(Midili et al, 2002)

2.2.1 Moisture ratio (M_R)

The moisture ratio in these model equations is defined as follows (Sacilik et al, 2005)

$$M_R = \frac{M - M_{exp}}{M_0 - M_{exp}} \quad (1)$$

Which was then simplified to M/M_0 because M_0 was much larger than M_{exp} (Goyal et al, 2007)

2.2.2. Correlation coefficient (R)

The correlation coefficient, R can be used to test the linear relation between measured and estimated values, which can be calculated from the equation

$$R^2 = \frac{\sum_{i=1}^{N}\left(M_{r_i} - Mr_{pre,i}\right) * \left(Mr_i - Mr_{exp,i}\right)}{\left[\sum_{i=1}^{N}(Mr_i - Mr_{pre,i})^2\right] * \left[\sum_{i=1}^{N}(M\,r_i - Mr_{exp,i})^2\right]} \quad (2)$$

Where, R^2 is called the coefficient of determination, $M_{r\ exp,i}$ stands for the experimental moisture ratio found in any measurement, $M_{rpre,i}$ is the predicted moisture ratio for this measurement and N is the total number of observations.

2.2.3. Mean Bias Error (MBE)

The mean bias error is given as:

$$MBE = \frac{1}{N}\sum_{i=1}^{N}(M\,r_{pre,i} - Mr_{exp,i}) \quad (3)$$

2.2.4. Root Mean square error (RMSE)

The root mean square error may be calculated by the following equation. It provides the information on the short term performance. The value of RMSE is always positive, represented as zero in the ideal case.

$$RMSE = [\frac{1}{N}\sum_{i=1}^{N}(M\,r_{pre,i} - Mr_{exp,i})^2\ ]^{\frac{1}{2}} \quad (4)$$

2.2.5. Reduced chi-square (χ^2):

The reduced chi-square may be calculated by the following equation where *n* is the number of constants. The lower are the values are the values of the reduced chi-square,the better is the goodness of fit.

$$\chi^2 = \frac{\sum_{i=1}^{N}(Mr_{exp,i} - Mr_{exp,i})^2}{N-n} \quad (5)$$

2.2.6. Efficiency (E.F.):

The model efficiency may be calculated by the following equation.

$$EF = \frac{\sum_{i=1}^{N}\left(MR_{i,exp} - MR_{i,expmean}\right)^2 - \sum_{i=1}^{N}\left(MR_{i,pre} - MR_{i,exp}\right)^2}{\sum_{i=1}^{N}\left(MR_{i,exp} - MR_{i,expmean}\right)^2} \quad (6)$$

III. RESULT & DISCUSSION:

Figure 1 shows the variation of moisture ratio with drying time of drying *Clarius Batrachus*. From the graph one can predict that the removal of moisture from the product at different temperature is exponential i.e. moisture is decreasing within the product with the passage of time. Drying time of *Clarius Batrachus* at 50^0C is approximately 3.5 times in compare to 70^0C. The curves between the drying rate and drying time at different temperatures are shown in the Figure 2 and the observations shows that drying rate is faster at higher temperature as compared to low temperature.

Curve fitting calculations have been done on the twelve drying models relating the drying time and moisture proportion with the test condition for drying temperatures of 50°C, 60°C, and 70°C and steady drying air speed of 3.6m/s. The results based on these statistical analysis models are shown in Table 2 -4. The acceptability of the drying model has been based on a value for the chi square and root mean square error (RMSE) should have low values, and high values for Efficiency. According to this, the most suitable model in describing drying process of *Clarius Batrachus* is the Midilli et al. model with root mean square error (RMSE), Chi Square and Efficiency as 0.0086, 8.00E-05and 0.9988 at 50°c respectively and 0.0067, 6.00E-05, 0.9995at 60°c, 0.0073, 7 E-05, 0.9993 at 70°c respectively. The changes in the moisture ratio and drying time are shown in the given Fig 1. This observation is accounted by the fact that on the initial stage of drying there is rapid decrease in moisture while at later stages the moisture content was almost negligible which was concluded by the fact that the weight was decreasing at a constant rate.

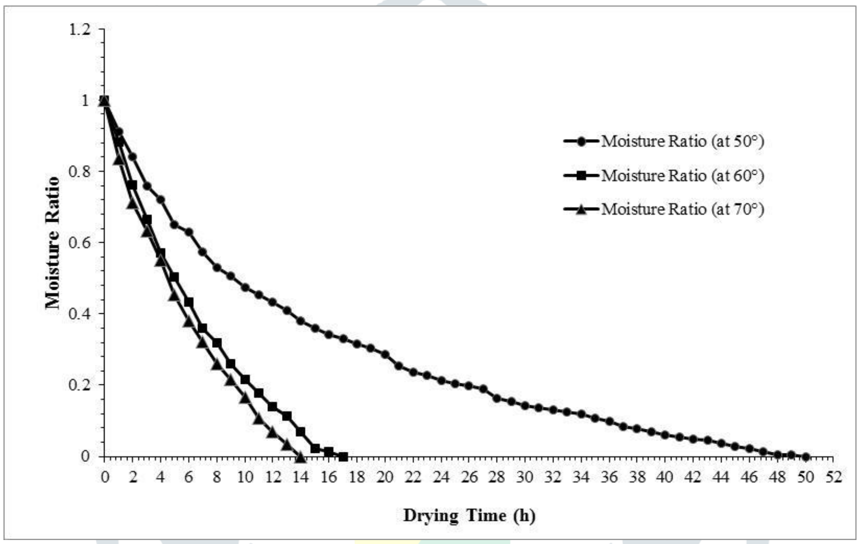

Fig.1.Relationship between the moisture ratio and drying time of drying *Clarius Batrachus*

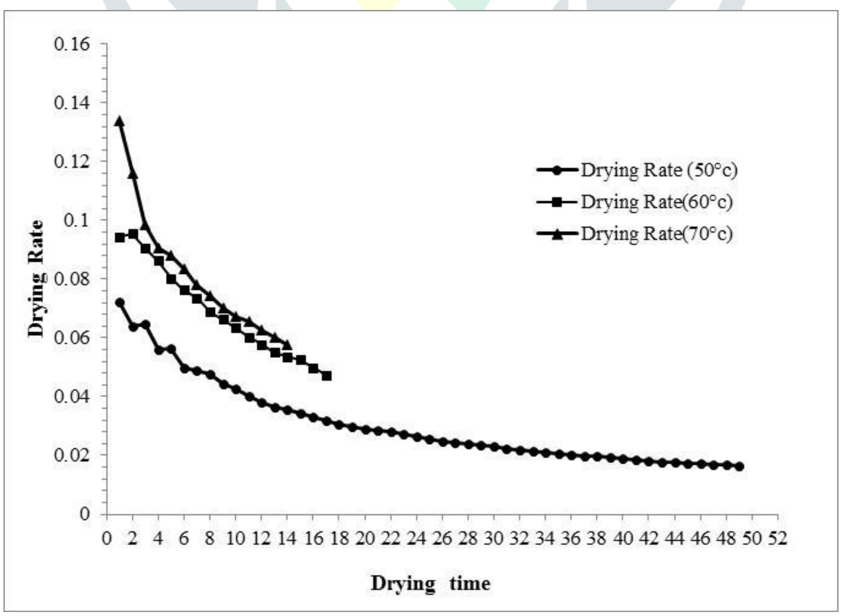

Fig 2.Drying rate of *Clarius Batrachus* versus drying time

Table 2 Result of statistical analysis on the modeling of *Clarius Batrachus* at 50°C

Model	Model Constant	Correlation Cofficient (R)	Mean Bias Error(MBE)	Root Mean Square Error(RMSE)	Chi Square (χ2)	Efficiency (E.F.)
Lewis	k=0.06844	0.99493	-0.0033	0.0255	0.0007	0.9908
Page	k=0.0913,n=0.9006	0.99715	-0.004	0.0191	0.0004	0.99422
Modified page	k=0.26161,n=0.26161	0.99493	-0.0037	0.0255	0.0007	0.99081
Henderson and Pabis	a=0.9459,k=0.0646	0.99684	-0.001	0.0201	0.0004	0.9936
Vagcioglu et al.	a=0.9532,k=0.0616 ,c=-0.015	0.99698	-0.00063	0.1391	0.0004	0.9939
Two term	a=0.108,ko=0.5482,b=0.8963 ,k1=0.0614	0.99789	-0.003	0.1162	0.0003	1.0043
Two term exponential	a=0.102,k=0.6019	0.99788	-0.003	0.0165	0.0003	0.9956
Wang & Singh	a=-0.049,b=0.006	0.97053	-0.017	0.061	0.0039	0.9556
Diffusion approach	a=0.1037,k=0.5323,b=0.1153	0.99789	-0.003	0.0164	0.0003	0.9956
Verma et al.	a=0.1036,k=0.5323,g=0.0614	0.99789	-0.003	0.0164	0.0003	0.9956
Modified Henderson and Pabis	a=0.2972,k=0.0614,b=0.5991, g=0.0614,c=0.108,h=0.5482	0.99789	-0.003	0.0164	0.0003	0.9957
Midilli and Kucuk	a=1.0143,k=0.1223,n=0.7575 ,b=-0.002	0.99943	3.00E-05	0.0086	8.00E-05	0.9988

Table 3 Result of statistical analysis on the modeling of *Clarius Batrachus* at 60°C

Model	Model Constant	Correlation Cofficient (R)	Mean Bias Error(MBE)	Root Mean Square Error (RMSE)	Chi Square (χ2)	Efficiency (E.F.)
Lewis	k=0.1523	0.99275	-0.003	0.0356	0.0013	0.983
Page	k=0.1076,n=1.1726	0.99672	-0.005	0.0239	0.0006	0.9934
Modified page	k=0.3903,n=0.3903	0.99275	-0.003	0.0356	0.00014	0.983
Henderson and Pabis	a=1.0363,k=0.158	0.9937	-0.007	0.0331	0.0012	0.9991
Vagcioglu et al.	a=1.1994,k=0.1054,c=-0.206	0.99957	-7.33E-08	0.0087	9.00E-05	0.9991
Two term	a=0.5182,ko=0.158,b=0.5182 ,k1=0.158	0.9937	-0.007	0.0331	0.0014	0.9864
Two term exponential	a=1.6837,k=0.2009	0.99673	-0.005	0.0239	0.0006	0.9933
Wang & Singh	a=-0.113,b=0.0033	0.99769	-0.004	0.0201	0.0005	0.9956
Diffusion approach	a=-10.97,k=0.2526,b=0.9524	0.99697	-0.005	0.023	0.0006	0.9938
Verma et al.	a=0.2414,k=0.1523,g=0.1523	0.99275	-0.003	0.0356	0.0015	0.983
Modified Henderson and Pabis	a=0.3454,k=0.158,g=0.158 ,b=0.3454,c=0.3454,h=0.158	0.9937	-0.007	0.0331	0.0017	0.9864
Midilli and Kucuk	a=1.0045,k=0.1328,n=0.9421 ,b=-0.01	0.99975	1.00E-05	0.0067	6.00E-05	0.9995

Table 4 Result of statistical analysis on the modeling of *Clarius Batrachus* at 70°

Model No.	Model Constant	Correlation Cofficient(R)	Mean Bias Error(MBE)	Root Mean Square Error(RMSE)	Chi Square (χ2)	Efficiency (E.F.)
Lewis	k=0.1735	0.99077	0.0003	0.0331	0.0012	0.9843
Page	k=0.1278,n=1.1616	0.99416	-0.003	0.0272	0.0009	0.9906
Modified page	k=0.4165,n=0.4165	0.99077	0.0003	0.0331	0.0013	0.9843
Henderson and Pabis	a=1.0221,k=0.1774	0.99114	-0.002	0.0326	0.0012	0.9855
Vagcioglu et al.	a=1.2417,k=0.1077,c=-0.267	0.99911	0.0005	0.0124	0.0002	0.998
Two term	a=0.511,ko=0.1774,b=0.5111 ,k1=0.1774	0.99114	-0.002	0.0326	0.0014	0.9855
Two term exponential	a=1.6597,k=0.2253	0.99439	-0.003	0.0261	0.0008	0.9913
Wang & Singh	a=-0.127,b=0.0041	0.99655	-0.005	0.0236	0.0006	0.9935
Diffusion approach	a=-14.38,k=0.2809,b=0.9646	0.99472	-0.003	0.0254	0.0008	0.9918
Verma et al.	a=-10.39,k=0.2829,g=0.2693	0.99472	-0.003	0.0254	0.0008	0.9918
Modified Henderson and Pabis	a=0.3407,k=0.1774,g=0.1774 ,b=0.3407,c=0.3407,h=0.1774	0.99114	-0.002	0.0326	0.0018	0.9855
Midilli and Kucuk	a=0.9978,k=0.1573,n=0.843,b=-0.017	0.99968	-4E-04	0.0073	7 E-05	0.9993

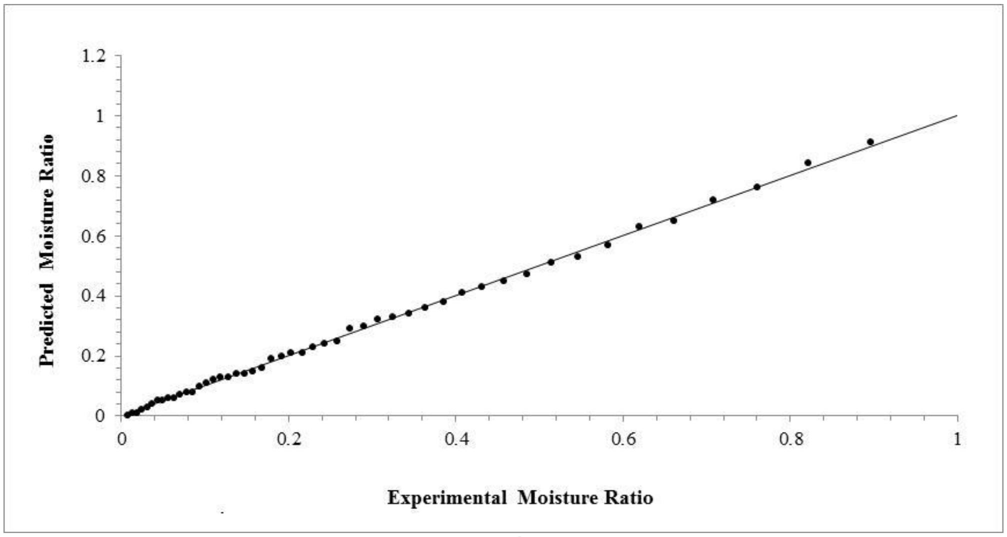

Fig.3.Experimental and predicted moisture ratio at 50°C for Midili et al

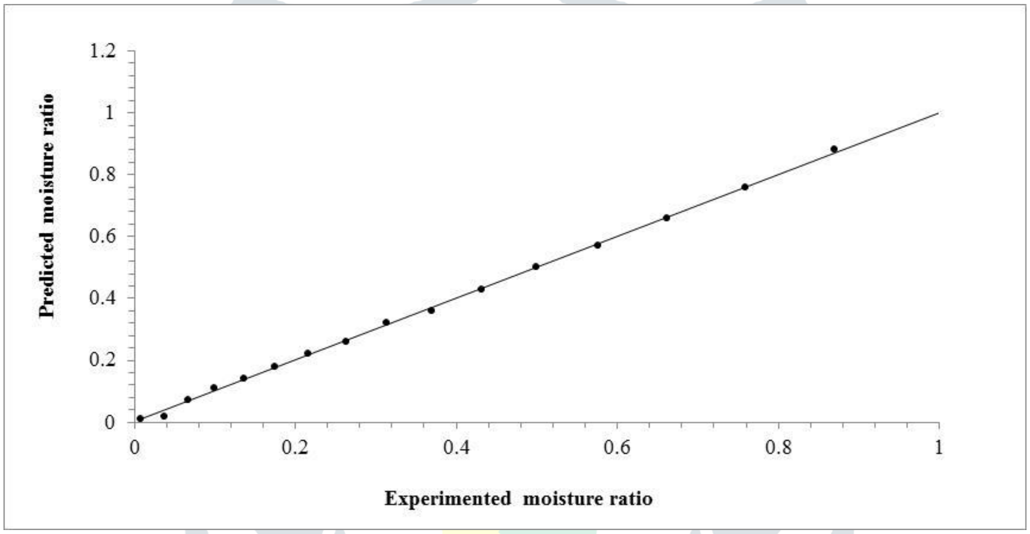

Fig. 4.Experimented and predicted moisture ratio at 60°c for Midili et al.

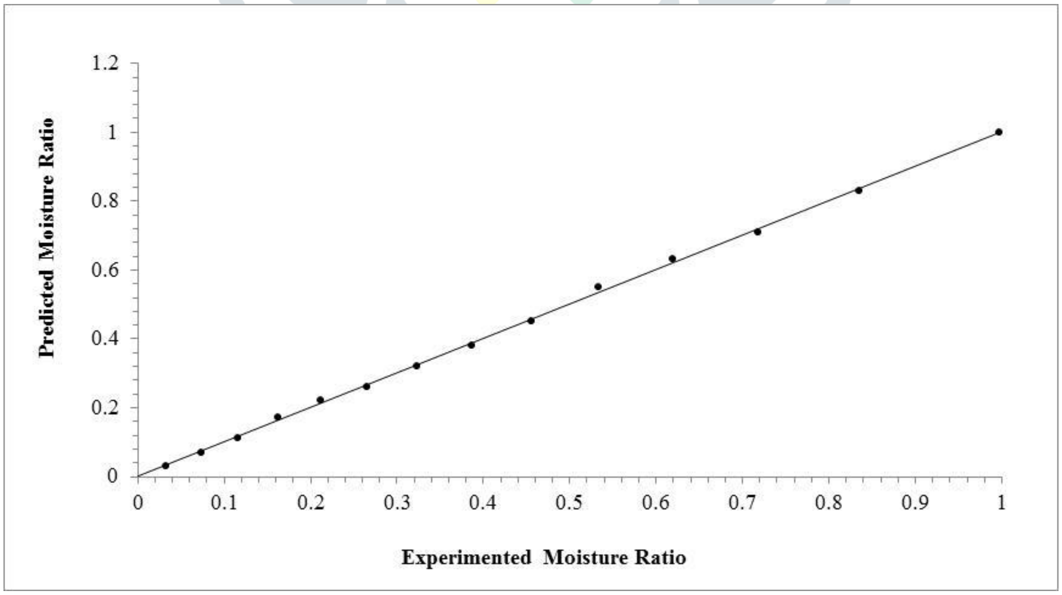

Fig.5. Experimented and predicted moisture ratio at 70°c for Midili et al.

IV. CONCLUSION:

In this experimental study the drying behavior of *Clarius Batrachus* was investigated. The drying behavior of *Clarius Batrachus* was described by twelve drying models. The observed data for Midlii et al. model found best in describing the drying behavior of *ClariasBatrachus*. The choice of model was based on the factor of high values of Efficiency i.e. approximately equal to one and low value for root mean square error(RMSE) and chi-square ($\varkappa^2$).The drying of *Clarius Batrachus* took place in the falling drying rate. The observed data was compared with previous papers to choose the best suitable model for drying.

REFERENCES

1. Agrawal, Y.C., Singh, R.P. 1997. Thin layer drying studies on short grain rough rice. ASAE, 3531.
2. Akpinar, E., Midilli, A., Bicer, Y. 2003. Single layer drying behaviour of potato slices in a convective cyclone dryer and mathematical modelling Energy Convers Manage, 44(10), 1689–705.
3. Akpinar, E.K., Midilli, A., Bicer, Y. 2003. Experimental investigation of drying behaviour and conditions of pumpkin slices via a cyclone-type dryer. J Sci Food Agric, 83(14), 1480–6.
4. Bala, B.K., Mondol, M.R.A. 2001. Experimental investigation on solar drying of fish using solar tunnel dryer. Drying Technology. 19(2), 427-436.
5. Chhinnan, M.S. 1984. Evaluation of few selected mathematical models for describing thin layer drying of in-shell pecans. Trans ASAE, 27(2), 610–5.
6. Diamente, L.M., Munro, P.A. 1993. Mathematical modelling of the thin layer solar drying of sweet potato slices. Solar Energy, 51, 271–6.
7. El-Sebaii, A.A., Aboul-Enein, S., Ramadan, M.R.I., El-Gohary, H.G. 2002. Empirical correlations for drying kinetics of some fruits and vegetables. Energy, 27, 845–59.
8. Erbay, Z., Icier, F. 2010. Thin-layer drying behaviors of olive leaves (Olea europaea L.). *Journal of Food Process Engineering*, *33*(2), 287-308.
9. Ertekin C, and Yaldiz O. 2004. Drying of eggplant and selection of a suitable thin layer drying model. J Food Eng., 63(3), 349–59.
10. Haghi, A. K., Amanifard, N. 2008.Analysis of heat and mass transfer during microwave drying of food products. Brazilian Journal of Chemical Engineering, 25(3), 491–501. doi:10.1590/s0104-66322008000300007
11. Henderson, S.M. 1974. Progress in developing the thin layer drying equation. Trans ASAE, 17, 1167–8.
12. Jain, D. 2006. Determination of convective heat and mass transfer coefficients for solar drying of fish. Biosyst. Eng. 94 (3), 429–435.
13. Jain, D., Pathare, P.B., 2007. Study the drying kinetics of open sun drying of fish, study the drying kinetics of open sun drying of fish. J. Food Eng. 78, 1315–1319.
14. Karathanos, V.T. 1999. Determination of water content of dried fruits by drying kinetics. J Food Eng, 39, 337–44.
15. Kassem, A.S. 1998. Comparative studies on thin layer drying models for wheat. In: 13th international congress on agricultural engineering, Morocco,2–6 February, 6.
16. Lui, Q., Bakker-Arkema, F.W. 1997. Stochastic modelling of grain drying. Part 2: Model development. J AgricultEng Res, 66, 275–80.
17. Midilli, A, Kucuk H, Yapar Z. 2002. A new model for single layer drying. Drying Technol, 20(7), 1503–13.
18. Midilli, A., Kucuk, H. 2003. Mathematical modelling of thin layer drying of pistachio by using solar energy. Energy Convers. Manage, 11, 11–22.
19. Mujaffar, S. and Sankat, C. K. 2015. Modeling the Drying Behavior of Unsalted and Salted Catfish (A rius sp.) Slabs. Journal of Food Processing and Preservation, 39(6), 1385-1398.
20. O_Callaghan, J.R., Menzies, D.J., Bailey, P.H. 1971. Digital simulation of agricultural dryer performance. J Agricul Eng Res,16, 223–44.
21. Overhults, D.D., White, G.M., Hamilton, M.E., Ross, I.J. 1973. Drying soybeans with heated air. Trans ASAE,16, 195–200.
22. Raghavan, G.S.V., Rennie, T.J., Sunjka, P.S., Orsat, V., Phaphuangwittayakul, W., Terdtoon, P., 2005. Overview of new techniques for drying biological materials with emphasis on energy aspects. Braz. J. Chem. Eng, 22 (2).
23. Rahman MS, Perera CO, Theband C. 1998 Desorption isoterm and heat pump drying kinetics of peas. Food Res Int, 30, 485–91.
24. Sacilik, K., Unal, G., 2005. Dehydration characteristics of Kastamonu garlic slices. Biosystems Engineering, 92 (2), 207–215.
25. Sarsavadia, P.N., Sawhney, R.L., Pangavhave, D.R., Singh ,S.P. 1999. Drying behavior of brined onion slices. J Food Eng, 40, 219–26.
26. Sharaf-Elden, Y.I., Blaisdell, J.L., Hamdy, M.Y. 1980 A model for ear corn drying. Trans ASAE,23,1261–5.
27. Togrul, I.T., Pehlivan, D. 2002. Mathematical modelling of solar drying of apricots in thin layers. J Food Eng,55, 209–16.
28. Verma, L.R., Bucklin, R.A., Endan, J.B., Wratten, F.T. 1985. Effects of drying air parameters on rice drying models. Trans ASAE,28,296–301.
29. Wang, CY, Singh RP. 1978. A single layer drying equation for rough rice. ASAE Paper No. 3001.
30. Westerman, P.W., White, G.M., Ross, I.J. 1973. Relative humidity effect on the high temperature drying of shelled cornTrans ASAE,16,1136–9.
31. White, G.M., Ross, I.J., Ponelert, R. 1981. Fully exposed drying of popcorn. Trans ASAE, 24, 466–8.

32. Yagcioglu, A., Degirmencioglu, A., Cagatay, F. 1999. Drying characteristic of laurel leaves under different conditions. In:Bascetincelik A, editor. Proceeding of the 7th international congress on agricultural mechanization and energy, 26–27 May, Adana, Turkey. Faculty of Agriculture, Cukurova Universty, 565–9.
33. Yagcioglu, A.K., Demir, V., Gunhan, T. 2001. Determination of the drying characteristics of laurel leaves. Ege University Research Fund Project Final Report, Faculty of Agriculture, Agricultural Machinery Department, Project No. 99ZRF 029, (in Turkish).
34. Yaldiz, O., Ertekin, C. 1991. Thin layer solar drying of some vegetables. Drying Technol, 9, 551–88.
35. Zhang, Q., Litchfield, J.B. 1991. An optimization of intermittent corn drying in a laboratory scale thin layer dryer Drying Technol, 9, 383–95.