# INTRODUCTION TO GRAPH THEORY AND ITS TYPES

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***Abstract:*** Graphs are of simple structures which are made from nodes, vertices or points which are connected by the arcs, edges or lines. Graphs are used to find the relationship between two objects which are connected through nodes and edges. Also there are many types of graphs which represent different properties of graphs. Graphs are used in many fields in modern times. In today's time graph theory will needed in all aspects.

**Keywords:** Father of graph theory, graphs, uses, type, paths, cycle, walk, Hamiltonian graph, Euler graph, colouring, chromatic numbers.

## 1. INTRODUCTION

Graph theory begins with very simple geometric ideas and has many powerful applications. Graph theory begins in 1736 when Leonhard Euler solved a problem that has been puzzling the good citizens of the town of Konigsberg in Prussia. In modern times Graph theory played very important role in many areas such as communications, engineering, physical sciences, linguistics, social sciences, and many other fields. On the basis of this variety of application it is useful to develop and study the subject in abstract terms and finding the results.

## 2. WHY DO WE STUDY GRAPH THEORY?

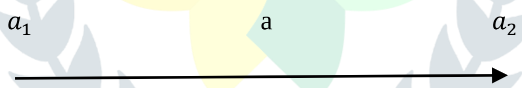
Graph theory is very important for "analysing things that we connected to another thing" which applies almost everywhere. It is mathematical structure which is used to study of graphs, to solve pairwise relation between two objects. A graph is made up of nodes, vertices or points which are connected by lines, edges or arcs.

## 3. DEFINATION OF GRAPH

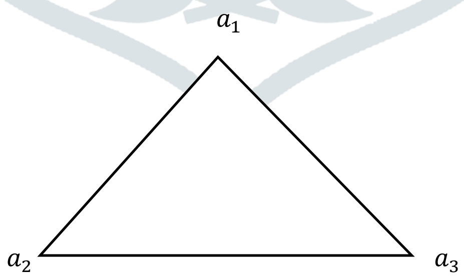
A Graph G consist of non-empty set V of nodes and collection of edges (not need to be a set) of unordered pair of nodes called edges. A graph is symbolically representing as G= (V, E). The order of a graph is number of its nodes and its edges.

## 4. TYPES OF GRAPH

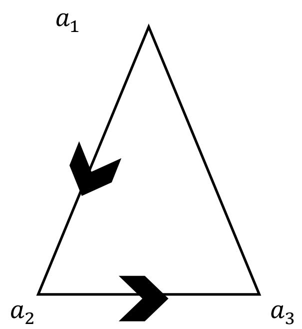
**4.1 Directed Graph** Directed Graph G consist of set V of nodes and set E of edges such that e belongs to E is associated with an ordered pair of nodes.



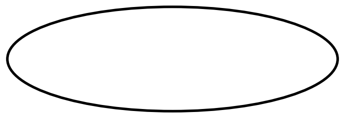

**4.2 Undirected Graph** Undirected Graph G consist of non-empty sets V of nodes and set E of edges such that e belongs to E is associated with an unordered pair of nodes.




**4.3 Mixed Graph** a Graph G= (V, E) is called mixed graph in which some edges are in ordered pair and some edges are in unordered pair.




**4.4 Self- loop:** An edge which is represented by an unordered pair in which two elements are not distinct.



**4.5 Proper edge** An arc that is not a self- loop is called proper arc.

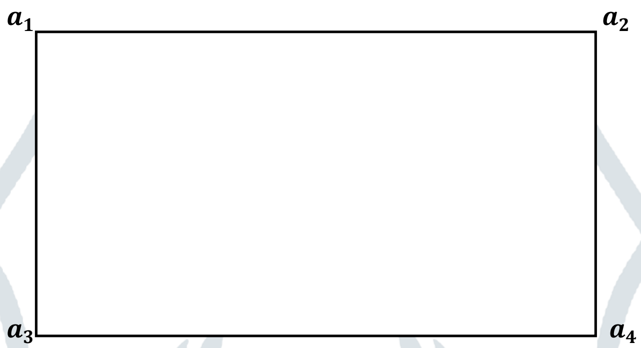
**4.6 Parallel edges** Two or more arcs that join the same pair of different vertices are called parallel edges.

**4.7 Multi-graph** A graph which has no loop is said to be multi-graph.

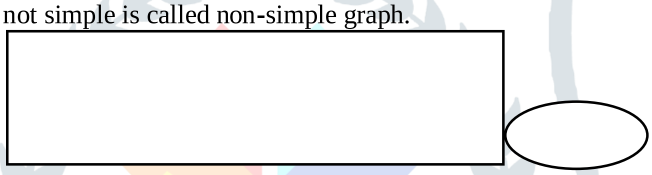
**4.8 Pseudo-graph** A graph which has at least one loop is called pseudo-graph.

**4.9 Originating and Ending** If a graph G be a directed edge is associated with the ordered pair of nodes (c and d) then the edge is said to be originating in node c and ending in node d.

**4.10 Simple- graph** An undirected graph which has no loops is called simple graph.




**4.11 Non-Simple Graph** A Graph which is not simple is called non-simple graph.



**4.12 Weighted-graph** A graph is said to be weighted graph if positive number is assigned to each edge. This positive number is called weight of graph.

**4.13 Finite-graph** A graph which has finite number of nodes and edges is called finite-graph.

**4.14 Infinite-graph** A graph is said to be infinite graph if it has infinite number of edges and nodes.

**4.15 Trivial graph** A graph which have one node and no edge is called trivial graph.

**4.16 Null-graph** A graph which have no node and no edge is called null-graph.

**4.17 Order of a graph** The number of nodes in the graphs is said to be the order of a graph.

**4.18 Size of a graph** The number of lines in the graph is said to be the size of the graph.

**4.19 Incidence** If G be a graph. An edge x belongs to E which connects the nodes c and d whether it is ordered or unordered is said to be incident to c and d.

**4.20 Adjacent edge** Two edges incident on the same terminating point edge is called adjacent edge.

**4.21 Underlying graph** It is undirected graph created using all of the nodes in and replacing all edges in with undirected edges.

**4.22 Orientation graph** If in simple graph an edges replaced by an edge is called orientation graph.

**4.23 Tournament** An orientation of complete graph is called Tournament.

**4.24 Bipartite graph** It is simple in which the set of nodes can be divided into two sets X and Y such that every edge is between a node in X and a node in Y, it is represented as G=(X, Y,E)

**4.25 Complete graph** A graph with n-nodes in which these at least one edge between every pair of nodes.

**4.26 Isomorphic graph** Two graphs G= (V, E) and G'= (V',E') are said to be isomorphic if there exist a one to one correspondence f called isomorphism from V to V'.

**4.27 Regular graph** A graph is said to be gRegular graph if each node of graph has same degree.

**4.28 Homomorphic graph** A graph G and G* is said to be homomorphic if they obtained from same graph or we can say from isomorphic graphs.

**5. DEGREE OF GRAPH**

Degree of graphs are of two types

**5.1 Out-degree** The out-degree of any node "a" is directed graph is the number of the edges which have "a" as a initial node. It is denoted by deg_(a).

**5.2 In-degree** The In-degree of any node "a" in directed graph is the number of the edges which have "a" as a terminal node. It is denoted by deg+(a).

**6. SUB-GRAPHS**

Let G and G' be two graphs and V(G) and V(G') be the sets of nodes of graphs such that first set of node is contained in another set i.e. V(G') is contained in V(G) then it is called sub-graph of a graph.

**7. ALGEBRA OF GRAPHS**

**7.1 Union of Graphs** The union of two graphs is denoted by $G_1 \text{U} G_2$ and it is defined as the union of nodes and edges in both the graphs but these are not repeated.

**7.2 Intersection of Graphs** The intersection of two graphs is denoted by $G_1 \cap G_2$. It is defined as the common portion of both the node and edges in two graphs.

**7.3 Complement of Graphs** It is defined as if $a_1$ node is connected to another nodes in one graph then in the complement of graph the $a_1$ is not connected to another nodes. If G is the graph then G* is its complement.

**8. WALK**

A walk is defined as the sequence of nodes $a_0\ a_1\ a_2\ \ldots \ldots a_n$ such that the nodes $a_i$ to $a_{i+1}$ are adjacent for i=1…..n. In the walk the nodes $a_0$ and $a_n$ with which it starts and terminate is called terminal nodes.

A walk in which all edges are different is said to be **trail**.

A walk in which all nodes are distinct is said to be **path**.

**8.1 Open Walk** If a walk start and terminate with different nodes is called an open walk.

**8.2 Closed Walk** If a walk start and terminate with same nodes is called an closed walk.9

**9. PATH**

In an open walk no node appear more than once.

**9.1 Simple Path** In Simple path all nodes are different.

**9.2 Trail** In trail all edges are different.

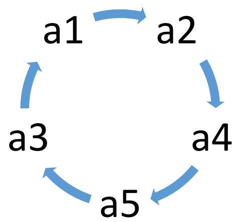
**9.3 Length of path** Length of path is defined as total number of edges in a path.

**Note** Nodes and edges are repeated in a path But no nodes and edges are repeated in simple path and in a trail edges are not repeated.

**10. CYCLE**

Cycle is a closed walk in which no nodes and edges are repeated.

**10.1 Simple-cycle** In this cycle only first and last node is repeated i.e.




**a1-a2-a4-a5-a3-a1**

**Note** Self- loop is also a cycle but a cycle is not a self-loop.

**11 CONNECTIVITY**

An undirected graph is said to be connected, if for any pair of nodes of graph, the two nodes are reachable from one another.

**11.1 Connected Graph** If there is at least one path between every pair of nodes in a Graph.

**12 PLANER GRAPH** If we draw a graph on a plane sheer in such a way that no nodes cross one another except at the end point.

**13 MAP** Map can be drawn in such a way that its edges do not intersect each other.

**14 REGION** Different Partitions of graph in a map is called regions of map.

**15 CRITICAL PLANER** If Graph is non-planer but we obtained a sub-graph from graph by removing a node is planer then graph then it is called Critical planer.

**16 CUT- EDGE** When we remove an edge from the graph. In result we get disconnected graph.

**17 COLOURING** It is the painting of a graph with colours in such a way that two adjacent nodes does not have same colours.

**18 CHROMATIC NUMBER** These are the minimum number of colours which are necessary to use the colour a graph and it is denoted by **c(G)**

**19 EULER PATH** In a Euler Path every edge includes at least once.

**19.1 Euler Circuit** It is closed Euler Path

**19.2 Euler Graph** A graph is said to be Euler Graph if it is a Euler path.

**20 BRIDGE** If a graph is connected graph and we cut one edge from the graph and graph become disconnected so the edge which we cut is known as Bridge.

**21 HAMILTONIAN PATH** Hamiltonian path is that path if it passes through each node exactly once. It may not visit all the nodes.

**21.1 Hamiltonian circuit** It is the circuit which contain each node exactly once except for first node which is also end node.

**21.2 Hamiltonian Graph** A graph which have Hamiltonian Path is said to be Hamiltonian graph.

**CONCLUSION**

In this paper we discuss about Introduction of Graph theory and its various types. This paper shows that why it is necessary to use Graphs in real life. Graph theory is very important in every area in modern times. By using Graph theory some complex problems become easy. There are lots of unsolved problems in graph theory solve one and become rich and famous like graph theory is used in traffic signal design and scheduling problem. So we get much valuable knowledge from Graph theory.

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