Structures of Q-intuitionistic fuzzy ordered filters in ordered Γ-ternary semiring

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Abstract

In this paper, the notion of structures of Q-intutionistic fuzzy ordered filter in ordered Γ- semiring and some their properties and structures are studied.

Key-words Q-intuitionistic fuzzy ordered filter, ordered Γ- semiring

1. Introduction

The notion of semiring was introduced by H.S. Vandiver[4], in 1934. The notion of Γ- ring was introduced by N. Nobusawa as a generalization of ring to 1964, Dutta and kar[2] introduced and studies some properties of ternary semirings which is a generalization of ternary ring. The notion of Q-intuitionistic fuzzy ordered filters in Γ- ternary semiring is introduced.

2. Preliminaries

Definition 2.1 *Let A and B be two Q-intuitionistic fuzzy subset of an ordered Γ-ternary semiring R α ∈ Γand q ∈ Q. Then the product of A ΓB of A and B is defined by*

$$\mu_{A\Gamma B}(a,q) = \begin{cases} \vee \mu_A(b,q) \wedge \mu_B(c,q) & \text{if } a = b\alpha c \\ a = b\alpha c & \\ 0 & \text{otherwise} \end{cases}$$

$$\mu_{A\Gamma B}(a,q) = \begin{cases} \vee \mu_A(b,q) \wedge \nu_B(c,q) & \text{if } a = b\alpha c \\ a = b\alpha c & \\ 1 & \text{otherwise} \end{cases}$$

Definition 2.2 *Let A and B be two Q-intuitionistic fuzzy subset of a ordered Γ-ternary semiring R, α ∈ Γ and q∈ Q. Then the product composition A◦B of A and B is defined by*

$$\mu_{AoB}(c, q) = \begin{cases} \vee (\mu_A(ai, q) \wedge \mu_B(bi, q) & \\ i \le i \le m & \\ 0 & \text{other wise} \end{cases}$$

$:C = \sum_{i=1}^{m} a_i \, \alpha bi, ai, bi \in R, \alpha \in \Gamma, q \in Q, k \in N$

$$\mu_{AoB}(c, q) = \begin{cases} \wedge \vee (\mu_A(ai, q) \wedge \mu_B(bi, q) & \\ i \le i \le m & \\ 0 & \text{other wise} \end{cases}$$

$:C = \sum_{i=1}^{m} a_i \, \alpha bi, ai, bi \in R, \alpha \in \Gamma, q \in Q, k \in N$

Definition 2.3 *Let A and B Q-intuitionistic fuzzy subset of an ordered Γ- ternary semiring R. Then the intuitionistic sum A⊕B of A and B is defined by*

$$\mu_{A\oplus B}(a,q) = \begin{cases} \vee \mu_A(b,q) \wedge \mu_B(b,q) & \text{if } c = a+b \\ c = a+b & \\ 0 & \text{otherwise} \end{cases}$$

$$\nu_{A\oplus B}(a,q) = \begin{cases} \wedge \vee \mu_A(a,q) \vee \nu_B(b,q) & \text{if } c = a+b \\ c = a+b & \\ 1 & \text{otherwise} \end{cases}$$

Definition 2.4 *Let* $\{A\}_{i\in N}$ *be an arbitrary family of Q-intuitionistic fuzzy subset of an ordered Γ-ternary semiring R. Then* $\cap A_i = \{\wedge \mu_{A_i}, \vee \nu_{A_i}\} = (\cup_{A_i} = (\vee \mu_{A_i}, \wedge \nu_{A_i})$.

3. Main Theorems

Theorem 3.1 If A, B and C are Q-intuitionistic fuzzy ordered filters in an ordered *Γ-ternary semiring R. Then AoBoC is a Q-intuitionistic fuzzy ordered filters in R.*

Proof: For any x, y∈R, we have

$\mu_{AoBoC}(x+y, q) = \vee[\wedge\mu_A(u_i,q)\wedge\mu_{BoC}(u_i,q)$: $x+y = \sum_{i=1}^{k} u_i\alpha v_l \in R, \alpha \in \Gamma, q \in Q, k \in N$.

$\geq \vee[\wedge_{1\leq i\leq m}\mu_A(a_i,q)\wedge\mu_{BoC}(b_i,q)]\wedge[\wedge_{1\leq i\leq m}\mu_A(c_i,q)\wedge\mu_{BoC}(d_i,q)]$

$x = \sum_{i=1}^{m} a_i\alpha b_i$, $y = \sum_{i=1}^{n} c_i\alpha d_i$, $a_i b_i c_i d_i \in R$, $\alpha \in \Gamma$, $q \in Q$ and m, n∈N.

$\{= \vee[\wedge\mu_A(a_i,q)\wedge \mu_{BoC}(b_i,q)]$: $x=\sum_{i=1}^{m} a_i\alpha b_i, \sum_{i=1}^{n} a_i\, b_i, \in R, \alpha\in\Gamma, q\in Q$ and m, n∈N.}

$\wedge\vee[\wedge\mu_A(c_i,q)\wedge\mu_{BoC}(d_i,q)]$: $y = \sum_{i=1}^{m} c_i\alpha d_i$, $c_i, d_i, \in R, \alpha\in\Gamma, q\in Q$ and m, n∈N.}

$= \mu_{AoBoC}(x, q)\wedge\mu_{AoBoC}(y,q)$.

$\nu_{AoBoC}(x+y, q) = \wedge [\vee_{1\leq i\leq k} \nu_A(u_i,q) \vee\nu_{BoC}(\nu_i, q)]$:

$x + y = \sum_{i=1}^{k} u_i\alpha\nu_i u_i$, $\nu_i \in R$, $\alpha\in\Gamma$, $q\in Q$, k∈N.

$\leq\vee[\vee_{1\leq i\leq k}\nu_A(a_i,q)\wedge\nu_{BoC}(b_i,q)]\vee[\vee_{1\leq i\leq k}\nu_A(c_i,q)\vee\nu_{BoC}(d_i,q)]$

$x=\sum_{i=1}^{m} a_i\alpha b_i$, $y = \sum_{i=1}^{n} c_i\alpha\, d_i$, $a_i\, b_i, c_i, d_i \in R$, $\alpha\in\Gamma$, $q\in Q$ and m, n∈N.

$=\wedge\{\vee[\nu_A(a_i,q)\vee\nu_{BoC}(b_i,q)]\}$: $x= \sum_{i=1}^{m} a_i\alpha b_i$, $a_i b_i \in R$, $\alpha\in\Gamma$, $q\in Q$ and m, n∈N.}

$\wedge\vee[\wedge\nu_B(c_i,q)\wedge\nu_{BoC}(d_i,q)]$:$y = \sum_{i=1}^{n} c_i\alpha d_i, c_i, d_i R$, $\alpha\in\Gamma$, $q\in Q$ and m, n∈N.}

$= \nu_{AoBoC}(x, q)\vee\nu_{AoBoC}(y,q)$.

$\mu_{A\circ B\circ C}(x\alpha y,\beta z,q)=\vee\{\wedge\mu_A(u_i,q)\wedge\mu_B(\nu_i,q):\mu_c(w_i,q)$: $=\sum_{i=1}^{m} x\alpha y\beta z) = \sum_{i=1}^{m} u, \alpha\nu_i\beta w_i$, $u_i, \nu_i, w_i \in R$ $\alpha, \beta \in \Gamma$, $q\in Q$, m∈N.}

$=\vee\{\wedge_{1\leq i\leq k} \mu_A(a_i,q)\wedge \mu_B(b_i,q)]\ \mu_c(c, \beta_z, q)$: $x\alpha y\beta_z, = \sum_{i=1}^{m} a_i\alpha b_i, \beta c, \beta z, a_i, b_i, c_i\beta_z \in R$, $\alpha, \beta\in\Gamma$, $q\in Q$ m∈N}

$\leq\vee\{\wedge_{1\leq i\leq k} \mu_A(a_i,q)\wedge\mu_B(b_i,q)\wedge \mu_C(c_i, q)$: $x=\sum_{i=1}^{m} a_i\alpha b_i, \beta c, ai, b_i, c_i, \in R$, $\alpha, \beta\in\Gamma$, $q\in Q$ m∈N}

$= \mu_{AoBoC}(x, q)$.

$\nu_{A\circ B\circ C}(x\alpha y,\beta_z,q)=\wedge_{1\leq i\leq k} \{\vee\nu_A(u_i,q)\vee\nu_B(\nu_i,q)\nu_c(w_i q)$: $x\alpha y\beta_z = \sum_{i=1}^{m} u, \alpha v_i\beta w, \nu_i, \nu_i\, w_i \in R, \alpha, \beta \in \Gamma, q \in Q$, $m \in N$}

$=\wedge\{[\wedge_{1\leq i\leq k} \nu_A(a_i,q)\nu_B(b_i,q)\mu_C(c_i,\beta z, q)$: $x\alpha y\beta z$, $q = \sum_{i=1}^{m} a_i\alpha b_i, \beta c, \beta z, a_i, b_i, c_i\beta_z \in R$, $\alpha, \beta\in\Gamma$, $q\in Q$ m∈N}

$\geq\wedge\{\vee_{1\leq i\leq k} \nu_A(a_i,q)\mu_B(b_i,q)\mu_C(c_i, q)$:$x=\sum_{i=1}^{m} a_i\alpha b_i$, $\beta c, a_i, b_i, c_i \in R$, $\alpha, \beta\in\Gamma$, $q\in Q$ m∈N}

$= \nu_{AoBoC}(x, q)$.

Hence $\mu_{AoBoC}(x\alpha y\beta z, q)\leq \mu_{AoBoC}(x, q)$ and $\nu_{AoBoC}(x\alpha y\beta z, q)\geq \nu_{AoBoC}(x, q)$.

Similarly, $\mu_{AoBoC}(y\alpha y\beta z, q)\leq\mu_{AoBoC}(x, q)$ and $\nu_{AoBoC}(x\alpha y\beta z, q)\geq \nu_{AoBoC}(y, q)$.

Also $\mu_{AoBoC}(x\alpha y\beta z, q)\leq\mu_{AoBoC}(z, q)$ and $\nu_{AoBoC}(x\alpha y\beta z, q)\geq \nu_{AoBoC}(z, q)$

Let $x \leq y$, then $\mu_A(x, q) \leq \mu_A(y, q)$ and $\mu_B(x, q) \leq \mu_B(y, q)$

Also $\nu_A(x, q) \leq \nu_A(y, q)$ and $\nu_B(x, q) \geq \nu_B(y, q)$

$\mu_{AoBoC}(x, q) = \vee_{1\leq i\leq k} \{\wedge \mu_A(a_i,q)\wedge\mu_{BoC}(b_i,q)$: $x=\sum_{i=1}^{m} a_i\alpha b_i, ai, b_i, \in R$, $\alpha, \beta\in\Gamma$, $q\in Q$ m∈N}

$\leq \vee_{1\leq i\leq k}\{\wedge \mu_A(c_i,q)\wedge\mu_{BoC}(d_i,q)$: $y = \sum_{i=1}^{m} c_i\alpha d_i, ci, d_i, \in R$, $\alpha, \beta\in\Gamma$, $q\in Q$ m∈N}

$= \mu_{AoBoC}(y, q)$.

$\nu_{A\circ B\circ C}(x, q) = \vee_{1\leq i\leq m} \{\wedge\nu_A(a_i,q)\wedge \nu_{BoC}(b_i,q)$: $x=\sum_{i=1}^{m} a_i\alpha b_i, ai, b_i, \in R$, $\alpha, \beta\in\Gamma$, $q\in Q$ m∈N}

$\geq \vee_{1\leq i\leq k} \{\wedge\nu_A(c_i,q)\wedge \nu_{BoC}(d_i,q)$: $x=\sum_{i=1}^{m} c_i\alpha d_i, b_i$, $c_i, d_i \in R$, $\alpha, \beta\in\Gamma$, $q\in Q$ m∈N}

$= \nu_{AoBoC}(y, q)$.

Hence AoBoC is Q intuitionistic fuzzy ordered filter of R.

Theorem 3.2

If A and B are Q intuitionistic fuzzy ordered filter of R, then the intuitionistic sum A⊕B is Q-intuitionistic fuzzy ordered filter of R.

Proof: For any x, y∈R.

$\mu_{A\oplus B}(x, q)\wedge \mu_{A\oplus B}(y, q) = \{\vee\{\mu_A(a, q) \wedge \mu_B(b, q)$: $x=a+b\}\} \wedge \{\vee\ \mu_A(c, q) \wedge \mu_B(d, q)$: $y = c + d\}\}$

$= \vee\{[\mu_A(a, q) \wedge \mu_B(b, q)] \wedge [\mu_A(c, q)\wedge\mu_B(d, q)]$: $x = a + b, y = c + d\}$

$= \vee\{[\mu_A(a, q) \wedge \mu_A(b, q)] \wedge[\mu_B(c, q) \wedge\mu_B(d, q)]$: $x = a + b, y = c + d\}$

$\leq\vee\{[\mu_A(a+c, q)\wedge \mu(b+d, q)$: $x=a+b, y = c+d\}$

$=\mu_{A\oplus B}(x+ y, q)$

$\nu_{A\oplus B}(x, q)\wedge \nu_{A\oplus B}(y, q) = \{\vee\{\nu_A(a, q) \wedge \nu_B(b, q)$: $x=a+b\}\} \wedge \{\vee\ \nu_A(c, q) \wedge \nu_B(d, q)$: $y = c + d\}\}$

$=\vee\{[\nu_A(a, q) \wedge \nu_B(b, q)] \wedge [\nu_A(c, q) \wedge \nu_B(d, q)]$: $x = a + b, y = c + d\}$

$=\vee\{[\nu_A(a, q) \wedge \nu_A(b, q)] \wedge [\nu_B(c, q) \wedge \nu_B(d, q)]: x = a + b, y = c + d\}$
$=\nu_{A\oplus B}(x+y, q)$
Also $\mu_{A\oplus B}(x, q) = \vee\{\mu_A(a, q) \wedge \mu_B(b, q): x = a+b\}$
$\geq \vee[\mu_A(a\alpha\beta z, q) \wedge \mu_B(b\alpha t\beta z, q): x\alpha y\beta z = a\alpha y\beta z + b\alpha y\beta z$
$\geq \vee[\mu_A(u, q) \wedge \mu_B(v, q) : x\alpha y\beta z = u + v\}$
$=\mu_{A\oplus B}(x\alpha y\beta z)$
$\nu_{A\oplus B}(x, q) = \vee\{\nu_A(a, q) \wedge \nu_B(b, q) : x=a+b\}$
$\geq \vee\{\nu_A(a\alpha\beta z, q) \wedge \nu_B(b\alpha t\beta z, q): x\alpha y\beta z = a\alpha y\beta z + b\alpha y\beta z\}$
$\geq \vee\{\nu_A(u, q) \wedge \nu_B(v, q): x\alpha y\beta z = u + v\}$
$= \nu_{A\oplus B}(x\alpha\beta z, q)$
Hence $\mu_{A\oplus B}(a\alpha\beta z, q) \leq \mu_{A\oplus B}(x, q)$ *and*
$\nu_{A\oplus B}(x\alpha\beta z, q) \leq \nu_{A\oplus B}(x, q)$
Similarly, we have
$\mu_{A\oplus B}(x\alpha y\beta, q) \leq \mu_{A\oplus B}(y, q)$ and $\nu_{A\oplus B}(x\alpha y\beta z, q) \leq \nu_{A\oplus B}(y, q)$ and also we have
$\mu_{A\oplus B}(x\alpha y\beta z, q) \leq \mu_{A\oplus B}(z, q)$ and $\nu_{A\oplus B}(x\alpha y\beta z, q) \leq \nu_{A\oplus B}(z, q)$
Let $x \leq y$ then $\mu_A(x, q) \leq \mu_B(y, q)$ and $\nu_A(x, q) \geq \nu_B(y, q)$
Also $\mu_B(x, q) \leq \mu_B(y, q)$ and $\nu_b(x, q) \geq \nu_B(y, q)$
Now $\mu_{A\oplus B}(x, q) = \vee\{\mu_A(a, q) \wedge \mu_B(b, q) : x = a+b\}$
$\leq \vee[\mu_A(c, q) \wedge \mu_B(d, q) : y = c+d\} = \nu_{A\oplus B}(y, q)$
$\nu_{A\oplus B}(x, q) = \wedge\{\nu_A(a,q) \wedge \nu_B(b, q): x = a+b)$
$\leq \vee\{\nu_A(c, q) \wedge \nu_B(d, q) : y = c+d\} = \nu_{A\oplus B}(y, q)$
Hence $A\oplus B$ is Q intuitionistic fuzzy ordered filter of R.

Theorem 3.3

If A, B and C are Q intuitionistic fuzzy ordered filter of R. Then (i) $A\cap B\cap C$ is a Q intuitionistic fuzzy ordered filter of R. (i) $A\Gamma B\Gamma C \supseteq A\cap B\cap C$.

Proof of (i) Let A, B and C are Q intuitionistic fuzzy ordered filter of R and let x, y∈R and a, $\beta\in\Gamma$, q∈Q. Then
$\mu_{A\cap B\cap C}(x+y,q) = \{\mu(x+y,q) \wedge \mu_{B\cap C}(x+y, q)$
$=\{\mu_A(x, q) \wedge \mu_A(y, q)\} \wedge \{\mu_B(x, q) \wedge \mu_B(y, q)\} \wedge \{\mu_C(x, q) \wedge \mu_C(y, q)\}$
$=\{\mu_A(x, q) \wedge \mu_B(x, q)\} \wedge \{\mu_A(y, q) \wedge \mu_B(y, q)\} \wedge \{\mu_C(x, q) \wedge \mu_C(y, q)\}$
$=\mu_{A\cap B\cap C}(x, q) \wedge \mu_{A\cap B\cap C}(y, q)$
$\nu_{A\cap B\cap C}(x+y, q) = \{\nu(x+y, q) \wedge \nu_{B\cap C}(x+y, q)\}$
$=\{\nu_A(x, q) \wedge \nu_A(y, q)\} \wedge \{\nu_B(x, q) \wedge \nu_B(y, q)\} \wedge \{\nu_C(x, q) \wedge \nu_C(y, q)\}$
$=\{\nu_A(x, q) \wedge \nu_B(y, q)\} \wedge \{\nu_A(y, q) \wedge \nu_C(y, q)\} \wedge \{\nu_C(x, q) \wedge \nu_C(y, q)\}$
$\nu_{A\cap B\cap C}(x, q) \wedge \nu_{A\cap B\cap C}(y, q)$ Now $\mu_{A\cap B\cap C}(x\alpha y\beta z, q)$
$= \{\mu(x\alpha y\beta z, q) \wedge \mu_B(x\alpha y\beta z, q) \wedge \mu_C(x\alpha y\beta z, q)$
$=\min\{\mu_A(x, q), \mu_A(y, q), \mu_A(z, q)\} \wedge \min\{\mu_B(x,q), \mu_B(y, q)\ \mu_B(z, q)\} \wedge \min\{\mu_C(x, q), \mu_C(y, q), \mu_C(z,q)\}$
$=\min\{\mu_A(x, q) \wedge \mu_A(y, q)\} \wedge \mu_A(z, q)\ \min\{\mu_B(x, q) \wedge \mu_B(y, q) \wedge \mu_B(z, q)\}\ \min\{\mu_C(x, q) \wedge \mu_C(y, q) \wedge \mu_C(z, q)\}$
$=\min\{\mu_{A\cap B\cap C}(x, q), \mu_{A\cap B\cap C}(y, q), \mu_{A\cap B\cap C}(z, q)\}$
$\nu_{A\cap B\cap C}(x\alpha y\beta z, q) = \{\nu_A(x\alpha y\beta z, q) \vee \nu_B(x\alpha y\beta z, q) \vee \nu_C(x\alpha y\beta z, q)$
$=\min\{\nu_A(x, q), \nu_A(y, q), \nu_A(z, q)\} \vee \min\{\nu_B(x,q), \nu_B(y, q)\ \nu_B(z, q)\} \vee \min\{\nu_C(x, q), \nu_C(y, q), \nu_C(z,q)\}$
$=\min\{\nu_A(x, q), \vee\nu_A(y, q), \vee\nu_A(z, q)\}$
$\vee \min\{\nu_B(x,q), \vee\nu_B(y,q)\ \vee\nu_B(z,q)\} \vee \min\{\nu_C(x,q)\ \vee\nu_C(y, q), \vee\nu_C(z,q)\}$
$= \min\{\nu_{A\cap B\cap C}(x, q), \nu_{A\cap B\cap C}(y, q), \nu_{A\cap B\cap C}(z, q)\}$
Let $x \leq y$. Then $\mu_{A\cap B\cap C}(x, q) = \mu_A(x, q) \wedge \mu_B(x, q)\ \mu_C(x, q)$
$\leq \mu_A(y, q) \wedge \mu_B(y, q) \wedge \mu_C(y, q) = \mu_{A\cap B\cap C}(y, q)$
$\nu_{A\cap B\cap C}(x, q) = \nu_A(x, q) \vee \nu_B(x, q) \vee \nu_C(x, q)$
$\geq \nu_A(y, q) \vee \nu_B(y, q) \vee \nu_C(y, q) = \nu_{A\cap B\cap C}(y, q)$

Proof of (ii)

If $\mu_{A\Gamma B\Gamma C}(x, q) = 1\ \mu_{A\Gamma B\Gamma C}(x, q) = 0$
Then there is nothing to prove suppose
$\mu_{A\Gamma B\Gamma C}(x, q) \neq 1$ and $\mu_{A\Gamma B\Gamma C}(x, q) \neq 0$
$\mu_{A\Gamma B\Gamma C}(a, q) = \vee_{a=x\alpha y\beta z}\{\mu_A(x, q) \wedge \mu_B(x, q)\mu_C(z,q)$
$\geq \{\mu_A(a, q) \wedge \mu_B(a, q)\mu_C(a, q)\} = \mu_{A\cap B\cap C}(a, q)$
$\nu_{A\Gamma B\Gamma C}(a, q) = \vee_{a=x\alpha y\beta z}\{\nu_A(x, q) \wedge \nu_B(y,q)\ \nu_C(z,q)\}$
$\geq \{\nu_A(a, q) \wedge \nu_B(a, q)\ \nu_C(a, q)\} = \nu_{A\cap B\cap C}(a, q)$ for all $a \in R$, $q \in Q$, $x, y, z \in R$, $\alpha, \beta\in\Gamma$
Hence $A\Gamma B\Gamma C \supseteq A\cap B\cap C$.

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