Magneto polar free convection flow in a porous medium in the Presence of radiation with slip effect

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ABSTRACT

In this paper we study the effects of different parameters on steady magneto polar free convection flow of an incompressible fluid in the presence of thermal radiation and uniform magnetic field of strength B_0 through a porous medium in slip flow regime. The suction velocity is considered to be variable and the fluid is assumed to be gray; emitting absorbing but not scattering medium. The results obtained have been presented, separately in two basic fluids air (Pr=0.71) and water (Pr=7,), numerically through graphs to observe the effects of different parameters and the physical aspect of the problem. We observe that on decreasing Gr (Thermal Grashof number), skin friction drops in air but rises in water.

KEY WORDS: Polar fluid, Free convection, steady, Radiation, Mass transfer.

1. INTRODUCTION

The flow through a porous medium is a common occurrence in industrial environment and so the heat transfer problems of viscous incompressible fluid through a porous medium has attracted the interest of many research workers in view of its applications in geophysics, astrophysics, aerodynamics, boundary layer control and so on. Geindreau and Auriault [5] studied the Magneto hydrodynamic flows in porous media. Kandasamy et al. Prasad and Reddy [16] studied the radiation effects on an unsteady MHD convective heat and mass transfer flow past a semi infinite vertical permeable moving plate embedded in a porous medium. Revankar [19] studied free convection effects on flow past an impulsively started or oscillating infinite vertical plate.Sahoo et al.[20] have studied MHD unsteady free convection flow past an infinite vertical plate with constant suction and heat sink. Magyari et al.[10] have discussed analytic solution for unsteady free convection in porous media. Muthukumaraswamy et al. [13] studied the flow past an impulsively started isothermal vertical plate with variable mass diffusion. Kandasamy et al.[7] have studied thermophoresis and variable viscosity effects on MHD mixed convective heat and mass transfer past a porous wedge in the presence of chemical reaction.

Several researchers have investigated radiative effects on heat transfer in nonporous and porous medium utilizing the rosseland or other radiative flux model. Sanyal and Adhikari [21] studied the effects of radiation on MHD vertical channel flow. Raptis and Perdikis [18] studied the unsteady flow through a highly porous medium in presence of radiation.These fluids are known as polar fluids in the literature and are more general than ordinary fluids. The dynamics of polar fluid has attracted considerable attention during the last few decades because traditional Newtonian fluids cannot precisely describe the characteristics of fluid flow with suspended particles. Lukaszewicz [9] gave a detailed study of such fluids. Aero et al. [1] derived and solved the flow equations of the fluid in which angular velocity of the fluid particles was considered. The theory of micro polar fluid and thermo micro polar fluid has been developed by Eringen [3,4] and they can be used to explain the characteristics of certain fluids such as exotic lubricants, colloidal suspensions, or polymeric fluids, liquid crystals and animal blood. The micro polar fluids exhibit certain microscopic effects arising from local structure and micro rotation of fluid elements. Ogulu [14] studied the influence of radiation/absorption on unsteady free convection and mass transfer flow of a polar fluid in the presence of uniform magnetic field. Patil and Kulkarni [15] studied the effects of chemical reaction on free convective flow of a polar fluid through a porous medium in the presence of internal heat generation. Rahman and Sattar [17] studied MHD convective flow of a micro polar fluid past a continuously moving vertical porous plate in the presence of heat generation/absorption. Kim [8] studied unsteady MHD convection flow of a polar fluid past a vertical moving porous plate in a porous medium.

The particle at the surface has a finite tangential velocity and slips along the surface. The flow regime is called the slip flow regime and its effect cannot be neglected. The fluid slippage phenomenon at the solid boundaries appear in many applications such as micro channels or nano channels and in applications where a thin film of light oils is attached to the moving plates or when the surfaces are coated with special coating to minimize the friction between them. Mehmood and Ali [12] studied the effects of slip conditions on unsteady MHD oscillatory flow of a viscous fluid in planer channel. Derek et al. [2] studied apparent fluid slip at hydrophobic micro channel walls. Jain and Gupta [6] studied unsteady magneto polar free convection flow in slip flow regime with variable permeability and constant heat flux. Mansour et al. [11] studied fluctuating thermal and mass diffusion on unsteady MHD convection of a micro polar fluid through a porous medium past a vertical plate in slip flow regime. Recently Hoshiyar singh et al. [22] have study the effect of thermal diffusion under boundary condition.

In the present paper, the objective is to investigate the effects of radiation parameter (R), magnetic parameter (M), permeability parameter (K), slip parameter(h_1), rotational parameter (α_1), couple stress parameter (β_1) and thermal Grashof (Gr) on the unsteady free convective magneto polar flow with variable suction velocity and jump in temperature in a slip flow regime. The effects, on velocity (u), angular velocity (ω), temperature (θ), skin friction (C_f) and rate of heat transfer (N_u), are shown graphically.

2. FORMULATION OF THE PROBLEM

Consider the problem of a Steady two-dimensional, MHD free convective, heat and mass transfer flow with radiation of a polar fluid through a porous medium over a vertical plate with slip boundary condition for velocity field and jump for temperature field. A transfer magnetic field of strength is applied. The plate is moving in its own plane with velocity U_0.
The permeability of the porous medium is considered to be constant and the suction velocity is considered to be of the form:

$$V = -V_0$$

Under these conditions and using the Boussinesq's approximation, governing equations of the flow are given by:

Continuity equation:

$$\frac{\partial v}{\partial y} = 0 \qquad \dots (2.1)$$

Linear momentum:

$$V\frac{\partial u}{\partial y} = g\beta(T - T_\infty) + g\beta'(C - C_\infty) + (\nu + \nu_r)\frac{\partial^2 u}{\partial y^2} - \frac{\nu}{K}u + 2\nu_r\frac{\partial \omega}{\partial y} - \sigma\frac{{\beta_0}^2}{\rho}u \qquad \dots (2.2)$$

Angular momentum:

$$V\frac{\partial \omega}{\partial y} = \frac{\gamma}{I}\left(\frac{\partial^2 \omega}{\partial y^2}\right) \qquad \dots (2.3)$$

Energy equation:

$$V\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\left(\frac{\partial^2 T}{\partial y^2}\right) - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} \qquad \dots (2.4)$$

Where u, ω, T and C are velocity , angular velocity, temperature and concentration of the fluid particles. g is acceleration due to gravity, β is coefficient of volumetric expansion, β' is coefficient of species concentration expansion, $\rho, \nu, \nu_r, k, c_p, \sigma, K$ are density, kinematic viscosity, rotational kinematic viscosity, , specific heat at constant pressure, electrical conductivity, mass diffusivity and permeability of the porous medium respectively. I is a scalar constant equal to moment of inertia of unit mass and

$$\gamma = C_a + C_d$$

Where C_a and C_d are coefficient of couple stress viscosities.

The initials and boundary conditions are as follows:

$$y = 0: u = U_0 + L_1\frac{\partial u}{\partial y}, \omega = \frac{-1}{2}\frac{\partial u}{\partial y}, T = T_\omega,$$

$$y \to \infty: u \to 0, \omega \to 0, T \to T_\infty, \qquad \dots (2.5)$$

The local radiant for the case of an optically thin gray gas is expressed by:

$$\frac{\partial q_r}{\partial y} = -4a^*\sigma^*\left(T_\infty{}^4 - T^4\right) \qquad \ldots(2.6)$$

We assume that the temperature difference within the flow is sufficiently small such thatT^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting the higher order, thus:

$$T^4 \cong 4T_\infty{}^3 - 3T_\infty{}^4 \qquad \ldots(2.7)$$

by using equations (2.7) in (2.6) we obtain:

$$\frac{\partial q_r}{\partial y} = -16a^*\sigma^*(T_\infty - T) \qquad \ldots(2.8)$$

Where σ^* is Stefan-Boltzmann constant and a^* is absorption coefficient. On introducing the following non-dimensional quantities:

$$y^* = \frac{V_0 y}{\nu}, u^* = \frac{u}{V_0}, \qquad \omega^* = \frac{\nu\omega}{V_0 U_0}, \theta = \frac{T - T_\infty}{T_\omega - T_\infty}, E_c = \frac{v_0^2}{C_p(T'_\omega - T'_\infty)}$$

equations (2.2) to (2.4), substituting (2.8) in (2.4), in non-dimensional form after dropping the asteriks are :

$$\frac{\partial u}{\partial y} = \theta G_r + cG_c + (1+\alpha_1)\frac{\partial^2 u}{\partial y^2} + 2\alpha_1\frac{\partial \omega}{\partial y} - \left[M^2 + \frac{1}{K}\right]u \qquad \ldots(2.9)$$

$$\frac{\partial \omega}{\partial y} = \frac{1}{\beta_1}\frac{\partial^2 \omega}{\partial y^2} \qquad \ldots(2.10)$$

$$\frac{\partial \theta}{\partial y} = \frac{1}{p_r}\frac{\partial^2 \theta}{\partial y^2} - \frac{R}{p_r}\theta \qquad \ldots(2.11)$$

With corresponding boundary conditions as:

$y = 0: u = h_1\frac{\partial u}{\partial y}, \omega = \frac{-1}{2}\frac{\partial u}{\partial y}, \theta = 1,$ $y \to \infty, u \to 0, \omega \to 0, \theta \to 0,$ ... (2.12)

Where

$$K^* = \frac{V_0{}^2 K}{\nu^2}(\text{Permeability parameter }), M^2 = \frac{\sigma\beta_0{}^2\nu}{\rho V_0{}^2}(\text{Magnetic parameter })$$

$h_1 = \frac{L_1 V_0}{\nu}$ (Velocity slip parameter), $E_c = \frac{v_0^2}{C_p(T'_\omega - T'_\infty)}$ (Ecart number)

$$G_r = \frac{\nu g\beta(T_\omega - T_\infty)}{V_0{}^2 U_0}(\text{Thermal Grashof number }), \alpha_1 = \frac{\nu_r}{\nu}(\text{Rotational viscosity parameter })$$

$$\beta_1 = \frac{I\nu}{\gamma} \quad (\text{ Couple stress parameter}), P_r = \frac{\mu c_p}{k}(\text{Prandtl number })$$

$R = \frac{16a^*\sigma^*\nu^2 T_\infty^3}{V_0^2 k}$ (Radiation parameter), $L_1 = \left(\left(\frac{2-m_1}{m_1}\right)L\right)$, m_1 being the maxwell's reflexion coefficient and L is the free path.

3. SOLUTION OF THE PROBLEM

We reduce the system of partial differential equations to ordinary differential equations by assuming:

$$f(y) = f_0(y) + E_c f_1(y) \qquad \ldots (3.1)$$

Where f stands for u, ω, θ, and c.

Substituting equation (3.1) in equations (2.9) to (2.12) and equating the like terms, we get:

$$u_0'' + \frac{1}{(1+\alpha_1)} u_0' - \frac{\left(M^2 + \frac{1}{K}\right)}{(1+\alpha_1)} u_0 = \frac{1}{(1+\alpha_1)}(-\theta_0 G_r - C_0 G_c - 2\alpha_1 \omega_0') \qquad \ldots (3.2)$$

$$u_1'' + \frac{1}{(1+\alpha_1)} u_1' - \frac{\left(M^2 + \frac{1}{K}\right)}{(1+\alpha_1)} u_1 = \frac{1}{(1+\alpha_1)}(-\theta_1 G_r - C_1 G_C - 2\alpha_1 \omega_1') \qquad \ldots (3.3)$$

$$\omega_0'' + \beta_1 \omega_0' = 0 \qquad \ldots (3.4)$$

$$\omega_1'' + \beta_1 \omega_0' + \beta_1 \omega_1 = -\beta_1 \qquad \ldots (3.5)$$

$$\theta_0'' + P_r \theta_0' - R\theta_0 = 0 \qquad \ldots (3.6)$$

$$\theta_1'' + P_r \theta_1' - (R - P_r)\theta_1 = -P_r \theta_0' \qquad \ldots (3.7)$$

Here primes denote differentiation with respect to y.

The corresponding boundary conditions can be written as:

$$y = 0: u_0 = 1 + h_1 u_0', \omega_0 = -\frac{1}{2} u_0', \ \theta_0 = 1, , u_1 = 1 + h_1 u_1',$$

$$\omega_1 = -\frac{1}{2} u_1', \theta_1 = 0$$

$$y \to \infty: u_0 \to 0, \ \omega_0 \to 0, \ \theta_0 \to 0, \quad u_1 \to 0, \quad \omega_1 \to 0, \ \theta_1 \to 0, \qquad \ldots (3.8)$$

Solving equations (3.2) to (3.7) with satisfying boundary conditions (3.8), and substituting back in (3.1), we get:

$$u = \{m_7 e^{x_8 y} + b_1 e^{x_3 y} + b_2 e^{-\beta_1 y}\} + E_c\{m_8 e^{x_7 y} + b_3 e^{x_8 y} + b_{10} e^{x_3 y} + b_{12} e^{-\beta_1 y} + b_6 e^{x_5 y} + b_7 e^{x_7 y} + b_8 e^{x_1 y}\},$$

$$W = m_1 e^{-\beta_1 y} + E_c\{m_2 e^{x_1 y} + b_{13} e^{-\beta_1 y}\},$$

$$\theta = m_3 e^{x_3 y} + E_c\{m_4 e^{x_5 y} + b_{14} e^{x_3 y}\},$$

4. SKIN FRICTION

Knowing the velocity field, the non dimensional skin friction (C_f)at the plate is given by:

$$C_f = \frac{\tau_\omega}{\rho U_0 V_0}$$

$$C_f = (1+\alpha_1)[(m_7x_7 + b_1x_3 - b_2\beta_1) + E_c(m_8x_7 + b_3x_5 + b_7x_3 - b_8\beta_1 + b_5x_7 + b_6x_1] \quad \ldots(4.1)$$

5. NUSSELT NUMBER

Another important physical parameter of interest viz. Nusselt number in dimensionless form is:

$$N_u = \left(\frac{\partial\theta}{\partial y}\right)_{y=0}$$

$$N_u = -[m_3x_3 + E_c(m_4x_4 + b_{14}x_3)] \quad \ldots(5.1)$$

Where,

$X_1 = \frac{-\beta_1-\sqrt{\beta_1}^2-4\beta_1}{2}$, $X_2 = \frac{-\beta_1+\sqrt{\beta_1}^2-4\beta_1}{2}$,

$X_3 = \frac{-P_r-\sqrt{P_r^2+4R}}{2}$, $X_4 = \frac{-P_r+\sqrt{P_r^2+4R}}{2}$,

$X_5 = \frac{-P_r-\sqrt{P_r^2+4(R-P_r)}}{2}$, $X_6 = \frac{-P_r+\sqrt{P_r^2+4(R-P_r)}}{2}$,

$X_7 = \frac{-1-\sqrt{1}+\left(M^2+\frac{1}{K}\right)(1+\alpha_1)}{2(1+\alpha_1)}$, $X_8 = \frac{-1+\sqrt{1}+\left(M^2+\frac{1}{K}\right)(1+\alpha_1)}{2(1+\alpha_1)}$,

$X_9 = \frac{-1-\sqrt{1}+4\left(M^2+\frac{1}{K}\right)(1+\alpha_1)}{2(1+\alpha_1)}$, $X_{10} = \frac{-1+\sqrt{1}+4\left(M^2+\frac{1}{K}\right)(1+\alpha_1)}{2(1+\alpha_1)}$,

$$b_1 = \frac{-m_3G_r}{(1+\alpha_1)(x_3-x_8)(x_3-x_{10})},$$

$b_2 = \frac{2\alpha_1 m_1\beta_1}{(1+\alpha_1)(\beta_1+x_8)(\beta_1+x_{10})}$, $b_3 = \frac{-m_7x_8}{(1+\alpha_1)(x_8-x_{11})(x_8-x_{12})}$, $b_4 = \frac{-b_1x_3}{(1+\alpha_1)(x_3-x_{11})(x_3-x_{12})}$,

$b_5 = \frac{b_3\beta_1}{(1+\alpha_1)(\beta_1+x_{11})(\beta_1+x_{12})}$,

$$b_6 = \frac{-m_4G_r}{(1+\alpha_1)(x_5-x_{11})(x_5-x_{12})},$$

$b_7 = \frac{Pm_3x_3G_r}{(1+\alpha_1)(x_3-x_5)(x_3-x_6)(x_3-x_{11})(x_3-x_{12})}$, $b_8 = \frac{-2\alpha_1 m_2x_1}{(1+\alpha_1)(x_1-x_{11})(x_1-x_{12})}$,

$b_9 = \frac{2m_1\alpha_1\beta_1{}^3}{(1+\alpha_1)(\beta_1+x_1)(\beta_1+x_2)(\beta_1+x_{11})(\beta_1+x_{12})}$, $b_{10} = b_5 + b_9$, $b_{11} = b_6 + b_{11}$, $b_{12} = b_7 + b_{11}$,

$b_{13} = \frac{m_1\beta_1{}^2}{(1+\alpha_1)(\beta_1+x_1)(\beta_1+x_2)}$, $b_{14} = \frac{-P_rm_3x_3}{(x_3-x_5)(x_3-x_6)}$,

$m_1 = -\frac{1}{2}[m_7x_7 - b_1x_3 - b_3\beta_1]$,

$m_2 = -b_{13} - \frac{1}{2}[m_8x_9 + b_3 + x_7 + b_{10}x_3 - b_{12}\beta_1 + b_6x_5 + b_7x_7 + b_8x_1]$,

$m_3 = \frac{1}{(1-x_3)}$, $m_4 = \frac{b_{14}(1-x_3)}{(x_5-1)}$, $m_5 = 1$, $m_6 = \frac{(1-b_1)(1-x_3h_1)-b_2-b_3(1+h_1\beta_1)}{(1-h_1x_{11})}$,

6. RESULTS AND DISCUSSION

In order to understand the physical importance of the flow and to find the effects of different parameters, calculations have been carried out for velocity, angular velocity, temperature ,concentration ,skin friction and the rate of heat transfer, for different values of the permeability parameter (k), the magnetic parameter (M), the thermal Grashof number (G_r) the velocity slip parameter(h_1),the rotational viscosity parameter(α_1), the couples stress parameter(β_1), the Prandtlnumber(P_r),. Results are also shown for particular cases of no slip ($h_1 = 0$).considered to be fixed.

In figure 1 R=0.2.It is observed that on decreasing k, and G_r angular velocity increases whereas on decreasing $M, h_1, \alpha_1$and $\beta_1$angular velocity decreases. Cases for no slip ($h_1 = 0$)also observed for both the basic fluids. In figure 2, temperature profiles are plotted against y for both air ($P_r = 0.71$)) and water($P_r = 7$). We observe that as R decrease. Also for negative of radiation (absorption) temperature rises.

In figure 3 skin friction is plotted against K for both the basic fluids air ($P_r = 0.71$) and water ($P_r = 7$) respectively. We noticethat for both air and water, on decreasing M and h_1 skin friction increases where as on decreasing R skin friction drops. Results differ for h_1, α_1, β_1 andG_r. For air, skin friction increases on the other hand decreasing α_1, β_1 and$G_r$drops the skin friction. For water the skin friction to drop where as decrease in$\alpha_1, \beta_1$and G_r increases the skin friction. Also for negative of radiation (absorption) skin friction drops further.

Nusselt number is plotted against t for both basic fluids air ($P_r = 0.71$), and water($P_r = 7$)in figure 4. We notice that when R decreases Nusselt number decreases on the other hand decreases in h_1 tends the Nusselt number to rise. Also for negative of radiation (absorption) Nusselt number falls further.

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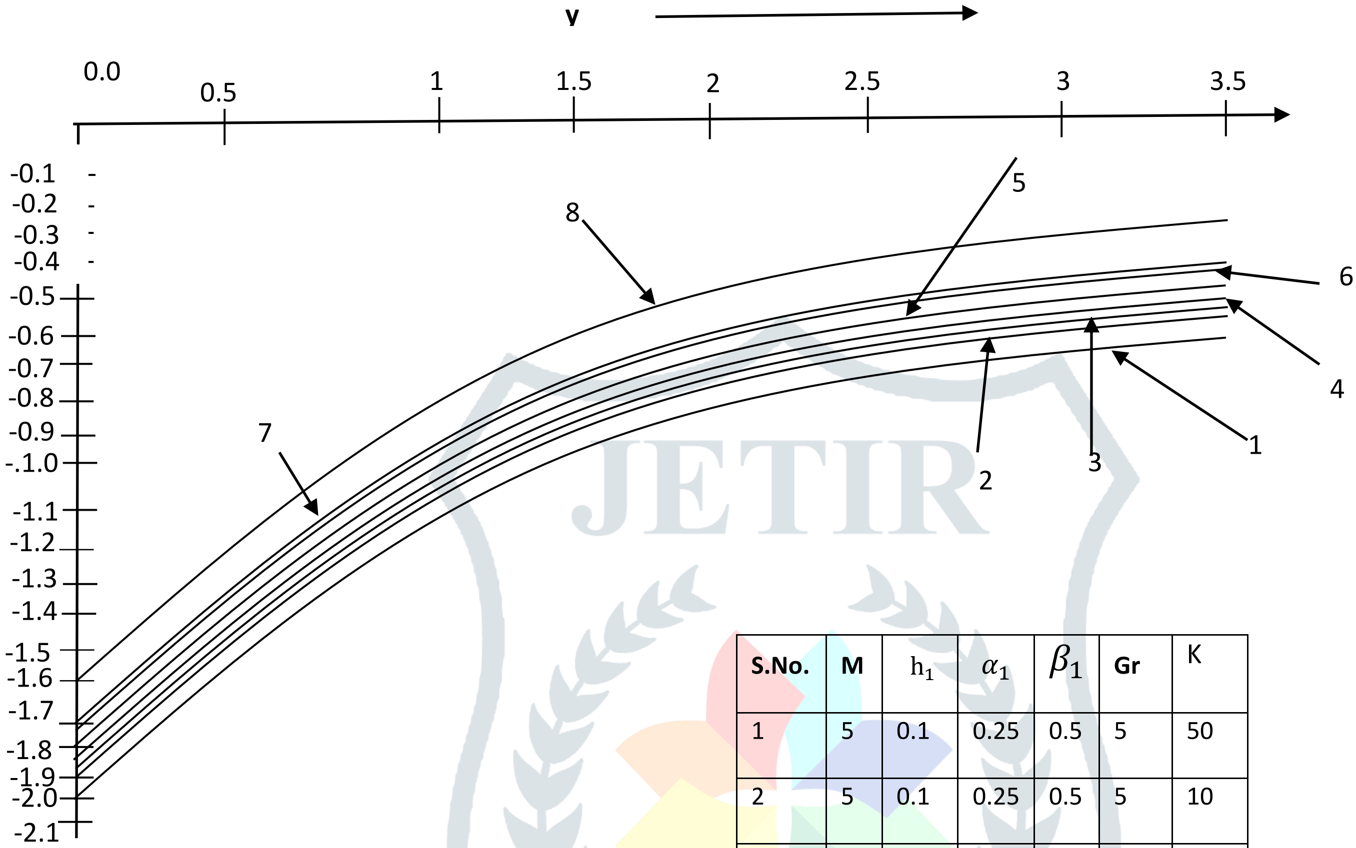
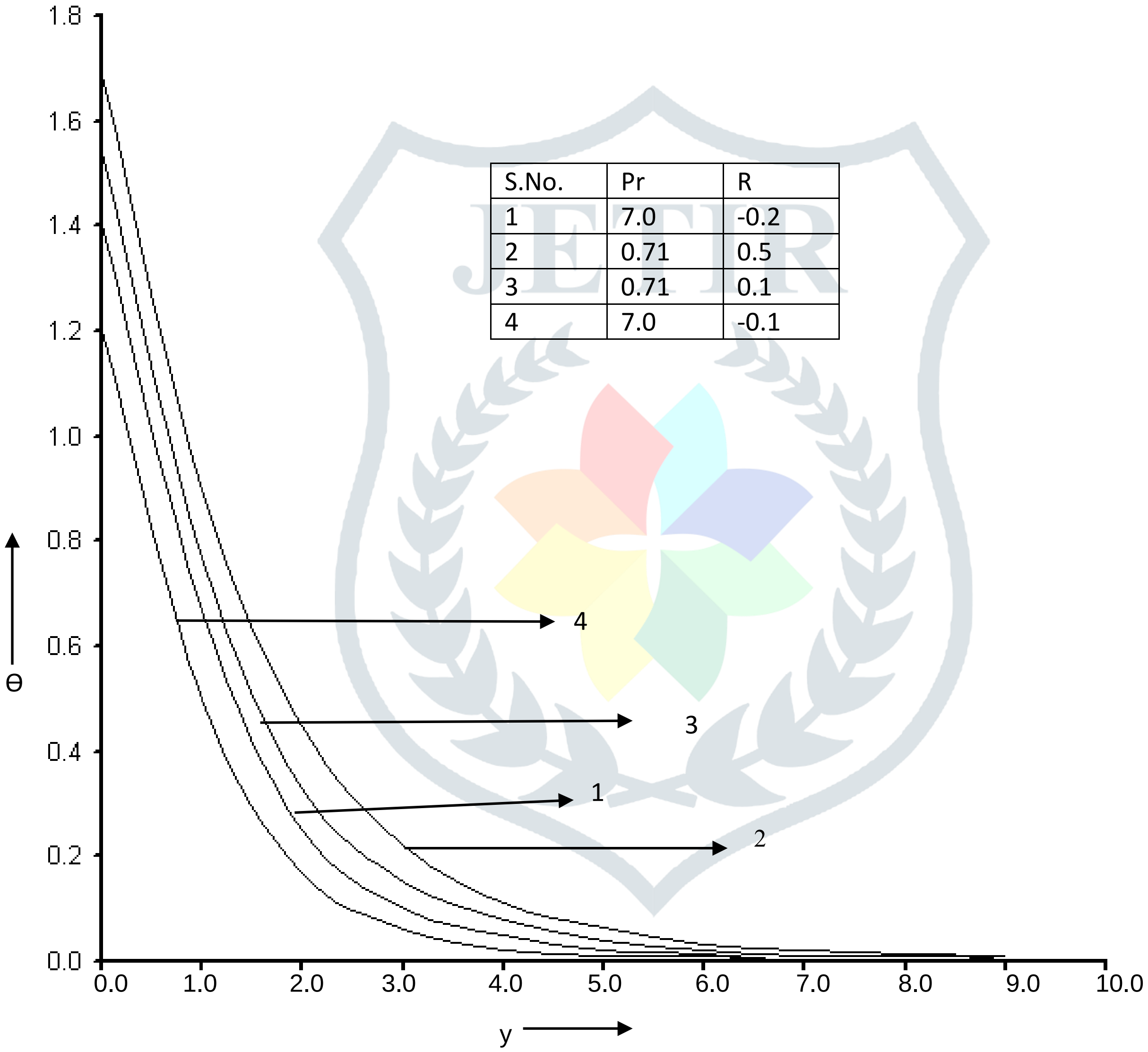


S.No.	M	h_1	α_1	β_1	Gr	K
1	5	0.1	0.25	0.5	5	50
2	5	0.1	0.25	0.5	5	10
3	8	0.1	0.25	0.5	5	50
4	5	0.05	0.25	0.5	5	50
5	5	0	0.25	0.5	5	50
6	5	0.1	0.25	0.5	5	50
7	5	0.1	0.25	0.5	5	50
8	5	0.1	0.05	0.5	5	50

Figure 1: Angular velocity profiles plotted against y for different values of K, M , $h_1, \alpha_1, \beta_1, G_r$.



S.No.	Pr	R
1	7.0	-0.2
2	0.71	0.5
3	0.71	0.1
4	7.0	-0.1

Figure 2: The Temperature profiles plotted against y for different values of R, p_r .

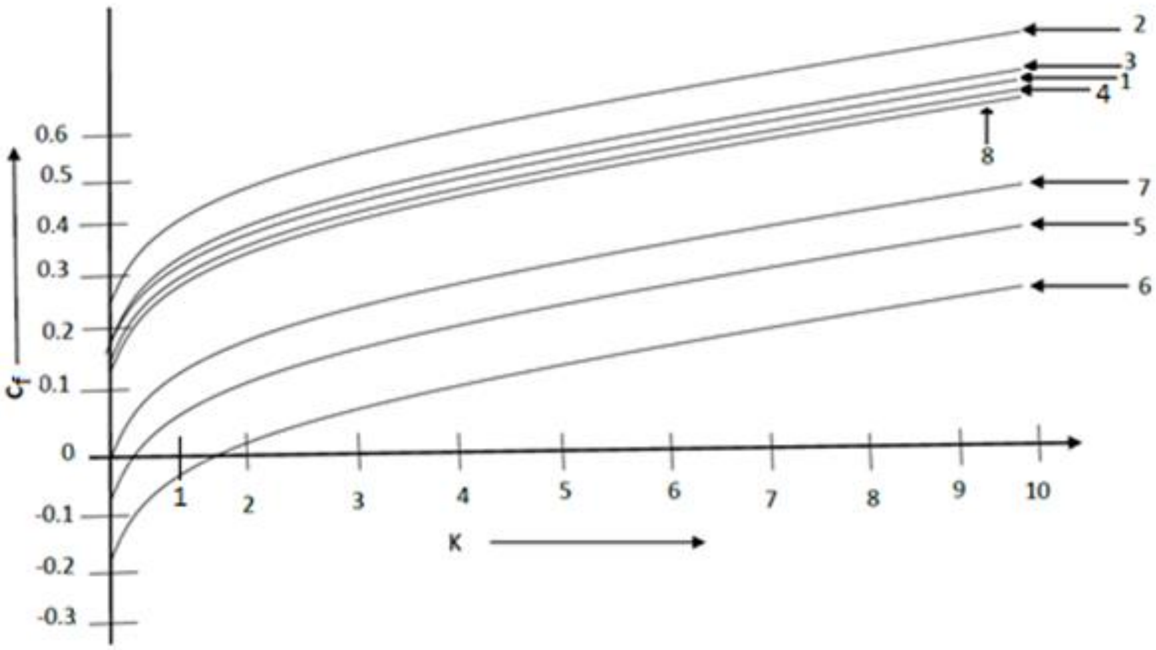


Figure 3. :Skin-friction water plotted against K for different values of M, h_1, α_1, β_1, G_r and R .

$P_r = 7$						
S.No.	M	h_1	α_1	β_1	Gr	R
1	5	0.1	0.25	0.5	5	0.05
2	4	0.1	0.25	0.5	5	0.05
3	5	0.05	0.25	0.5	5	0.05
4	5	0	0.25	0.5	5	0.05
5	5	0.05	0.25	0.5	5	0.05
6	5	0	0.25	0.5	5	0.05
7	5	0.1	0.1	0.5	5	0.05
8	5	0.1	0.25	0.5	5	0.05

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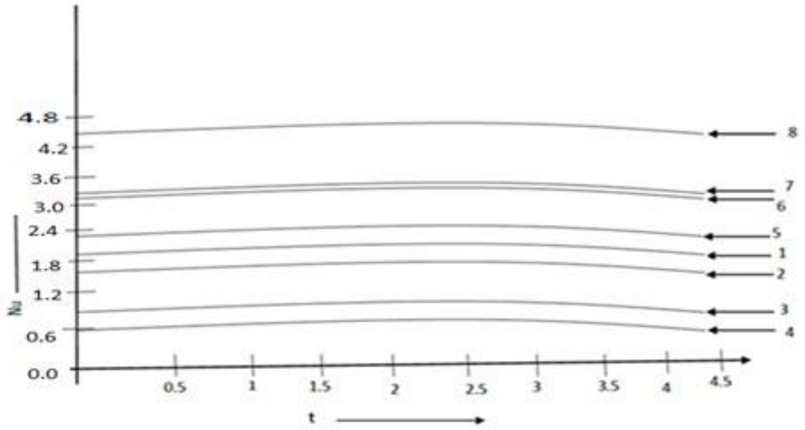

S.No.	P_r	h_1	R
1	0.71	0.1	0.2
2	0.71	0.1	0.05
3	0.71	0.1	0
4	0.71	0.1	-0.05
5	0.71	0.1	0.2
6	0.71	0	0.2
7	7	0.1	0.2
8	7	0.1	0.05

Figure 4 :Nusselt number plotted against t for different value of h , P_r ,R.