SUPPLY CHAIN INVENTORY MODEL FOR SINGLE VENDOR SINGLE BUYER WHEN SHORTAGES ARE ALLOWED TO BUYER

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***Abstract**:* The collaborative inventory models are studied where single buyer and single vendor for deteriorating items and demand follows exponential distribution function depends on time when shortages are permissible for buyer. The supply chain inventory model is constructed to maximize profit for optimal cycle time. Moreover, joint inventory model also determine the optimal profits when buyer and vendor take joint decision. The model is illustrated with numerical examples and observed that both buyer and vendor earn significant profit in supply chain inventory system.
***Keywords*: Supply Chain, Optimal Strategy, Deterioration, Shortages, Exponential Demand**

I. INTRODUCTION

Supply chain deliberates the strategy to compose independent members such as suppliers, buyers, vendors, retailers etc, each with its own objectives functions and individual costs. Therefore, a collaboration mechanism is required to progress the whole system's performance. This system make the decision to fulfill the satisfactory results under the condition that none of the members' profit is worse compared with the decentralized plan and system's profit achieves its maximum as in integrated plan. However, the supply chain management is more important, when the integrated inventory undergo decay or deterioration. Deterioration is defined as vary, spoil, decay, inefficiency or losing the original rate in a product that converts in the declining convenience from the original one. As well as integrated system is also affected by the shortages occur during supply chain from buyer's or vendor's prospective.

Shah et al. (2010) investigated the deteriorating items integrated inventory strategy under supply chain when demand is quadratic. The model considers shortages to retailer, partial backlogging, and multiple deliveries and observed that the several orders decreases the total cost at integrated supply chain. Yu (2010) developed a collaborative inventory for buyer and supplier subject to maximize the overall profits of the entire coordination when shortages are allowed for buyer's prospective, fixed replenishment rate deteriorating inventory system and price sensitive demand. Rad et al. (2011) considered collaborative strategy to developed inventory model for buyer and supplier when shortages are permitted for the buyer with shipment strategy and determined the optimal value of backorder quantity and number of delivered items to minimize integrated costs of buyer and seller. Bhowmick and Samanta (2012) discussed the integrated inventory by considering demand is dependent on price by assuming partial backlogged items with nonlinear backorder rate and decreasing amount of shortages to maximize the expected total profit. Kaur and Sharma (2012) discussed the supply chain model for buyer and vendor when shortages are allowed with partial backlogging with references of interest charged and interest earned. Roy et al. (2012) derived joint inventory model for producer and buyer for constant demand where shortages and backlogged items are considered for buyer and determined integrated costs of buyer and producer were minimum for optimum production rate and shortage period for both cases. Seifert et al. (2012) investigated the models when no collaboration between various systems, collaboration between two members only of the entire supply chain. In this constructed supply chain model supplier and retailer preferred to take decision without coordination with the producer, this contradiction explained the status of price of only agreements. Hsu and Hsu (2013) derived supply chain model when buyer's and vendor's collaboration is considered with defective manufacture procedure where the defective proportion of each unit of manufacturing batch is stochastic and shortages are permissible for completely backordered to minimize the expected yearly collaborative cost. Singh et al. (2013) constructed collaborative inventory model for single buyer, supplier and vendor under integrated production procedure when shortages were permitted to buyer, stock dependent demand rate and determined the optimal cycle time to minimize joint costs of supply chain inventory. Hematyar and Chaharsooghi (2014) derived the inventory model under supply chain system for single retailer and producer with customer returns and insurance contract by considering shortages by retailer. Collaborative inventory model found that uncertain high demand and to reduce order quantity products returned to retailer. Singh et al. (2014) discussed three echelon supply chain problem for inventory single supplier, single manufacturer and multiple buyers under inflation by allowing partial backlogging for retailers where deterioration over the time follows a two-parameter weibull distribution. Chen (2017) developed dynamic collaborative inventory model for single buyer and supplier, discussed two different business strategies such as retailer's inventory for price mode and vendor's inventory for consignment agreement case under the condition of shortages and when shortages are fully backlogged. Therefore, we consider the deteriorating items inventory when buyer has shortages during the supply chain.

II. NOTATIONS

$D(t) = a e^{bt}$, exponential demand function of time, where a > 0, 0<b<1

$I_b(t)$ = Buyer's inventory level at any instant of time t

$I_v(t)$ = Vendor's inventory level at any instant of time t

A_b = Buyer's ordering cost per order

A_v = vendor ordering cost per order

C_b = Purchasing cost of buyer per unit

C_2 = Shortage cost of buyer per unit

θ = Buyer's deterioration rate

x_b = Fixed holding cost of buyer
y_b = Varying holding cost of buyer
x_v = Fixed holding cost of vendor
y_v = Varying holding cost of vendor
p = Buyer's selling price per unit
n = Number of orders placed by buyer.
TP_b = Total profit of buyer
TP_v = Total profit of vendor
TP = Joint profit of vendor and buyer (TP_b+TP_v)
$t_1 = v_1*T/n$
t_2 = Time when inventory level reaches to zero (Decision variable)
T = Decision variable of vendor's cycle time

III. ASSUMPTIONS

Under the mentioned assumptions supply chain inventory models are developed:

1. Product's demand is decreasing function of exponential distribution depending on time.
2. Single buyer and single vendor are considered.
3. Shortages are allowed for buyer.
4. Lead-time is zero.
5. Deteriorated units cannot repair or replace throughout the cycle time and deterioration is dependent on time for buyer's inventory.
6. Time varying holding cost is considered for buyer and vendor.

IV. MATHEMATICAL MODEL:

Let inventory level be $I_b(t)$ at time t ($0 \le t \le T/n$) and buyer's inventory is shown in Figure-1.

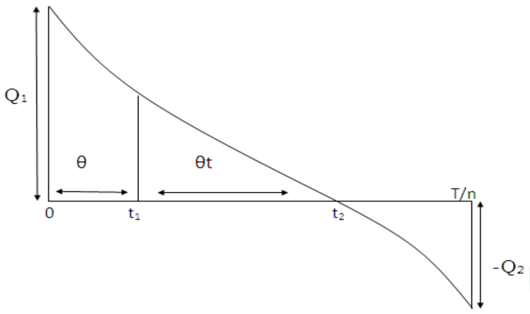

Figure-1

To develop inventory model we discuses two cases where vendor and buyer take decision without collaboration and with collaboration strategy.

Inventory level of buyer and vendor are depleted by exponential demand. The differential equations for vendor and buyer are given by:

$$\frac{dI_b(t)}{dt} + \theta I_b(t) = -a e^{bt} \quad ; \quad 0 \le t \le t_1 \tag{1}$$

$$\frac{dI_b(t)}{dt} + \theta t I_b(t) = -a e^{bt} \quad ; \quad t_1 \le t \le t_2 \tag{2}$$

$$\frac{dI_b(t)}{dt} = -a e^{bt} \quad ; \quad t_2 \le t \le \frac{T}{n} \tag{3}$$

$$\frac{dI_v(t)}{dt} = -a e^{bt} \quad ; \quad 0 \le t \le T \tag{4}$$

with the boundary conditions:

$$I_b(0) = Q_1, \quad I_b\left(\frac{t_2}{n}\right) = 0, \quad I_b\left(\frac{T}{n}\right) = -Q_2 \quad and \quad I_v(T) = 0$$

Their solutions are given by

$$I_b(t)=Q_1(1-\theta t)-a\left(t+\frac{b}{2}t^2+\frac{\theta}{2}t^2+\frac{b\theta}{3}t^3\right)+a\theta t\left(t+\frac{b}{2}t^2\right) \quad (5)$$

$$I_b(t)=a\left\{\begin{matrix}(t_2-t)+\frac{b}{2}(t_2^2-t^2)+\frac{\theta}{6}(t_2^3-t^3)\\ +\frac{b\theta}{8}(t_2^4-t^4)-\frac{\theta t^2}{2}(t_2-t)-\frac{b\theta t^2}{4}(t_2^2-t^2)\end{matrix}\right\} \quad (6)$$

$$I_b(t)=a\left\{(t_2-t)+\frac{b}{2}(t_2^2-t^2)\right\} \quad (7)$$

$$I_v(t)=a\left\{(T-t)+\frac{b}{2}(T^2-t^2)\right\} \quad (8)$$

(By neglecting higher power of θ)

Substituting $t=t_1$ in equations (5) and (6) and simplifying, we get

$$Q_1=\frac{1}{(1-\theta t_1)}\left\{\begin{matrix}a\left(\frac{\theta}{2}t_1^2+\frac{b\theta}{3}t_1^3\right)-a\theta t_1\left(t_1+\frac{b}{2}t_1^2\right)\\ +a\left(\begin{matrix}t_2+\frac{b}{2}t_2^2+\frac{\theta}{6}(t_2^3-t_1^3)\\ +\frac{b\theta}{8}(t_2^4-t_1^4)-\frac{\theta}{2}t_1^2(t_2-t_1)\\ -\frac{b\theta}{4}t_1^2(t_2^2-t_1^2)\end{matrix}\right)\end{matrix}\right\} \quad (9)$$

Putting the value of Q_1 in equation (5) we get

$$I_b(t)=\frac{(1-\theta t)}{(1-\theta t_1)}\left\{\begin{matrix}a\left(\frac{\theta}{2}t_1^2+\frac{b\theta}{3}t_1^3\right)-a\theta t_1\left(t_1+\frac{b}{2}t_1^2\right)\\ +a\left(\begin{matrix}t_2+\frac{b}{2}t_2^2+\frac{\theta}{6}(t_2^3-t_1^3)\\ +\frac{b\theta}{8}(t_2^4-t_1^4)-\frac{\theta}{2}t_1^2(t_2-t_1)\\ -\frac{b\theta}{4}t_1^2(t_2^2-t_1^2)\end{matrix}\right)\end{matrix}\right\}$$

$$-a\left(t+\frac{b}{2}t^2+\frac{\theta}{2}t^2+\frac{b\theta}{3}t^3\right)+a\theta t\left(t+\frac{b}{2}t^2\right) \quad (10)$$

Substituting $t=\frac{T}{n}$ in equation (7) and simplifying, we get

$$Q_2=a\left\{\left(\frac{T}{n}-\frac{t_2}{n}\right)+\frac{b}{2}\left(\frac{T^2}{n^2}-\frac{t_2^2}{n}\right)\right\} \quad (11)$$

V. BUYER'S RELEVANT COSTS:

Holding Cost:

$$HC_b=n\left\{x_b\left(\int_0^{t_1}I_b(t)dt+\int_{t_1}^{t_2}I_b(t)dt\right)+y_b\left(\int_0^{t_1}t\,I_b(t)dt+\int_{t_1}^{t_2}t\,I_b(t)dt\right)\right\} \quad (12)$$

Deterioration Cost:

$$DC_b=nC_b\theta\left\{\int_0^{t_1}I_b(t)dt+\int_{t_1}^{t_2}tI_b(t)dt\right\} \quad (13)$$

Shortage Cost:

$$SC_b=-nC_2\int_{t_2}^{\frac{T}{n}}I_b(t)dt \quad (14)$$

Ordering Cost

$$OC_b=nA_b \quad (15)$$

Sales Revenue:

$$SR_b = n\,p \int_0^{\frac{T}{n}} D(t)dt$$

$$= n\,p \int_0^{\frac{T}{n}} a\,e^{bt} t$$

$$= n\,p\,a\left(\frac{T}{n} + \frac{b}{2}\frac{T^2}{n^2}\right) \qquad (16)$$

(By neglecting higher power of b)

Total Profit:

$$TP_b = \frac{1}{T}\left[SR_b - HC_b - DC_b - SC_b - OC_b\right] \qquad (17)$$

VI. VENDOR'S RELEVANT COSTS:

Holding Cost:

$$HC_v = x_v\left\{\int_0^T I_v(t)dt - n\left(\int_0^{t_1} I_b(t)dt + \int_{t_1}^{t_2} I_b(t)dt + \int_{t_2}^{\frac{T}{n}} I_b(t)dt\right)\right\}$$

$$+ y_b\left\{\int_0^T t\,I_v(t)dt - n\left(\int_0^{t_1} t\,I_b(t)dt + \int_{t_1}^{t_2} t\,I_b(t)dt + \int_{t_2}^{\frac{T}{n}} t\,I_b(t)dt\right)\right\} \qquad (18)$$

Ordering cost:

$$OC_v = A_v \qquad (19)$$

Sales Revenue:

$$SR_v = C_b \int_0^T D(t)dt$$

$$= C_b \int_0^T a\,e^{bt} dt$$

$$= C_b\,a\left(T + \frac{b}{2}T^2\right) \qquad (20)$$

(By neglecting higher power of b)

Total Profit:

$$TP_v = \frac{1}{T}\left[SR_v - HC_v - OC_v\right] \qquad (21)$$

VII. SITUATION-I: BUYER AND VENDOR TAKE DECISION WITHOUT COLLABORATION:

Here the buyer and vendor make decision without collaboration

Buyer's maximum profit TP_b can be determined by following conditions:

$$\frac{dTP_b}{d\,t_2} = 0 \text{ and } \frac{dTP_b}{dT_b} = 0 \text{ where } T_b = \frac{T}{n} \qquad (22)$$

Provided it satisfies the condition

$$\begin{vmatrix} \frac{\partial^2 TP_b}{\partial t_2^2} & \frac{\partial^2 TP_b}{\partial t_2 \partial T} \\ \frac{\partial^2 TP_b}{\partial T \partial t_2} & \frac{d^2 TP_b}{\partial T^2} \end{vmatrix} > 0 \qquad (23)$$

This solution (n, t_2 ,T) maximizes TP_v

Then the total profit without collaboration is given by;

TP = max(TP_b + TP_v) (24)

VIII. SITUATION-II: BUYER AND VENDOR TAKE DECISION WITHOUT COLLABORATION

Here the buyer and vendor make decision without collaboration

Moreover, the optimum values of T must satisfy the following condition

$\frac{dTP}{dt_2}=0$ and $\frac{dTP}{dT}=0$ for T provided it satisfies the condition (25)

$$\begin{vmatrix} \frac{\partial^2 TP}{\partial t_2^2} & \frac{\partial^2 TP}{\partial t_2 \partial T} \\ \frac{\partial^2 TP}{\partial T \partial t_2} & \frac{d^2 TP}{\partial T^2} \end{vmatrix} > 0 \tag{26}$$

where total profit (TP) with collaboration is given by;

$$TP = TP_b + TP_v \tag{27}$$

IX. NUMERICAL EXAMPLE

In order to illustrate our proposed model, we considering a = 1000, b = 0.05, x_b = 10, y_b = 0.03, x_v = 8, y_v = 0.01, A_b = 140, A_v = 1500, C_b = 35, C_2=15, p = 45, v_1 = 0.4 in appropriate units. The optimal values of T and profits for buyer and vendor are given in Table-1. The second order conditions given in equation (23) and equation (26) are also satisfied. The graphical representations of the concavity of the profits for independent and joint profits are also shown (Figure-2 to 5).

The optimal total profit TP = Rs. 74614.21at n=3 for buyer's profit TP_b* = Rs. 43896.70, T* = 0.7564, t_2^* = 0.4311and TP_v= 30717.51when buyer and vendor take decision without collaboration. While when buyer and vendor take joint decision then the optimal total profit TP^* = Rs. 75125.89 at n=1, T^* = 0.6548, t_2^* = 0.5742, with buyer's profit TP_b = Rs. 42495.90 and TP_v= 32629.99.

Table-1
Without and With collaboration decisions

	Without	With
n	3	1
t_2	0.4311	0.5742
T	0.7564	0.6548
Buyer's Profit	43896.70	42495.90
Vendor's Profit	30717.51	32629.99
Total Profit	74614.21	75125.89

Table-2
Sensitive Analysis

	Para-meters	Without Collaboration			With Collaboration		
		TPb	TPv	TP	TPb	TPv	TP
-20%	a	35012.38	24161.58	59173.96	33768.72	25855.59	59624.31
-10%		39452.91	27433.52	66886.43	38129.95	29237.11	67367.06
10%		48343.16	34012.17	82355.33	46873.18	36019.27	82892.44
20%		52792.04	37316.24	90108.28	51253.87	39417.55	90671.42
-20%	θ	43910.35	30724.51	74634.85	42494.91	32705.04	75199.95
-10%		43903.45	30720.95	74624.41	42496.81	32664.74	75161.56
10%		43889.96	30713.82	74603.78	42499.54	32587.09	75086.63
20%		43889.96	30713.82	74603.78	42499.54	32587.09	75086.63
-20%	A_b	44013.83	30733.97	74747.79	42559.09	32607.66	75166.75
-10%		43953.63	30732.17	74685.80	42528.76	32616.46	75145.23
10%		43842.44	30693.7	74536.14	42468.46	32633.98	75102.44
20%		43790.60	30670.47	74461.06	42438.47	32642.70	75081.17
20%	x_b	43954.30	30813.18	74767.49	42393.91	33307.40	75701.32
-10%		43902.96	30727.32	74630.28	42442.81	32941.63	75384.44
10%		43817.35	30673.21	74490.56	42500.12	32365.23	74865.35
20%		43781.17	30644.72	74425.90	42517.25	32131.67	74648.92
-20%	A_v	43896.66	31151.84	75048.50	42690.09	32914.07	75604.17
-10%		43896.66	30915.61	74812.27	42592.57	32765.65	75358.22
10%		43896.66	30518.99	74415.65	42407.62	32491.71	74899.33
20%		43896.66	30320.68	74217.34	42320.71	32362.89	74683.60
-20%	x_v	43896.66	31309.41	75206.07	42732.66	32558.76	75291.42
-10%		43896.66	31013.35	74910.01	42626.00	32572.28	75198.28
10%		43896.66	30421.24	74317.90	42350.66	32716.43	75067.09
20%		43896.66	30175.04	74071.70	42181.84	32845.69	75027.53

Sensitive analysis is carried out by changing the values of given parameters a, A_b, A_v, x_b, x_v and θ respectively, one parameter at a time and the reaming parameters are kept constant. Based on the results of Table-2 we observe that total profit increases when buyer and vendor take joint decision instead as compared to independent decision. When a increases/decreases then total profit will increase/decrease, while if $A_{b,}$ x_b, x_v, A_v and θ increase/decrease then total profit will decrease/increase in independent and joint decision.

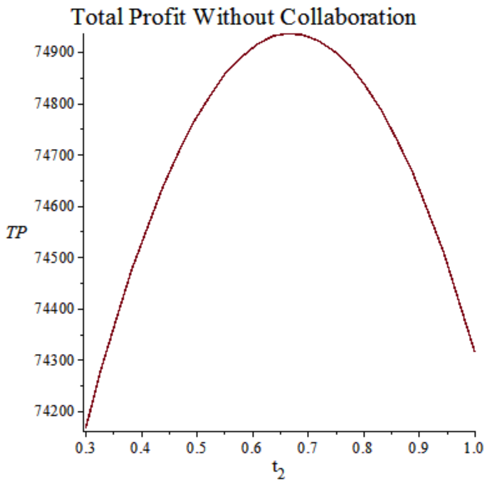

Figure-2

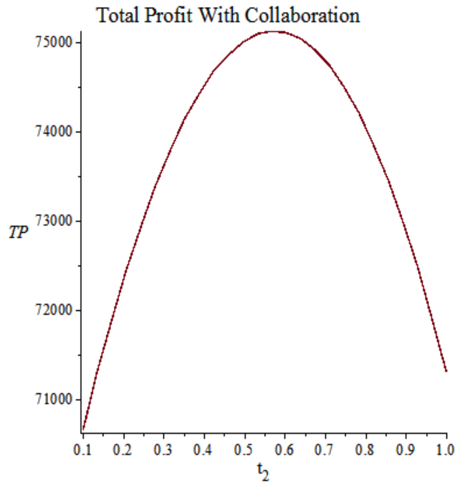

Figure-3

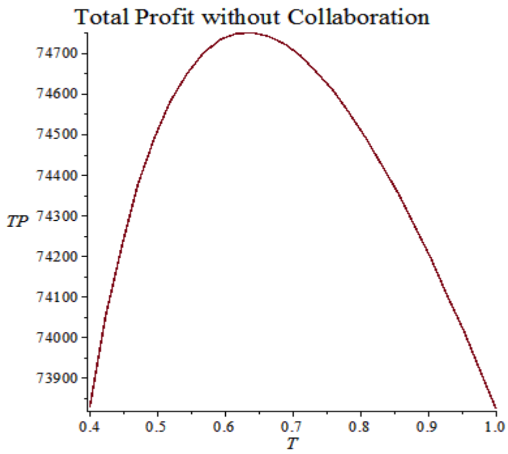

Figure-4

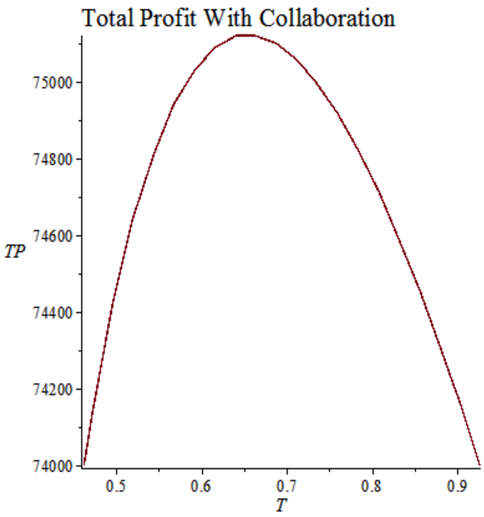

Figure-5

X. CONCLUSION

The result shows that the optimal cycle time is significantly decreased and total profit significantly increased when buyer and vendor consider joint decision policy under supply chain as compared to independent decision taken by buyer and vendor. We also observe that the vendor's profit is increased and number of times order placed by buyer during cycle time is also decreased when buyer and vendor take joint decision.

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