NONLINEAR DYNAMIC ANALYSIS OF 2DOF SYSTEM FOR VIBRATION ABSORPTION AND ISOLATION IN FOUNDATION

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Abstract: Most of mechanical systems exhibit nonlinear behavior and a great class of nonlinearities. In case of two degree of freedom system, normal mode vibrations are free vibrations that depend only on the mass and stiffness of the system and how they are distributed. In the present paper, mainly the nonlinearity is considered due to the non-symmetric arrangement of spring. Here, meaning of non- symmetric arrangement of the spring means, the stiffness of three/two springs are not equal. The general relations have been derived to eliminate the nonlinearity of the system due to difference in spring stiffness by varying the masses or initial displacement. It is observed that for mode 1 i.e. same displacement in same direction, the middle spring does not affect the frequencies of both the masses. The study has been done in four steps. First step is development of mathematical model by considering the nonlinearity, second is numerical solution of by matlab, third by experimental verification and finally by simulation in software. The mathematical model had been developed by Euler-Lagrangian method. The numerical technique Runge- Kutta-Fourth-Order method is used to solve the coupled nonlinear differential equations iteratively. Various techniques like Time-displacement response plot, Orbits plots, and Poincare map used to detect nonlinear behavior. The programmed code has been developed in the MATLAB software to solve the nonlinear equation of motion and to get the different plots.

***IndexTerms* - Nonlinear, self excited, two degree of freedom.**

I. INTRODUCTION

In contrast to the situation for linear systems, nonlinear systems still present formidable analytical and experimental problems. Nonetheless, these problems need to be overcome, as many Engineering structures and Mechanical systems contain a great class of nonlinearities which have significant effects on their dynamical behaviour. The work of Q. Feng is on the Stick- slip phenomenon in a two degree of freedom mass-on-belt friction dynamical system. Two reconstructed deterministic and stochastic discrete models have been established. The numerical calculations of an example have shown that chaos may occur in this system, and noise can change the nonlinear behaviour of the system. Since the friction characteristic consists of two qualitatively different parts with a non-smooth transition, the resulting motion also has non-smooth behaviour. Therefore, stick-slip systems belong to the class of non-smooth systems. [1] Popp and Stelter and Feeny and Moon observed stick-slip chaos in simple oscillators whose nonlinearity is only due to dry friction. In these systems, the motion collapses during stick-slip, leading to a one-dimensional map in the Poincare section. [2, 3, 4] Popp also pointed out that simulation time by Poincare mapping model is only 1/1000 of the simulation time by ordinary computer method. [5] Qingjie Cao and Yeping Xiong worked on an archetype two degree of system with irrational type nonlinearity, which provides a strategy nonlinear dynamics research using the original nonlinearity without any truncation. The system behaviour is strongly nonlinear even when the spring provides linear resistance, due to the geometric configuration of the mechanism. [6] The practical occurrences of the nonlinear phenomena such as periodic, sub-harmonic, quasi-periodic and chaotic are explained in the different books of Authors (A. H. Nayfeh and B. Balachandran, F. C. Moon, H.S. Steven, J.M.T. Thompson and H.B. Stewart). The authors want to give more importance to all these books because a different tool to identify the nonlinear behaviour of the system has been explained very nicely with plots of experiments.[7, 8, 9, 10] The approach of using Euler-Lagrange method and numerical integration technique Runge-Kutta-Fourth-Order method have been used by H.K.Yadav et al. [11,12] In this paper the authors have worked on the nonlinearity of the system and the converted nonlinear system into linear system by varying the masses and amplitudes.

II. PROBLEM FORMULATIONS

The work of M.L.Kansara, J.A.Gohil and H.K.Yadav throws lights on the nonlinearity that induced in the two degree of freedom spring mass system due to non-symmetric arrangement of the springs. The paper describes that, when springs are arranged in the symmetric manner according to their stiffness (i.e. k1-k2-k1/k1-k1-k1), system acts linearly and Experimental – Theoretical - Numerical results verifies. When the springs are arranged in non-symmetric manner according to their stiffness (i.e. k1-k2-k3), system tends to behave nonlinearly and Experimental - Theoretical results do not verifies but Experimental - Numerical results verifies.[13] In the present paper authors have converted the nonlinearity (induced by arrangement of springs) of the system into linearity by varying the masses/displacement of the masses. **Fig.1** (a) shows two degree of freedom spring mass system. The motion of the system is completely described by the coordinates $x_1(t)$ and $x_2(t)$ which define the positions of the masses m_1 and m_2 at any time t from the respective equilibrium positions. The free-body diagrams of the masses m_1 and m_2 are shown in **Fig.1** (b) Experimental model of two degree of freedom system is shown in **Fig.1**(c).

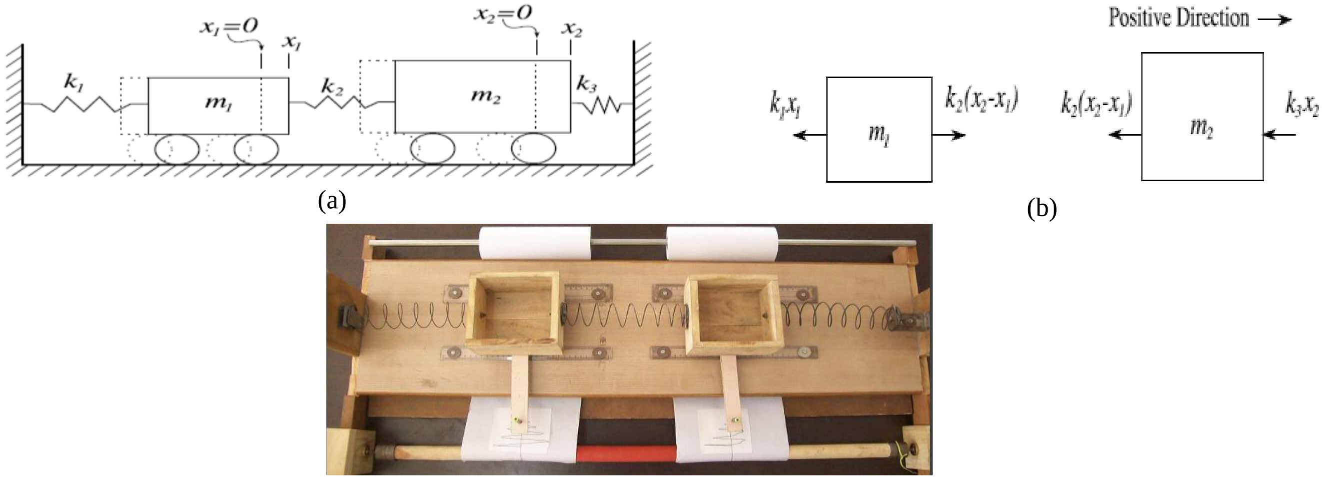


Figure 1 (a) spring-mass system (b) free-body diagrams of the masses m_1 and m_2 (c) Experimental setup of two degree of freedom spring mass system.

III. EQUATION

The equation of motion of system can be written as following equations,

$$m_1\ddot{x}_1 + (k_1 + k_2)x_1 - k_2 x_2 = 0 \quad (1)$$

$$m_2\ddot{x}_2 + (k_2 + k_3)x_2 - k_2 x_2 = 0 \quad (2)$$

For the **MODE 1** the displacement of both the masses are in same direction. Authors have derived the relation for the conversation of nonlinear system into linear system can be written as:

$$m_1 = \frac{k_1}{k_3} m_2 \quad (3)$$

For the **MODE 2** the displacement of both the masses are in opposite direction. Authors have derived the relation to convert the nonlinear system into linear system can be written as:

$$m_2 = \frac{k_3 + 2k_2}{k_1 + 2k_2} m_1 \quad (4)$$

The above relation is used for varying the masses and converting the nonlinear system into linear system for mode 2. Authors have also derived the relation for converting the nonlinear system to linear system by **varying amplitude** of the masses and that can be written as follows,

$$x_2 = \frac{x_1(m_1(k_2+k_3) - m_2(k_2+k_3)) \pm \sqrt{[m_1(k_2+k_3) - m_2(k_2+k_1)]^2 x_1^2 + 4m_2k_2(x_1^2 m_1 k_2)}}{2k_2m_2} \quad (5)$$

IV. METHOD OF SOLUTION

In order to use the MATLAB program, the two coupled second order differential equations, Eq. 1 and Eq. 2 are to be expressed as a system of coupled first order differential equations.

4.1 Numerical Integration

The Eq. 1 and Eq. 2 are solved by using the explicit type numerical integration technique Runge-Kutta-Fourth-Order method to convert the second order differential equation of spring mass system into first order differential equation. H.K.Yadav et al. [11] explains the numerical integration technique Runge-Kutta-Fourth-Order method is used to solve the coupled nonlinear differential equations iteratively.

$$y_1 = x_1,\ y_2 = \dot{x}_1,\ y_3 = x_2,\ y_4 = \dot{x}_2$$

$$m\dot{y}_2 + k_1y_1 + k_2(y_1 - y_3) = 0 \quad (6)$$

$$m\dot{y}_4 + k_3y_3 - k_2(y_1 - y_3) = 0 \quad (7)$$

$$\dot{y}_2 = \frac{-k_1y_1 - k_2(y_1 - y_3)}{m} \quad (8)$$

$$\dot{y}_4 = \frac{-k_3y_3 + k_2(y_1 - y_3)}{m} \quad (9)$$

$$\begin{Bmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \\ \dot{y}_4 \end{Bmatrix} = \begin{Bmatrix} y_2 \\ \dfrac{-k_1y_1 - k_2(y_1 - y_3)}{m} \\ y_4 \\ \dfrac{-k_3y_3 + k_2(y_1 - y_3)}{m} \end{Bmatrix} \quad (10)$$

4.1.1 Solution Procedure

The solution procedure for coupled nonlinear differential equation is explained with the flow chart as shown in **Fig. 2**. The programming is done in MATLAB software and different response plots are obtained. The parameters used for obtaining results analytically, experimentally and numerically have shown in the **Table 3.1**.

Table 3.1 Descriptive Statics

Sr. no	Properties	Values
1.	Mass (m_1)	0.815 kg
2.	Mass (m_2)	0.815 kg
3.	Stiffness k_1 (Soft spring)	41.5 N/m
4.	Stiffness k_2 (Semi hard spring)	138 N/m
5.	Stiffness k_3 (Hard Spring)	157.8 N/m

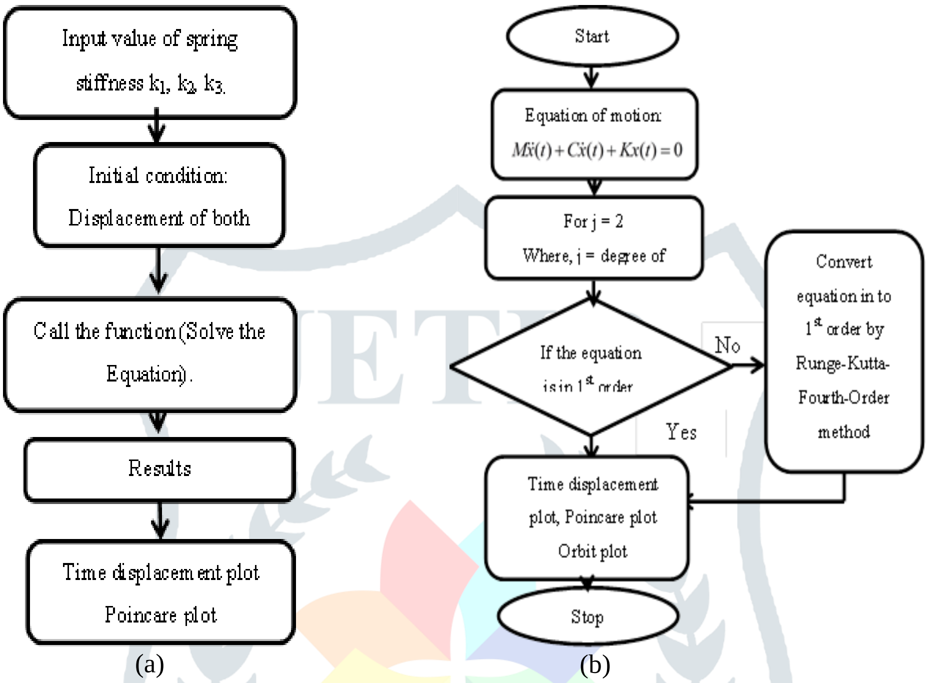


Figure 2: Flow chart of (a) main program (b) call function

4.2 Simulations

The model of two degree of freedom system has been created and dynamically analysed in the computational software.

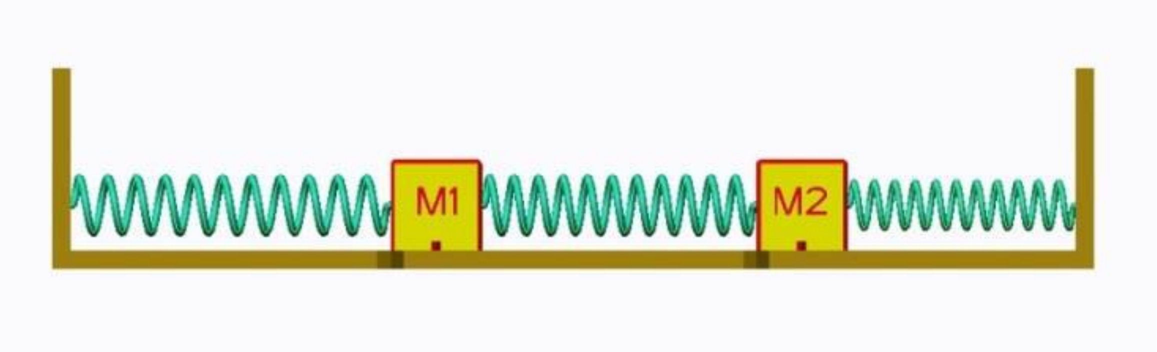
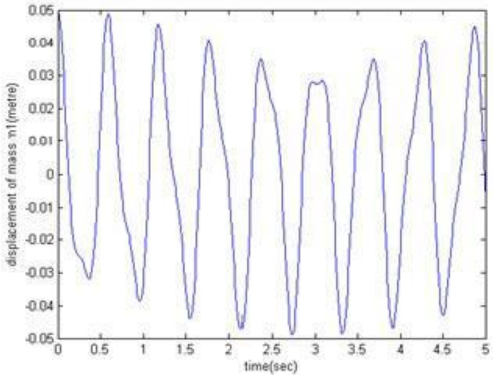


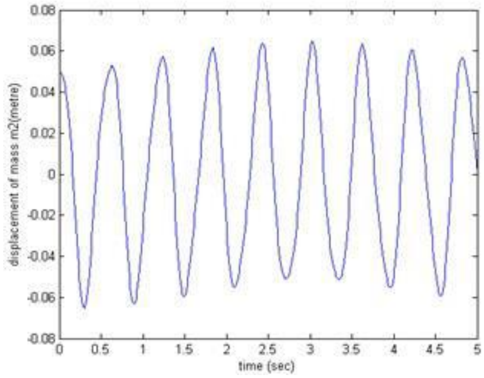
Figure 3: Model of two degree of freedom spring mass system

V. RESULT AND DISCUSSION

First, the author represents the nonlinear system. From the time displacement plot of mass m_1 & m_2 **Fig. 4(a)**, **Fig. 4(b)** shows the nonlinear motion of m_1 and m_2. The same can be confirmed from the Poincare plot of mass m_1 and mass m_2 shows in the **Fig. 4(c)** and **Fig. 4(d)** because it can be observed that more than one loop are overlapped with each other. **Fig. 4(e)** shows the orbit plot for the mode 1. Now for mode 2, from the time displacement plot of mass m_1 & m_2 **Fig. 4(f)**, **Fig. 4(g)** shows the nonlinear motion of m_1 and m_2. The same can be confirmed from the Poincare plot of mass m1 and mass m2 shows in the **Fig. 4(h)** and **Fig. 4(i)**, **Fig. 4(j)** shows the orbit plot for the mode 2. Nonlinear motion of the masses can be understand by these all the plots.



(a)



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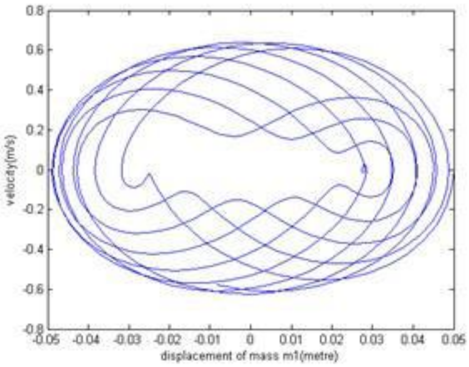

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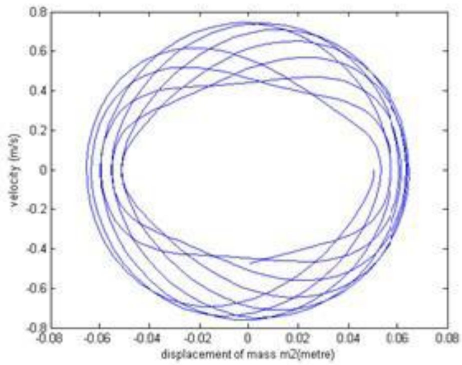

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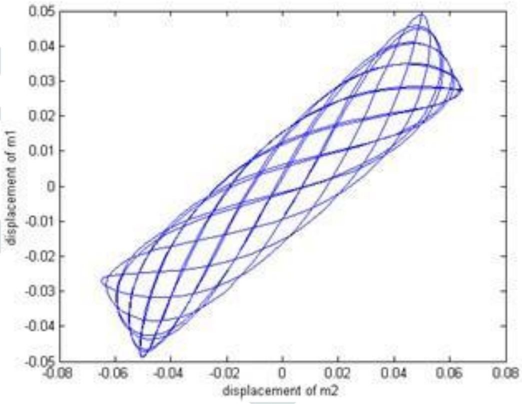

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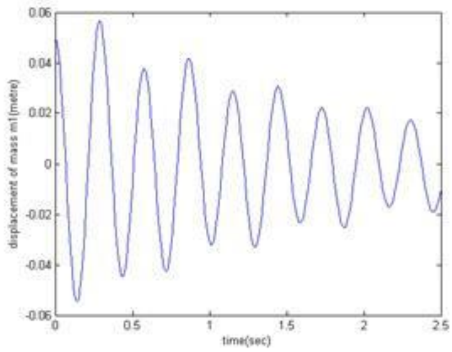

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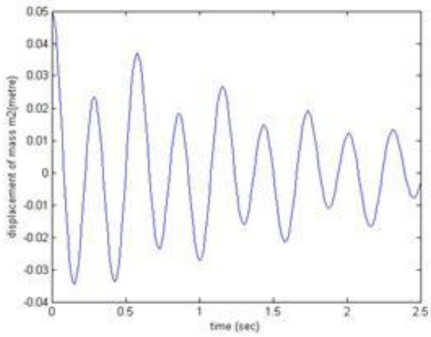
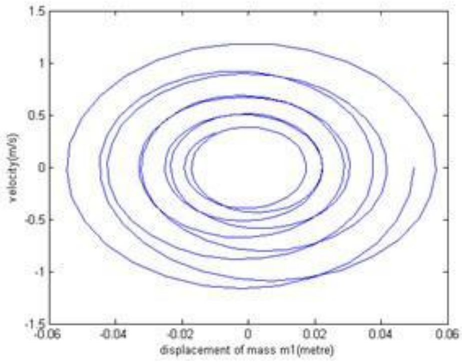
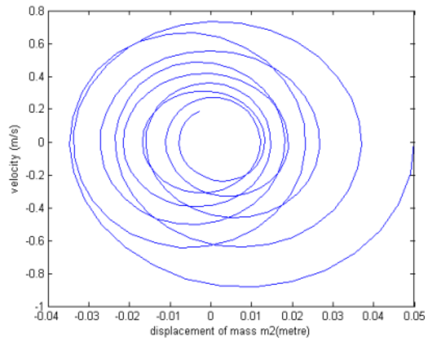


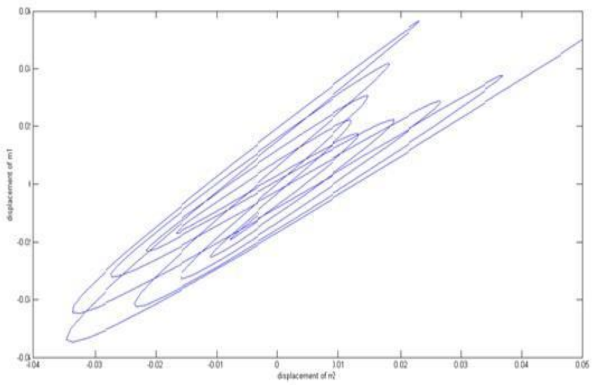
(g)



(h)



(i)



(j)

Figure 4: Time displacement plot for mode 1 (a) for mass m_1 (b) for mass m_2. Poincare plots for mode 1 (c) for mass m_1 (d) for mass m_2 (e) Orbit plots for mode 1. Time displacement for mode 2 (f) for mass m_1 (g) for mass m_2 Poincare plot for mode 2 (h) for mass m_1 (i) for mass m_2 (j) orbit plot for mode 2.

Now, in order to convert that system into linear system following two methods can be implementing. In first method if masses having equal initial displacement than from **Eq. 3** & **Eq. 4**, one can change value of masses in term of the stiffness of springs. So, if both mass having 50 mm initial displacement in mode 1 & mode 2 and mass m_1 is 0.815 kg given than value of mass m_2 is 3.099 kg and 1.114 kg for mode 1 & mode 2 respectively.

Table 4.1 Result obtained by varying masses

Mode	Displacement of mass m_1 (mm)	Displacement of mass m_2 (mm)	Mass m_1 (kg)	Mass m_2 (kg)	Theoretical frequency (cps)	Experimental frequency (cps)
1	50	50	0.815	3.099	1.135	1.1499
2	50	50	0.815	1.114	3.14	3.11

In second method if the all the values of stiffness of springs and the value of masses are given than, by changing the initial displacement, the system can linear by using **Eq. 5** for mode 1 and mode 2. So, if the value of both masses is 0.815 kg & stiffness are as it is, and initial displacement of mass m_2 is 100 mm than from **Eq. 5** the value of initial displacement for mass m_1 becomes 150 mm and -66.376 mm for mode 1 & mode 2 respectively.

Table 4.2 Result obtained by varying masses

Mode	Displacement of mass m_1 (mm)	Displacement of mass m_2 (mm)	Mass m_1 (kg)	Mass m_2 (kg)	Theoretical frequency (cps)	Experimental frequency (cps)
1	150	100	0.815	0.815	1.64	1.667
2	-66.376	100	0.815	0.815	3.44	3.48

As shown in the Table 1 & Table 2, the Experimental and theoretical frequencies for both the masses are nearly to each other. So the system behaves linearly. Also experimentally and theoretically all the results are verified.

Fig. 5 represents the linear behaviour of masses by using changing the value of mass m_2. From the time displacement plot of mass m_1 & m_2 **Fig. 5(a)**, **Fig. 5(c)** are the MATLAB plots which shows the linear motion of mass m_1 and m_2 because it can be observed that motion are completely sinusoidal. The same can be confirmed from the simulated graph **Fig. 5(b)**, and **Fig. 5(d)**. Experimental graph **Fig.(e)**, and **Fig. (f)** Poincaré plot of mass m_1 and mass m_2 shows in the **Fig. 5(g)** and **Fig. 5(h)** indicates the linear behaviour because only one loop can be observed. **Fig. 5(i)** shows the orbit plot for the mode 1 which indicate that both masses move in same direction.

Similarly it can be observed for the mode 2, for the mode 1 & mode 2 by varying the amplitude of masses, that mass m_1 and m_2 behaves linearly.

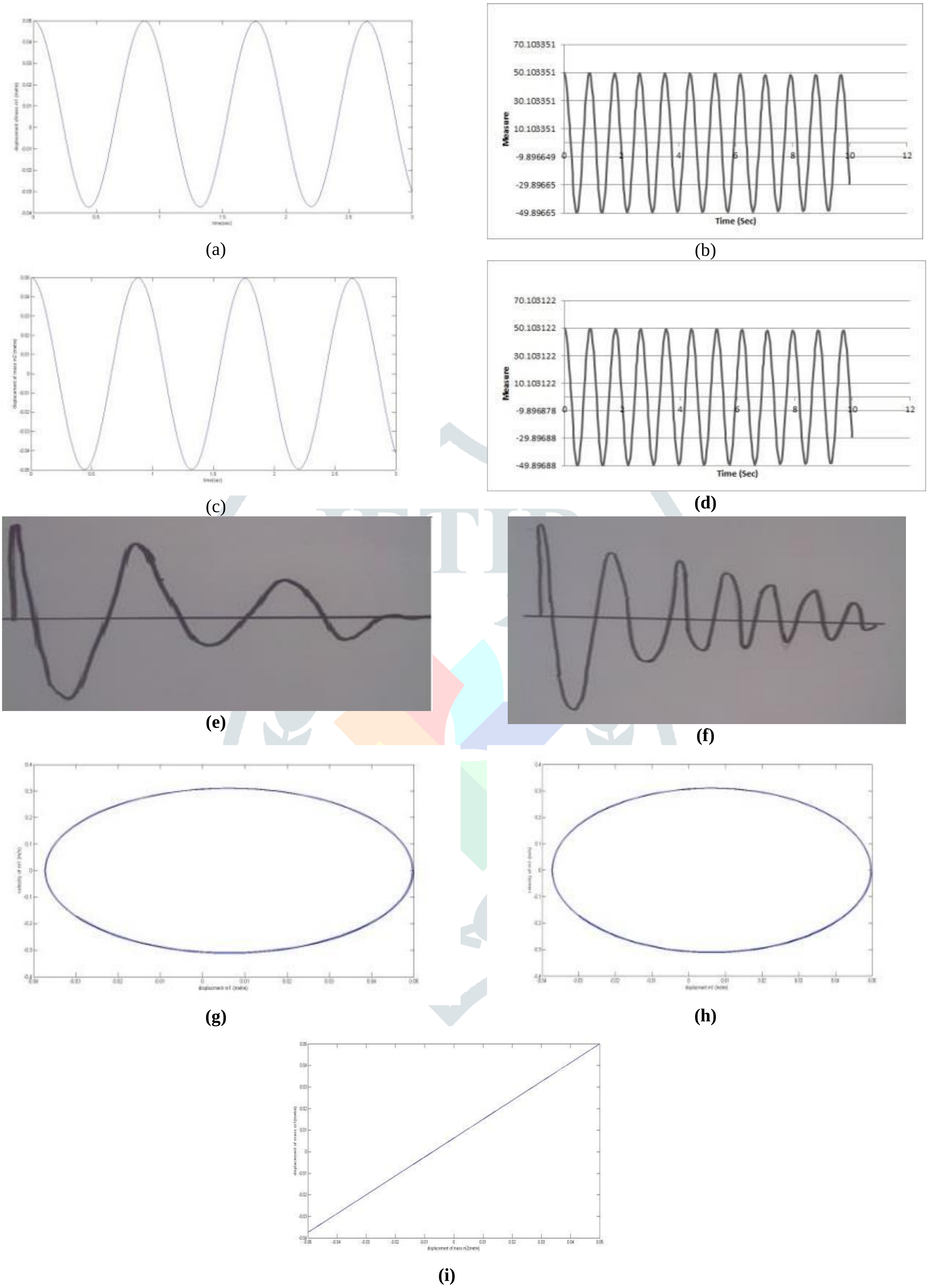


Figure 5: Time displacement plot for mode 1. (a) For mass m_1 (numerical). (b) For mass m_1 (simulated). (c) For mass m_2 (numerical). For mass m_1 (simulated). (d) For mass m_1 (Experimental). (e) For mass m_2 (Experimental) (f) Poincare plots for mode 1. (g) For mass m_1 (h) for mass m_2 (i) Orbit plot.

VI. CONCLUSION

From the above discussion it can be concluded that if all the parameters are chosen arbitrarily than the system behaviour becomes chaotic. But that chaotic behaviour can be change by changing different parameters relative to each other. In this paper the authors suggested that linear motion can be achieved by changing the values of masses and initial displacements.

VII. ACKNOWLEDGEMENT

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