ODD SUM LABELING OF JOINS OF H GRAPH

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Abstract : Let G= (V, E) be a graph with p vertices and q edges. An injective function $f:V(G)\to\{0,1,2,3...q\}$ is an odd sum labelling if the induced edge labelling f^* defined by $f^*(uv)=f(u)+f(v)$ for all uv in $E(G)$ is a bijective function and $f^*:E(G)\to\{1,3,5,7,...2q-1\}$. A graph which is odd sum labelling is called odd sum graph. We in this paper investigate on the H_n graph for $n\geq 3$ by joining with similar H_n graph for $n\geq 3$ and we call the graph to be a join of H graph. We prove that such a graph is odd sum labelling graph and also prove some properties connecting those H_n graphs. We have also obtained the sum of the edges joining the H_n graph.

Key words: Odd sum labelling , Odd sum graph, H_n graph, Joins of H_n graph
2000 Mathematics Subject Classification :05C78

I. INTRODUCTION

A graph G is a finite nonempty set of objects called vertices and edges. We consider all graphs here as finite, simple and undirected. Gallian[1] has given a dynamic survey of graph labelling. The origin of graph labelings can be attributed to Rosa. Odd sum labelling is one of the kind of labelling techniques in graph theory. Odd sum labelling was introduced by S. Arockiaraj and P. Mahalakshmi ,P.Namasivayam [2,3,4] . Further study was done by R.Gopi on odd sum labelling on tree related graphs[5]. Motivated towards the odd sum labelling of graphs. We wish to study on joins of H_n graph and proved that the joins of H_n graph is odd sum labelling and further studied on the some properties of such labelling. We also find the sum of edges of joins o H_n graph in this paper.

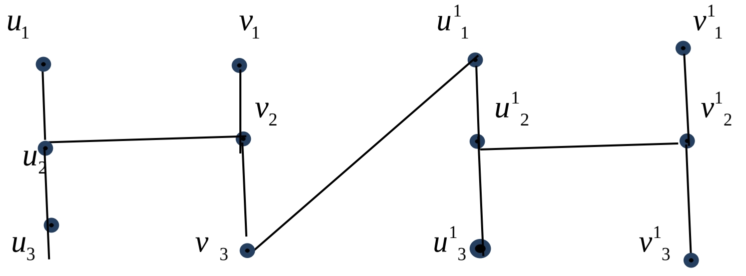
II. PRELIMINARIES

Definition 2.1: The H graph of the path P_n, denoted by H_n is the graph obtained from two copies of P_n with vertices $v_1,v_2,...v_n$ and $u_1,u_2,...u_n$ by joining the vertices $v_{\frac{n+1}{2}}$ and $u_{\frac{n+1}{2}}$ if n is odd and the vertices $u_{\frac{n}{2}}$ and $v_{\frac{n}{2}+1}$ if n is even.

Definition.2.2: An injective function $f:V(G)\to\{0,1,2,3...q\}$ is an odd sum labelling if the induced edge labelling f^* defined by $f^*(uv)=f(u)+f(v)$ for all uv in $E(G)$ is a bijective function and $f^*:E(G)\to\{1,3,5,7,...2q-1\}$. A graph which is odd sum labelling is called odd sum graph

We now try to join a H_n graph denoted by H_{n_1} with a H graph denoted by H_{n_2} by an edge where $n_1=n_2$ and we call it as 1-join of H_n graph.

Figure.1: 1- Join of H_3 graph



Similarly we can construct M-joins of H_n graph by attaching H_{n_1} graph with an edge e_1 with H_{n_2} graph, H_{n_2} graph is attached by an edge e_2 with H_{n_3} and so on and $H_{n_{M-1}}$ by an edge e_{M-1} with H_{n_M} such that $n_1=n_2=...=n_M$

III **MAIN RESULTS**.

Theorem .3.1 : 1-Join of H_n is odd sum labelling graph

Proof: Consider the graph $G = 1 - Join\ of\ H_n$.

We know the H_n graph of the path P_n, denoted by H_n is the graph obtained from two copies of P_n with vertices $v_1, v_2, \ldots v_n$ and $u_1, u_2, \ldots u_n$ by joining the vertices $v_{\frac{n+1}{2}}$ and $u_{\frac{n+1}{2}}$ if n is odd and the vertices $u_{\frac{n}{2}}$ and $v_{\frac{n}{2}+1}$ if n is even

We now join a H graph denoted by H_{n_1} with a H graph denoted by H_{n_2} by an edge where $n_1 = n_2$ which we call it as 1-join of H graph. Now the 1-Join of H_n graph consists of 4n vertices and 4n-1 edges.

Now let us define the vertices of 1-Join of H_n graph as follows

$$V(G) = \{u_1, u_2, \ldots u_n, v_1, v_2, \ldots v_n\} \cup \{u^1{}_1, u^1{}_2, \ldots u^1{}_n, v^1{}_1, v^1{}_2, \ldots v^1{}_n\}$$

$$\begin{aligned} E(G) = &\{u_i u_{i+1}, 1 \le i \le n-1\} \cup \{v_i v_{i+1}, 1 \le i \le n-1\} \\ &\cup \{u^1{}_i u^1{}_{i+1}, 1 \le i \le n-1\} \cup \{v^1{}_i v^1{}_{i+1}, 1 \le i \le n-1\} \\ &\cup \left\{u_{\frac{n+1}{2}} v_{\frac{n+1}{2}}\right\} \cup \left\{u^1{}_{\frac{n+1}{2}}, v^1{}_{\frac{n+1}{2}}\right\} \cup \{v_n u^1{}_{n+1}\} \text{, for n is odd} \end{aligned}$$

$$\begin{aligned} E(G) = &\{u_i u_{i+1}, 1 \le i \le n-1\} \cup \{v_i v_{i+1}, 1 \le i \le n-1\} \\ &\cup \{u^1{}_i u^1{}_{i+1}, 1 \le i \le n-1\} \cup \{v^1{}_i v^1{}_{i+1}, 1 \le i \le n-1\} \\ &\cup \left\{u_{\frac{n}{2}} v_{\frac{n}{2}+1}\right\} \cup \left\{u^1{}_{\frac{n}{2}}, v^1{}_{\frac{n}{2}+1}\right\} \cup \{v_n u^1{}_{n+1}\} \text{, for n is even} \end{aligned}$$

Now let us label the vertices as follows

$f(u_i) = i - 1$ for $1 \le i \le n$

$f(v_i) = i - 1$ for $n + 1 \le i \le 2n$, where we consider and label $f(v_i)$ from 1 to n

$f(u^1{}_i) = i - 1$ for $2n + 1 \le i \le 3n$, where we consider and label $f(u^1{}_i)$ from 1 to n

$f(v^1{}_i) = i - 1$ for $3n + 1 \le i \le 4n$, where we consider and label $f(v^1{}_i)$ from 1 to n

Then the induced edge labelling is obtained as

$f^*(u_i u_{i+1}) = f(u_i) + f(u_{i+1})$

$f^*(v_i v_{i+1}) = f(v_i) + f(v_{i+1})$

$f^*(u^1{}_i u^1{}_{i+1}) = f(u^1{}_i) + f(u^1{}_{i+1})$

$f^*(v^1{}_i v^1{}_{i+1}) = f(v^1{}_i) + f(v^1{}_{i+1})$

$f^*(u_i v_i) = f(u_i) + f(v_i)$ for n is odd and $i = \frac{n+1}{2}$

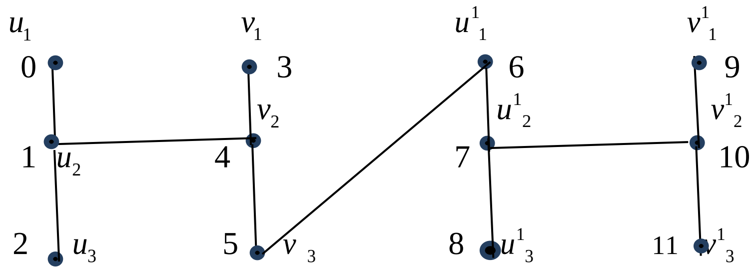
$f^*(u^1{}_i v^1{}_i) = f(u^1{}_i) + f(v^1{}_i)$ for n is odd and $i = \frac{n+1}{2}$

$f^*(u_i v_j) = f(u_i) + f(v_j)$ for n is even and $i = \frac{n}{2}, j = \frac{n}{2} + 1$

$f^*(u^1{}_i v^1{}_j) = f(u^1{}_i) + f(v^1{}_j)$ for n is even and $i = \frac{n}{2}, j = \frac{n}{2} + 1$

Hence we can find that the induced edge labelling is odd sum labeling and hence 1-Join of H_n is odd sum labelling graph. Hence the proof of the theorem.

Figure.2 : Odd sum labelling of 1- join of H_3 graph



Theorem.3.2: M-Join of H_n graph is odd sum labelling graph.

Proof: Consider M-Join of H_n graph. From the proof of theorem 3.1 we have seen that 1-join of H_n graph is odd sum labelling graph. Now let us prove the theorem by induction on the number of joins of H_n graph. Let us assume that the theorem is true for M=k i.e the theorem is true for k-join of H_n graph. To prove that the theorem is true for M=k+1. We understand that M=k+1 joins is the combination of k-joins of H_n graph and 1-join of H_n graph. We have assumed that k-joins of H_n graph is odd sum labelling graph and already we know that 1-join of H_n graph is odd sum labelling graph. Hence M=k+1 joins of H_n graph is also odd sum labelling graph. Hence the proof of the theorem.

Definition :3.4: For a M-join of H_n graph let us define the difference of the labels between two vertices u_i and v_i as $d\left(f\left(u_i\right), f\left(v_i\right)\right) = f\left(v_i\right) - f\left(u_i\right)$ for $1 \le i \le n$

Theorem.3.5: For a M-join of H_n graph, $d\left(f\left(u^M{}_i\right), f\left(v^M{}_i\right)\right) = n$ for $1 \le i \le n$. In Particular $d\left(f\left(u_i\right), f\left(v_i\right)\right) = n$.

Proof: Consider M-join of H_n graph. Following the labelling procedure adopted in theorem.3.1 we find the labels for the vertices that is assigned to the M-join of H_n graph.

The difference of the labels between u_i and v_i is defined as

$d\left(f\left(u_i\right), f\left(v_i\right)\right) = f\left(v_i\right) - f\left(u_i\right)$ for $1 \le i \le n$. Labels for the vertices u_i is defined as $f\left(u_i\right) = i-1$ for $1 \le i \le n$ and the labels for the vertices v_i is defined as $f\left(v_i\right) = i-1$ for $n+1 \le i \le 2n$, where we consider the label for $f(v_i)$ from 1 to n. In general the difference of each labels assigned for u_i and v_i is n.

Similarly for each of the joins of H_n graph the difference between the corresponding vertices u_i and v_i of joins of H_n graph is also found to be n. Hence $d\left(f\left(u^M{}_i\right), f\left(v^M{}_i\right)\right) = n$. Hence the proof of the theorem.

Theorem.3.6: For a M-Join of H_n graph, $1 \le i \le n$ we have the following result

(1) $f\left(u^M{}_i\right) = M\left(f\left(v_i\right) + n\right)$

(2) $f\left(v^M{}_i\right) = M\left(f\left(v_i\right) + n\right) + n$

Proof: For a M-Join of H_n graph, $1 \le i \le n$ we prove the result

(1) $f\left(u^M{}_i\right) = M\left(f\left(v_i\right) + n\right)$ by choosing M=1 and n=3

By the labelling procedure adopted in theorem.3.1 we find that for 1-Join of H_3 graph

L.H.S of (1) is $f\left(u^1{}_i\right) = i-1$ for $2n+1 \le i \le 3n$

For n=3

$f\left(u^1{}_i\right) = i-1$ For $7 \le i \le 9$

Hence the possible labels are 6,7,8

Similarly R.H.S of (1) is $M\left(f\left(v_i\right)+n\right) = 1\left(f\left(v_i\right)+n\right) = i-1+3 = i+2$ for $4 \le i \le 6$

Hence the possible labels are 6,7,8. Hence the result is true. Similarly we can prove for any M-joins and for any value of n that the result (1) $f\left(u^{M}{}_{i}\right)=M\left(f\left(v_{i}\right)+n\right)$ is true.

Similarly we can prove the result (2) $f\left(v^{M}{}_{i}\right)=M\left(f\left(v_{i}\right)+n\right)+n$ by fixing M=1,n=3

L.H.S of (2) $f\left(v^{1}{}_{i}\right)=i-1$ for $10\le i\le 12$

Hence the possible labels are 9,10,11

Also R.H.S of (2) is $M\left(f\left(v_{i}\right)+n\right)+n=1\left(f\left(v_{i}\right)+3\right)+3=f\left(v_{i}\right)+6$ for $3\le i\le 5$

Hence the possible labels are 9,10,11

Hence the result is true. Similarly we can prove for any M-Joins and for any value of n that the result (2) $f\left(v^{M}{}_{i}\right)=M\left(f\left(v_{i}\right)+n\right)+n$ is true. Hence the proof of the theorem.

Theorem.3.7: For a M-join of H_n graph the sum of the edges denoted by $S(n)$ is given by

$$S(n)=\left[\left((M+1)(2n)\right)-1\right]^2$$

Proof : Given M-Join of H_n graph . Now to prove that the sum of the edges as $S(n)=\left[(M+1)(2n)-1\right]^2$. Let us choose the number of joins required and the number of vertices of M-joins of H_n graph. Let us choose n=3 and M=1. Then we have to prove that sum of the edges $S(3)=\left[\left((1+1)(2(3))\right)-1\right]^2=(11)^2=121$. We understand from the labelling procedure adopted in theorem.3.1 for the vertices we have the labels for 1-join of H_3 graph as 0 to 11. The induced edge labelling for the labelled vertices are 1,3,5,7,9,11,13,15,17,19,21. Whose sum is 121. Hence the theorem is true for n=3 and M=1.

Following the similar labelling techniques for different values of n and the number of joins M we can obtain the corresponding sum using the formula $S(n)=\left[\left((M+1)(2n)\right)-1\right]^2$. Hence the proof of the theorem.

IV RESULTS

In this paper we have constructed M-joins of H_n and proved that it is an odd sum labelling graph and proved some properties connecting M-joins of H_n graph and also found the formula for finding the sum of the edges of M-joins of H_n graph.

V CONCLUDING REMARKS

We are investigating on some more graphs which can be proved odd sum labelling graph and also we are interested in finding the generalisation of the some more graphs in our future works.

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