AN INVESTIGATION ON REDUCTION OF VORTEX INDUCED LOADS ON MARINE RISERS

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Abstract : Estimation and reduction of vortex induced loads on risers caused by ocean currents on long cylindrical structures like marine risers is a critical issue for offshore ventures. The complex vortex induced motions of the marine risers makes it difficult for researchers to predict the life and reliability of these risers. This paper investigates the nature of vortex induced vibrations created by ocean currents and ways to suppress the adverse effects by making a change in structural configuration. An Unsteady RANS simulation is used for the estimation of vortex induced loads on a long cylindrical marine riser. Shear Stress Transport (SST) k-ω turbulence model is used in this investigation to study the changes in turbulent characteristics accurately. The impact of the ocean currents on structure is investigated on a bare cylindrical structure and one with modified structural configuration. The intensity of Vortex Induced Vibration (VIV) at different conditions is analysed in terms of Strouhal number. It is found that the change in structural configuration can significantly reduce magnitude of vortex instabilities around the cylindrical marine structure. The results were found to be in reasonable agreement with the available data in the literature. The investigation in this paper can be applied to improve understanding of the vortex shedding phenomenon on marine risers and to estimate the vortex induced forces on marine risers.

***IndexTerms* Marine Riser, Strouhal Number, Vortex induced vibration, Turbulence, Structural configuration.**

I. INTRODUCTION

Marine Users are conduits to transfer materials from the seafloor to production and drilling facilities atop the water's surface, as well as from the facility to the seafloor. Subsea risers act as pipeline developed for this type of vertical transportation. Similar to pipelines or flow lines, risers transport hydrocarbons, as well as production materials, such as injection fluids, control fluids and gas lift. Usually insulated to withstand seafloor temperatures, risers can be either rigid or flexible. The formation of complicated vortex induced instabilities in and around deep marine risers is a grave challenge for the designers. Since the formation and development of VIVs cause serious safety problems to the entire offshore setup, the reduction of these vortex instabilities emerged as an area of intensive research. Constant ocean currents that flow across long cylindrical marine structures lead to initiation of vortex shedding of fluid layers from the sides of the marine structure due to a low pressure zone created in the wake of the structure. The fluid layers get detached from the either sides of the cylindrical structure resulting in so called Von Karman Vortex Street. The structure is then subjected to both drag and lift forces. The lift force on the structure is instrumental in structural vibrations across the flow direction. In due course when the vortex shedding frequency matches with the natural frequency, the structure vibrates violently. This vibration challenges the safety and life of the structure. The suppression of these vortex induced instabilities is a major problem for design engineers of marine risers. The numerical investigation on marine risers with and without structural modifications can impart precise information of the changes in fluid flow characteristics all around the structure. The reduction and control of VIVs is essential to save marine risers against all ocean currents.

Vortex Induced Vibrations can cause severe oscillations of circular structures when encountering the constant ocean currents. This may lead to fatigue damage and eventually result in total collapse of the marine structure. Many researchers have contributed immensely to the study of vortex shedding and its suppression methods. Saha*et al.* [2003] has experimented with different velocities to study the fluid flow characteristics for a fluid flow past a square cylinder. Zdrakovich [1989] has introduced various surface protrusions, shrouds and near wake stabilizers. Kwon and Choi [1996] carried out numerical investigation to study the influence of a splitter plate placed behind a circular cylinder in suppression of vortex shedding and associated vortex induced vibrations. Baek and Karniadakis [2009] conducted a study by making a slit on the structure parallel to the incoming flow and observed that modification of structure is very effective in suppressing vortex shedding. Hover *et al.* [2001] have carried out experimental investigation with thin wires attached to the surface of a cylinder in reducing the VIVs at various Reynolds numbers and reported an earlier onset of frequency resonance. Huang [2013] reported that the helical grooves made on the surface of structure were found to reduce the amplitudes of across flow vibration with a peak amplitude reduction of 64%. Lee KeeQuen [2014] had used flexible risers to find the effectiveness of helical strakes in reducing the VIVs and suggested an optimum design of strakes. Experimental investigation was conducted by *Gao* et *al.* [2016] on a flexible riser with and without helical strakes. A uniform current was produced, to study the suppression of vortex induced vibration using strakes with different heights and pitches. The experimental results revealed that the response characteristic of a bare riser was different from those of a riser with helical strakes, and the suppression performance depends on the geometry of the helical strakes. Lou *et al.* [2017]conducted an experimental investigation to study the effectiveness of multiple control rods, in different configurations around two risers in suppressing wake interference and riser VIV. They measured strain responses in the cross-flow and in-line directions. The analysis showed a significant reduction in VIV, indicating that the control rods effectively suppressed across line vibration of the riser. It is reported that for all configurations, as the Reynolds number increases beyond 500, both the lift and drag coefficients undergoes tremendous changes. Compared to a plain cylinder, a homogeneous fairing system (no gaps) can help reduce the drag force coefficient by 15 %. Assi*et al.* [2006] presented experimental results concerning flow-induced oscillations of circular cylinders arranged in tandem. They reported that the introduction of a tandem cylinder intervene the formation of vortices and reduce the VIV in comparison to isolated cylinders. Mackowski and Williamson [2013] experimentally examined the amplitude of a bluff body undergoing vortex-induced vibration (VIV) supported by linear and various nonlinear structural forces. The study predicted the response of the nonlinear structural system using knowledge of a standard linear VIV system. Blevins and Cougrans [2009] carried out investigations with vortex induced vibration of an elastically supported cylinder in water as a function of flow velocity, damping, mass ratio, inline and transverse degrees-of-freedom, Reynolds number, and strake configuration. It was observed that the entrainment or lock-in band of reduced velocities where lightly damped cylinders oscillate periodically transverse to the flow begins near the stationary cylinder shedding resonance (fs=0.9 to 1.0 fn). The selective review of recent research by Bearman (2011) on vortex induced vibrations of isolated circular cylinders and vibration of circular cylinders in a tandem arrangement claimed that the response of a cylinder free to react in the in-line and transverse directions is contrasted with that of a cylinder responding in only one direction. From literature it can be found that very few similar studies are conducted on flow past circular cylindrical structures provided with hemispherical dimples on surface at moderate Reynolds numbers to reduce vortex induced vibration.

The vortex shedding phenomenon is highly transient in nature and three dimensional numerical simulation is required for the analysis. Hence unsteady Reynolds Averaged Navier Stokes equations coupled with *SST k-ω* turbulence model are used for simulations with moderate computational resources. *SST k-ω* is considered to be highly effective in evaluating the flow parameters near and away from the surface of structure. The numerical simulation on chimney structure with modified structural configurations can provide in depth investigation on fluid flow characteristics in the wake. The reduction of effect of vortex shedding on marine risers is a topic of intense research due to the economic and environmental dimensions. Changes in structural configurations can be used in marine risers to protect them from vortex induced vibrations. The hemispherical dimples are acting as a surface depression which modifies the cylinder surface. The influence of dimples in the reduction of ocean current loads and VIVs can be estimated in terms of Strouhal Number, a dimensionless number which characterises the vibration in a structure.

In the present work numerical investigation has been carried out to study reduction of vortex shedding and associated Aeolian vibrations by analyzing the flow parameters in terms of Strouhal number for a flow past a circular marine riser modified with and without dimples on the outer surface. The numerical investigation mainly focuses on the changes in magnitude of VIVs during flow of water. The effect of modification on structural configuration in the suppression of VIV's in a flow field is investigated systematically. A comparison of resultant Strouhal number at same velocity for marine riser with and without structural modifications has been presented in this paper. Numerical simulation of turbulent subsonic fluid flow past a marine riser with and without structural configurations was carried out using SST *k-ω* turbulence model at different flow velocities using ANSYS CFD.

II. GOVERNING EQUATIONS

The governing equations used in the numerical simulation of incompressible fluids are continuity equation and three momentum equations. As the turbulence model employed is SST *k-ω*, the two transport equations for *k* (turbulent kinetic energy) and *ω* (frequency of energy dissipation) are also solved. A Boussinesq approximation is also employed to eliminate the closure problem in turbulence by replacing the terms involving correlations of velocity fluctuations with the gradient of mean velocities. The Reynolds stresses can be estimated accurately from Boussinesq approximations and a clear insight of the flow field is achieved.

Continuity Equations

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \qquad [1]$$

X- Momentum Equation

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu((\frac{\partial^2 u}{\partial x^2})+(\frac{\partial^2 u}{\partial y^2})+(\frac{\partial^2 u}{\partial z^2})) \qquad [2]$$

Y- Momentum Equation

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu((\frac{\partial^2 v}{\partial x^2})+(\frac{\partial^2 v}{\partial y^2})+(\frac{\partial^2 v}{\partial z^2})) \qquad [3]$$

Z- Momentum Equation

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\nu((\frac{\partial^2 w}{\partial x^2})+(\frac{\partial^2 w}{\partial y^2})+(\frac{\partial^2 w}{\partial z^2})) \qquad [4]$$

Transport equation for k

$$\frac{\partial(\rho k)}{\partial t}+div(\rho kU)=div\left|(\mu+\frac{\mu_t}{\sigma_k})grad(k)\right|+P_k-\beta^*\rho k\omega \qquad [5]$$

Transport equation for *ω*

$$\frac{\partial(\rho\omega)}{\partial t}+div(\rho\omega U)=div\left[(\mu+\frac{\mu_t}{\sigma_{\omega,i}})grad\omega\right]+\gamma_2(2\rho S_{i.j}\bullet S_{i,j}-\frac{2}{3}\rho\omega\frac{\partial u_i}{\partial x_j}S_{i,j}-\beta_2\rho\omega^2+2\frac{\rho}{\sigma_{\omega,2}\omega}\frac{\partial k}{\partial x_k}\frac{\partial\omega}{\partial x_k} \qquad [5]$$

Boussinesq approximation:

$$-\overline{\rho u_i^{'}}\,\overline{u_j^{'}}=\mu_t(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i})-\frac{2}{3}\rho k\delta_{i,j} \qquad [6]$$

Strouhal Number (St):

$$St.=fD/U \qquad [7]$$

where f is Strouhal frequency, D is Characteristic length and U is velocity.

III. MARINE RISER

Rapid development on marine exploration signifies the intensive study of offshore structures like floating platforms, semisubmersibles, floating OTEC and marine risers. Marine riser is generally used as a fluid-conveyed curved pipe while drilling crude oil, natural gas, hydrocarbon, petroleum materials, and mud and then transporting those to the production lines. It acts as a link between the platform and the well head on the seabed. They are inherently long, slender, extensible, and flexible tubular structures and liable to undergo the large amplitude motion subject to the severe environmental forces such as vortex shedding, current and wave forces.

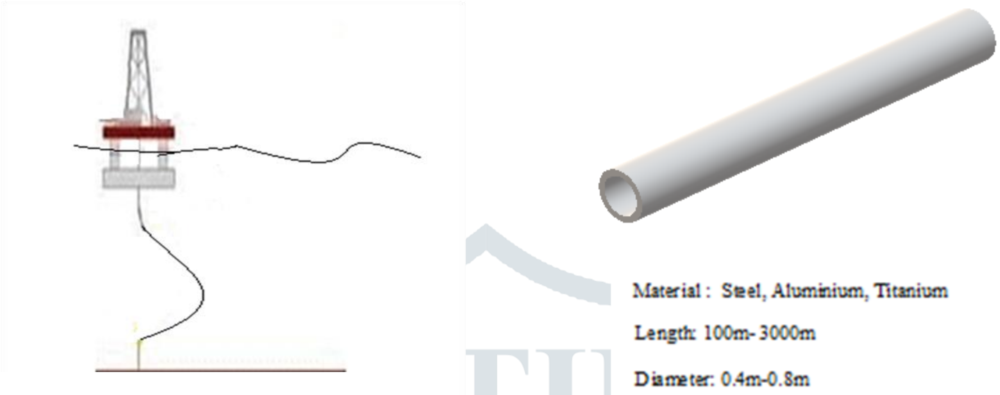


Figure.1 Marine Riser

A marine riser of height 180 m as shown in Figure 1 is considered for analysis. Outer diameter *(d)* of the shell is constant and equal to 0.4 m. To counter ocean current loads, dimples of size 0.04 m in diameter are provided all around the structure in this analysis. A section of Aluminum structure of height 0.5 m is used for analysis of VIV and its suppression.

IV. SIMULATION PARAMETER

3D numerical simulations have been performed on a marine riser with and without dimpled surface. The diameter (*d*) and length (*L*) of the marine riser were 0.4 m and 180 m respectively. The dimples of diameter 0.04 m arranged all around the cylinder surface were used to modify the structural configuration. The fluid flow was assumed to be incompressible as the Mach number was less than 0.3.

IV.1. Computational domain and mesh

A domain of length 40*d* and width 20*d* where *d* is the diameter of the riser was used as flow field as shown in Figure 2 after conducting domain independence study. The height of the section of the riser taken for analysis is $L = 0.5$ m

Figure 2 shows the two dimensional view of the domain and boundary conditions that were applied in the flow field. The optimum domain size and number of grids required in the domain was determined after conducting numerical investigation. The results of these simulations showed that flow parameters like drag and lift forces have a certain dependency on the grid resolution and domain size. It was observed that the magnitude of lift force remains unchanged for number of cells beyond 1.2 million and hence a grid of 1.2 million cells was chosen for the present computational study.

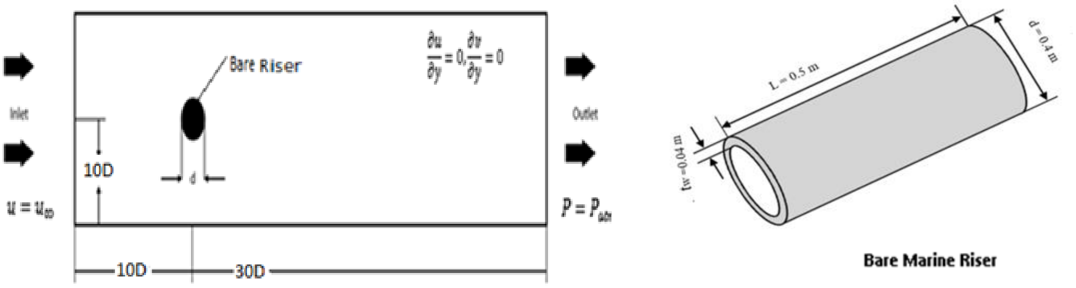


Figure 2. Computational domain for a bare marine Riser

IV.2 Boundary Conditions

Computational domain showing the boundary types and boundary conditions is shown in Figure 2. Boundary conditions of 'velocity inlet' and 'pressure outlet' were applied at inlet and outlet respectively. At bottom and top boundaries, symmetry boundary conditions were assumed.

Table1. Flow properties

Properties	$P_\infty(Pa)$	$\rho_a(kg/m^3)$	$U_\infty(m/s)$
Values	101300	992	1

V. RESULTS AND DISCUSSION

V.1 Flow characteristics for a fluid flow across a bare marine riser

Numerical simulations were carried out to study the flow parameters for water currents past a bare marine riser. The effect of ocean current loads on riser was analysed in terms of different flow parameters. It was observed that the VIV increased marginally with increase in ocean current load. The results are in reasonable agreement with the experimental data for circular cylinders available in the literature.

A flow velocity of 1 m/s was used in the simulation. Figure 3(a) shows that the fluid flow oscillations has a definite pattern in the wake of the cylinder inducing a structural vibration perpedicular to the flow direction. Figure 3(b) shows that drag force on cylinder has a large variation in the initial stages and attains a steady low level at a later stage.

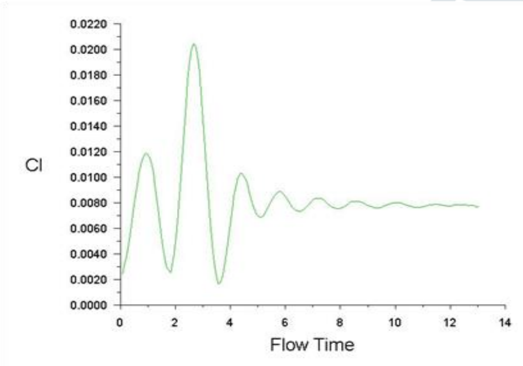


Figure 3(a). Coefficient of Lift with flow time

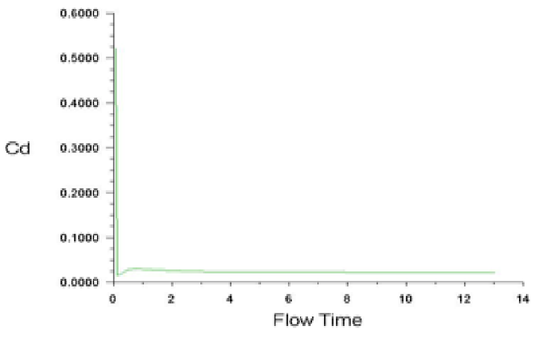


Figure 3(b). Coefficient of Drag with flow time

Figure 4 shows the pressure distribution throughout the domain. A low pressure zone formed in the wake of the cylinder has resulted in the flow separation from the structure and vortex shedding.The fluid undergoes alternate attachments and detatchments resulting in vortex and shear instabilities. Again, the fluid becomes slower and slower and the fluid layer near the cylinder surfaces tends to get separated from either surfaces of the cylinder alternatively, producing VIV of the structure due to the adverse pressure gradient. The pressure plot drawn clearly shows that downstream of the riser, there is an abrupt change in the pressure in the domain resulting in a highly turbulent wake. The distribution of static pressure shows the formation of a wider low pressure zone in the wake of the cylinder. The pressure attains a maximum value when it hits the cylinder surface but suddenly undergoes a large drop in the rear of the cylinder.

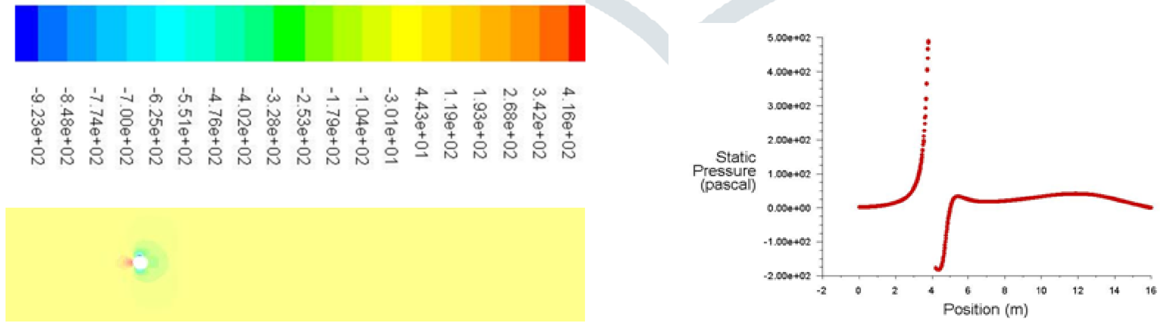


Figure4. Pressure distributions in the central plane

Figure 5 shows the velocity distributions in the central plane. The flow on striking the marine riser shows some abrupt changes and velocity is only restored to its initial value slowly. Similar to the pressure distribution, abrupt velocity fluctuations are also noticed in the wake of the cylinder to mark the formation of vortices in the wake. The fluid layer separation and the formation of vortices from the sides of the cylindrical structure is mainly responsible for the turbulent conditions. The velocity profile clearly shows that the velocity attains a stagnation state on striking the cylinder and slowly restores magnitude towards the end of the domain.

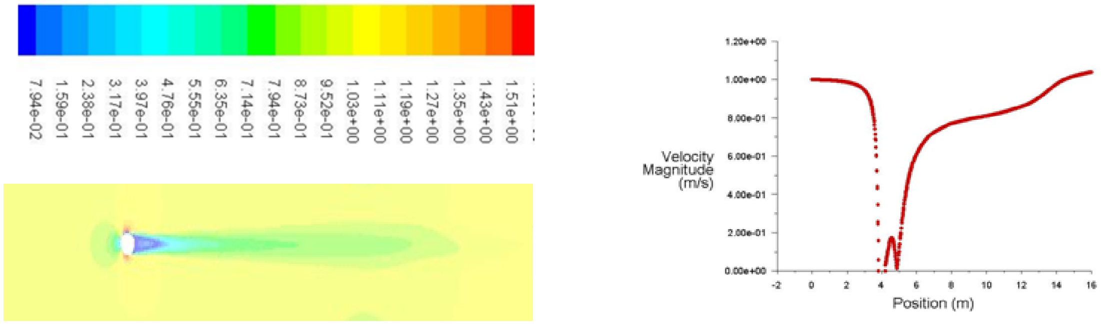


Figure 5.Velocity distributions in the central plane

Figures 6(a) and (b) depict vortex street formed where distinct vortices are found to be detached from both surfaces of the cylinder alternatively. Initially large vortices are formed and as it moves away from the riser resembles as a stream of vortices and finally diffuses into the medium. This effect is due to increase in the internal energy of the fluid and the flow becomes more and more turbulent as it moves downstream of the domain..

The Strouhal frequency and Strouhal number with flow time are shown in Figures 7 (a) and (b). The Strouhal frequency was calculated with the help of Fast Fourier Transform (FFT) and the a non dimensional Strouhal number is estimated using the structural diameter and inlet velocity of the flow. The Strouhal number is computed as 0.22 for a water current velocity of 1 m/s. This value is in reasonable agreement with the numerical works on smaller diameters available in the literature

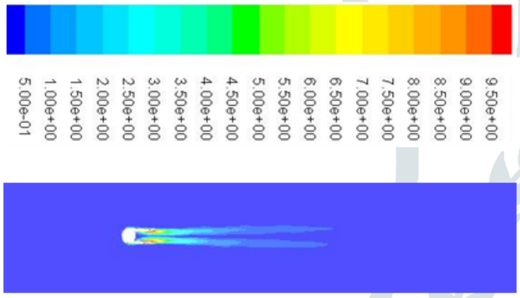


Figure 6(a). Contours of vorticity magnitude

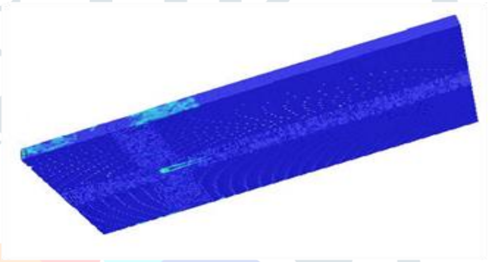


Figure 6(b). 3Dview of contours of vorticity magnitude

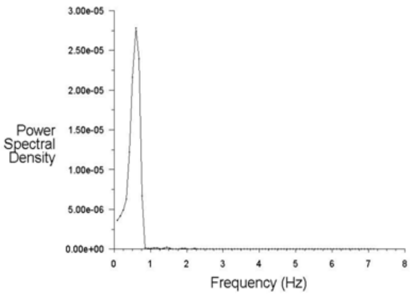


Figure 7(a). Strouhal Frequency

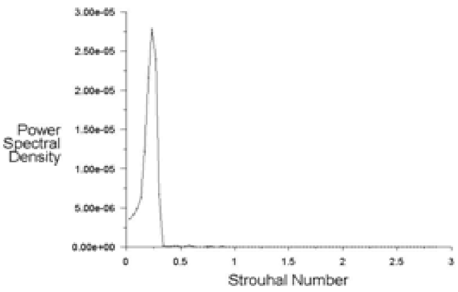


Figure 7(b). Strouhal Number

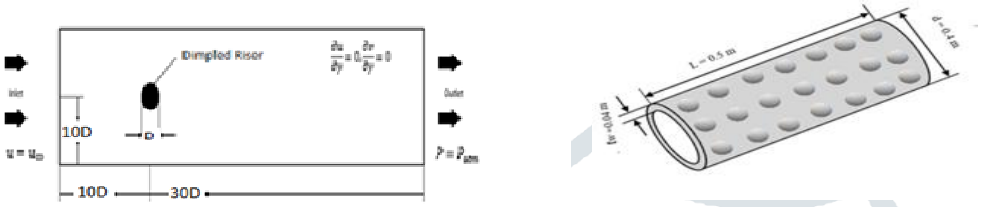
The different flow parameters in a flow field past a bare marine riser is shown in Table II. The Strouhal number is close to the experimental values available in the literature for a fluid flow past a cylinder of smaller dimensions. The increased rate of formation of vortices from the structure mainly contributes to this increase of Strouhal number and if this value is close to the natural frequency of the struture, it may end up with the collapse of the structure. Hence the suppression of VIV becomes a major design problem. Many researchers in this field have experimented with the surface protrusion devices and slightly reduced the magnitude of VIVs. One of the method that can reduce VIV is to create dimple surfaces on the outer surface of the riser and is investigated in the present work.

Table II. Flow field characteristics for a fluid flow past a bare marine riser

Drag (N)	Coefficient of Drag	Lift (N)	Coefficient of Lift	Strouhal Frequency (Hz)	Strouhal Number
38.189	*0.022*	12.985	0.0076	0.55	0.22

V.2 Flow over Circular Riser with dimples on the surface at a velocity of 1m/s

3D numerical investigation was carried out for a section of marine riser with dimples all around the surface. Dimples of 0.1d (d is thediameter) were provided on the surface. A riser section of 0.5 m is used for analysis. Figure 8 shows cross sectional view of the domain. The domain is exactly of same size for a bare riser. The numerical simulation with constant velocity was carried out without any changes in boundary conditions used for bare marine riser.



Marine Riser with dimples on surface

Figure 8 Computational domain for a Marine riser with dimples

The variation in the flow field was closely monitored by observing changes in various flow parameters. When the fluid flows past the dimpled structure, fluid layer close to the cylinder surface undergoes churning effect by the presence of dimples on surface. Small vortices close to the surface are found to be diffusing with one another leading to partial cancellation of the vortices all around the structure. It is observed that small vortices formed near the surface have negligible effect and both lift and drag forces developed are much smaller than that of a bare riser.

The dimple pattern of the surface induces a swirling motion for the fluid around structure. The swirling motion of the fluid disrupts the vortex motion around the structure to a great extent. It is observed that the strength and direction of these swirls largely contributes to the cancellation of the formation of vortex shedding at different points all around the surface. The flow past a dimpled cylinder shows significant variation when compared with that of a fluid flow past a bare riser. There is tremendous interaction between the rotating fluid masses all around the cylindrical structure, leading to significant reduction of the vortex induced vibration and vortex induced fatigue damages.

Figure 9 indicates that the values of coefficient of lift and drag observed with structure with dimple surface with that of a bare cylinder. C_L values are undergoing abrupt changes when the fluid flows past the riser. The nature of the curve indicates a similar pattern; but smoother when compared to that of a bare riser. C_D values registering a sharp decline at the initial stages assumes a constant nature toward the end. It can be observed that C_D shows a marginal decrease in magnitude whereas C_L registers a sharp reduction.

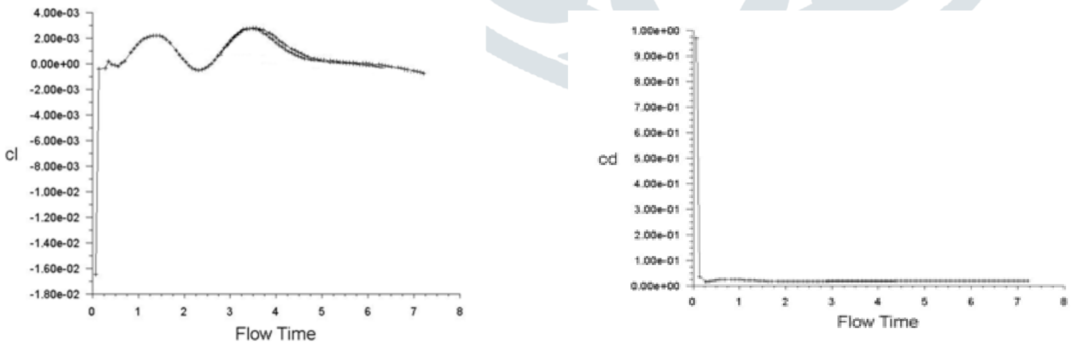


Figure 9. Coefficient of Lift and Drag with flow time

Figure 10 describes the pressure distribution in the central plane of the flow field. A well-defined low pressure zone is diminished in the wake of the cylinder due to the presence of dimple surface. Pressure plot demonstrates that the pressure regains the original magnitude immediately downstream of the structure and has a constant nature in the wake region in place of abrupt changes observed for a bare cylinder.

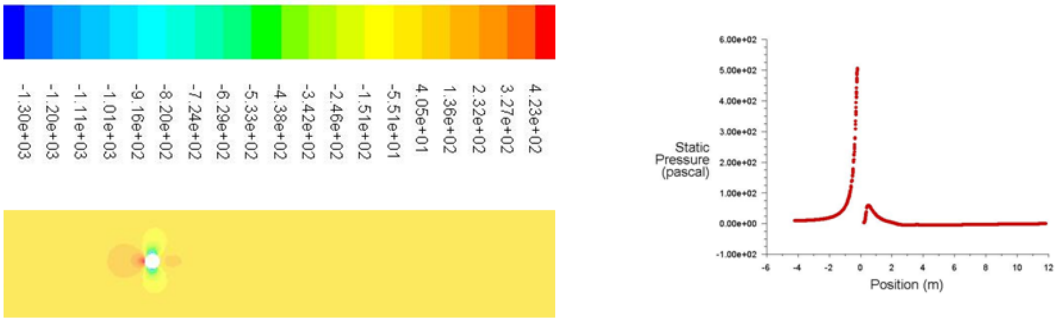


Figure 10. Pressure distributions in the central plane

Figure 11 shows the velocity distribution in the central plane for a fluid flow past a cylinder with dimples. The velocity plot clearly depicts the smooth restoration of velocity in the wake.

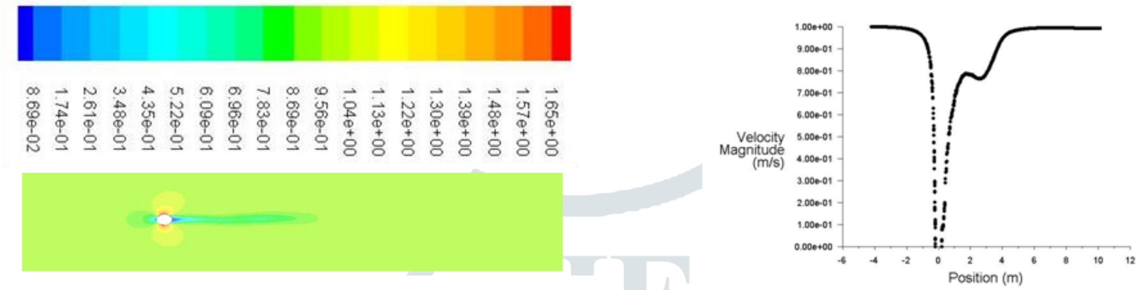


Figure 11. Velocity distributions in the central plane

The contours of vorticity magnitude shown in the Figures 12(a) and (b) clearly show that the vortex shedding fails to form a definite structure during the fluid flow past the structure with dimpled surface. Vortices are found to be extremely small so that its contribution to the formation of VIV is negligible.

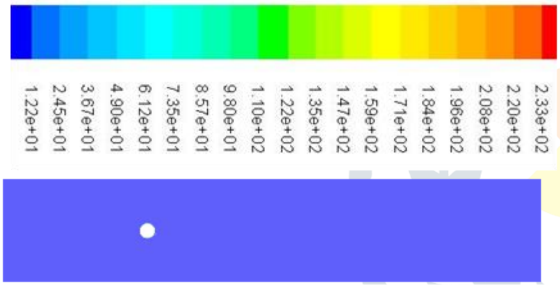


Figure 12(a). Contours of vorticity magnitude in central

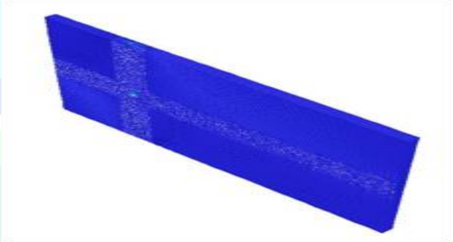


Figure 12(b). 3D view of contours of vorticity magnitude

The resultant Strouhal number and Strouhal frequency are shown in Figures 13(a) and (b) respectively. The Strouhal frequency is estimated with the help of Fast Fourier Transform. The value shows a significant reduction in the frequency of VIV when compared with that of a bare chimney. The reduction in these parameters is mainly due to the presence of dimples on the surface.

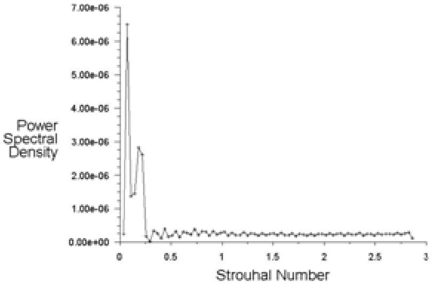


Figure 13(a). Strouhal number

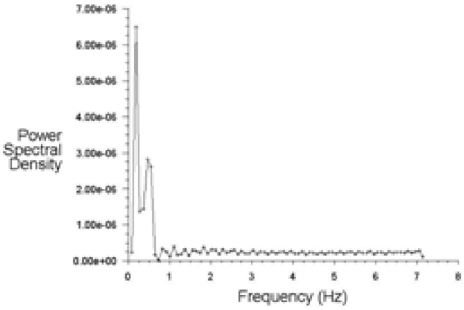


Figure 13(b). Strouhal frequency

The flow parameters for a fluid flow past a modified marine riser are shown in Table III. Significant reduction in lift along the flow field is the major contributor to the suppression of Aeolian vibration. The coefficient of lift also marked a similar decreasing trend. Both Strouhal frequency and Strouhal number showed sharp decline in their values due to the elimination of vortices. There is no drag crisis as reported by Huang (2011) and here, drag registered a decrease. This fact makes this method superior to use of helical strakes for suppressing VIV. The decrease in magnitude of Strouhal frequency ensures reduction in VIV to a much lesser value. Since the effect of hemispherical is omni-directional, it can affect the fluid flow all around the structure. The diffusion of fluid masses leaving the structure takes place instantaneously. As a result, the frequency of vortex shedding is reduced to a low value as shown in the table. A reduction of 67.4% in Strouhal number is realised with this modification of structure.

Table III. Flow field characteristics for a fluid flow past a marine riserwith dimples on surface

Drag (n)	Coefficient of Drag	Lift (n)	Coefficient of Lift	Strouhal frequency (Hz)	Strouhal number
33.75	*0.0198*	-1.3816	0.0007	0.182	0.072

VI. CONCLUSIONS

Numerical investigations of an incompressible fluid flow across a marine riser with and without modified surface were carried out using unsteady RANS simulations with SST *k-ω* turbulence model. Estimation of the flow parameters such as coefficient of lift, coefficient of drag and Strouhal number has shown a decline in values by the provision of dimples all around the surface. The change in the structural configuration has considerable effect on the flow parameters. The decrease in the lift force resulted in the reduction of vortex induced loads. The small vortices formed around the structure found to get mutually diffused away due to the presence of dimples on the surface. The dimples were found to intercept the emerging vortices and prevent their development even at an early stage. Fluid seems to flow very close to the riser surfaces and formation of vortices is prevented. Strouhal frequency registered a sharp decline of approximately 67.4%. The drag crisis was eliminated and drag seen to reduce considerably. This investigation predicts that the dimpled surfaces can suppress the Aeolian vibration and protect whole length of marine riser from structural failures in its life time.This numerical investigation finds application in the suppression of undesirable vibrations of marine risers and safeguard economic and environmental concerns.

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