Difference Labeling of Some Special Graphs

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ABSTRACT: A graph with vertex V and edge E is said to have difference labeling if for an injection f from V to the non-negative integers together with weight function f* on E given by f*(uv)=|f(u)-f(v)| for each edge uv in E. A graph with difference labeling defined on it is called a labeled graph.
In this paper we investigate difference labeling on a Sunlet graph $\mathbf{S_n}$, Sparkler graph $\mathbf{P_m^{+n}}$**,** Shell graph $\mathbf{C_{(n,n-3)}}$**,** Star graph $\mathbf{K_{(1,n)}}$**,** Bistar graph $\mathbf{B_{(m,n)}}$**,** Subdivision of Star graph $\mathbf{S(K_{(1,n)})}$**,** Subdivision of Wheel graph $\mathbf{S(W_n)}$**,** Triangular Snake graph $\mathbf{T_n}$**,** $\mathbf{Z\text{-}P_n}$ graph**.**

KEYWORDS: Difference labeling , Common weight decomposition.

1. Introduction

In this paper, we consider only finite simple undirected graph. The graph G has vertex set V=V(G) and edge set E=E(G). The set of vertices adjacent to vertex u of G is denoted by N(u), for notation and terminology we refer to Bondy and Murthy[2]. A difference labeling of a graph G is realized by assigning distinct integer values to its vertices and then associating with each edge uv the absolute difference of those assigned to its end vertices. The concept of difference labeling was introduced by G.S.Bloom and S.Ruiz[1] and was further investigated by Arumugam and Meena[1]. Vaithiyalingam and Meena[7] further investigated difference labeling of Crown graph C_n* and grid graph P_m*P_n, Pyramid graph fire cracker, banana trees.

Definition: 1.1: Let G=(V,E) be a graph. A difference labeling of G is an injection f from V to the set of non-negative integer with weight function f* on E given by f*(uv)=|f(u)-f(v)| for every edge uv in G. A graph with difference labeling defined on it is called a labeled graph.

Definition: 1.2: A decomposition of labeled graph into parts, each part containing the edge having a common weight is called a common-weight decomposition.

Definition: 1.3: A Common weight decomposition of G in which each part contains m edges is called m-equitable.

Definition: 1.4: The Sunlet graph $\mathbf{S_n}$ is a graph obtained from a cycle C_n attached a pendent edge at each vertex of the n-cycle. It has 2n vertices and 2n edges.

Definition: 1.5: The Sparkler graph $\mathbf{P_m^{+n}}$ is a graph obtained from a path P_m and appending n edges to an end point. It has m+n vertices and m+n-1 edges.

Definition: 1.6: A Shell graph is defined as a cycle C_n with (n-3) chords sharing a common end point called a apex. Shell graphs are denoted as $\mathbf{C_{(n,n-3)}}$**.** It has n vertices and n+3 edges.

Definition: 1.7: A Star graph is a complete bipartite graph if a single vertex belongs to one set and all remaining vertices belong to the other set. It is denoted by $K_{1,n}$. It has n+1 vertices and n edges.

Definition: 1.8: A Bistar $\mathbf{B_{(m,n)}}$ is a graph obtained by making adjacent two central vertices of $K_{1,m}$ and $K_{1,n}$. It has m+n+2 vertices and m+n+1 edges.

Definition: 1.9: A wheel $\mathbf{W_n}$**,** n≥3 is a graph obtained by joining all vertices of cycle C_n to a further c called the centre. It has n+1 vertices and 2n edges.

Definition: 1.10: The Triangular Snake $\mathbf{T_n}$ is obtained the path P_n by replace each of the path by a triangle. It has 2n+1 vertices and 3n edges.

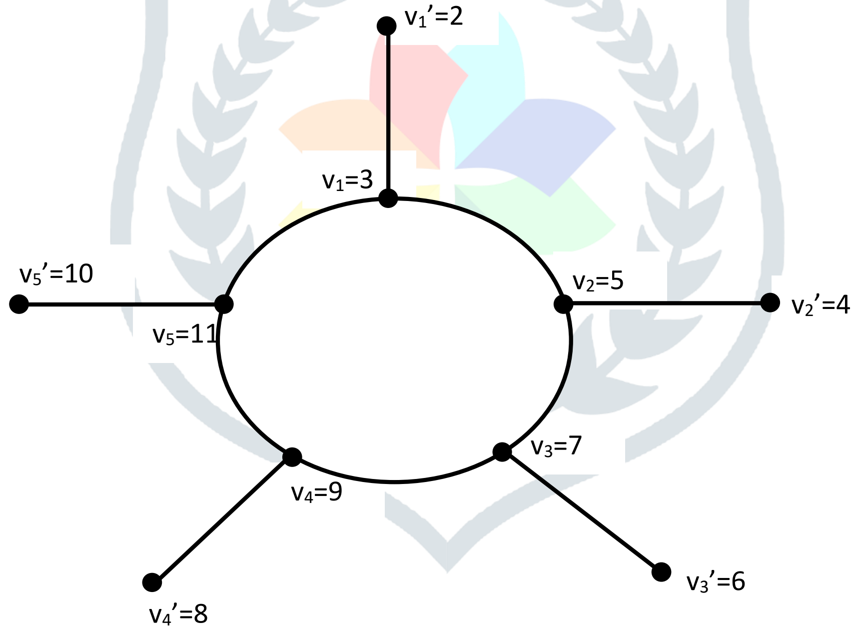
Definition: 1.11: In a pair path P_n, i^{th} vertex of a path P_1 is joined with $i+1^{th}$ vertex of a path P_2. It is denoted by Z-P_n.

II. MAIN RESULTS

Theorem:2.1
The Sunlet graph S_n is a labeled graph with common weight decomposition.

Proof:
Let G=S_n be a Sunlet graph. Let V(G)={$v_1,v_2,\ldots\ldots,v_n,v_1',v_{2'},\ldots\ldots,v_n'$}
Let E(G)={ v_iv_i', $1\leq i\leq n$}U{v_iv_{i+1}, $1\leq i\leq n-1$}U{v_nv_1}
| V(G)| = |2n|
Now we define a function f for its vertices to the set of integer as follows
$f(v_i) = 2i+1$,$1\leq i\leq n$
$f(v_i')=2i$, $1\leq i\leq n$
Define the weight function f* on the edge of G as **f*=|f(u)-f(v)|.** Then f* decomposed the edge G as when n=5. The total edge is 10.
The decomposition of G as **$nP_1U(n-1)P_2+P_{2(n-1)}$.** The sunset graph decomposes by its weight function. Therefore the Sunset graph is a labeled graph.

Example : S_{10}



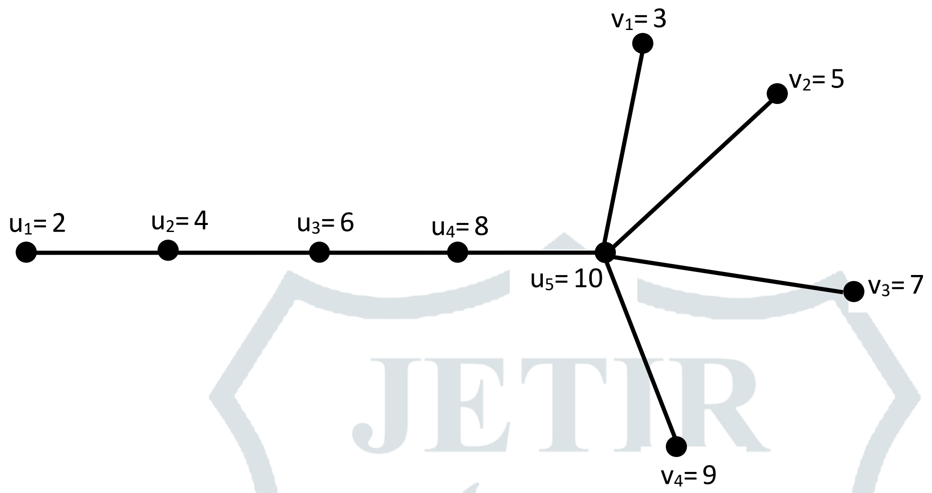
Theorem: 2.2
The Sparkler graph $\mathbf{P_m^{+n}}$ is a labeled graph with common weight decomposition.

Proof:

Let G=$\mathbf{P_m^{+n}}$ be a sparkler graph. Let V(G)={u_i,v_j / $1\leq i\leq m$, $1\leq j\leq n$}
Let E(G)={u_iu_{i+1} / $1\leq i\leq m-1$}U{u_mv_j / $1\leq i\leq n$}
| V(G)| = |m+n|
|E(G)| = |m+n-1|
Now define a function f for its vertices to the set of integers as follows.
Let $f(u_i)=2i$, $1\leq i\leq m$
$f(v_j)=2j+1$, $1\leq j\leq n$

Define the weight function **f*** on the edge G as **f*=f(uv)=|f(u) - f(v)|.** Then **f*** decomposed the edge G as **(m-1)P2+** $\sum_{i=1}^{n} P_{2i-1}$**.**
The Sparkler graph decomposes into parts as shown above.
The Sparkler graph is a labeled graph.

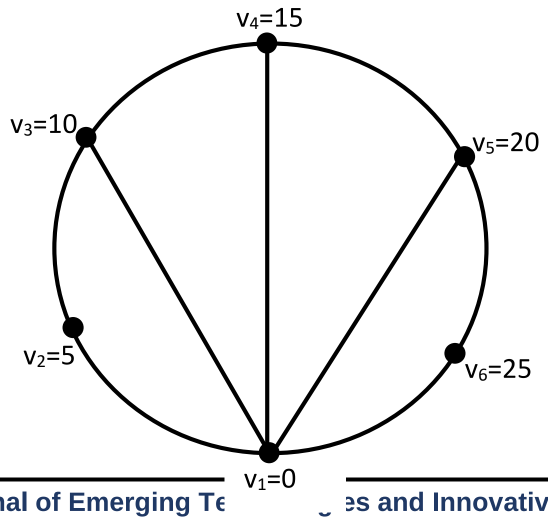
Example: $\mathbf{P_5^{+4}}$



Theorem2.3
The Shell graph is a labeled graph with common weight decomposition.

Proof:
Let $G=C_{(n,n-3)}$ be a Shell graph. Let $V(G)=\{v_1,v_2,\ldots\ldots.,v_n\}$
Let $E(G)=\{\ v_iv_{i+1},\ 1\leq i\leq n-1\}U\{v_1v_{2i+1},\ 1\leq i\leq n-3\}U\{v_nv_1\}$
$|V(G)|=|n|$
Now define the weight function f: V(G) ⟶ {1,2,……,n}
Let $f(v_i)=0$
$f(v_i)=5j,\ \ 2\leq i\leq n\ \ ,1\leq j\leq n$
Define the weight function **f*** on the edge G as **f*=f(uv)=|f(u) - f(v)|.** Then **f*** decomposed the edge G as **(n-2)P5+** $\sum_{i=1}^{n-2} P_{5i}$**+P5(n-1).**
The Shell graph decomposes into parts as shown above.
The Shell graph is a labeled graph.

Example: $\mathbf{C_{(6,3)}}$



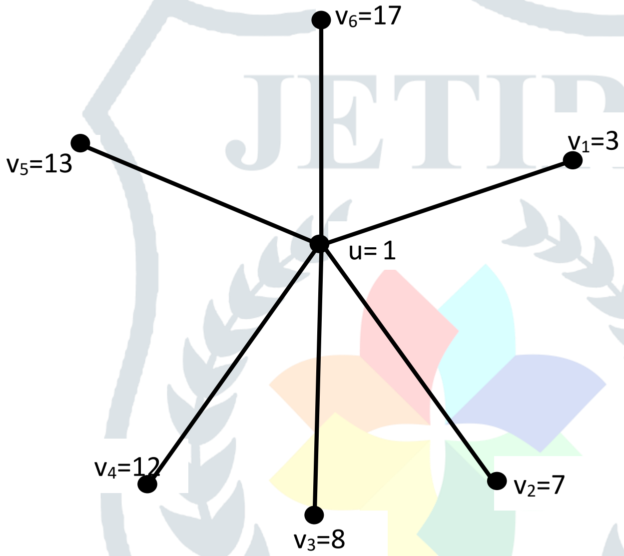
Theorem:2.4

A Star graph $K_{(1,n)}$ is a labeled graph with common weight decomposition.

Proof:

Let G= $K_{(1,n)}$ be a Star graph. Let V(G)={u,v_1,v_2,…….,v_n}
Let E(G)={uv_i, / 1≤i≤n}
|V(G)|=|n+1|
Now define the weight function f: V(G) ⟶{1,2,……,n}
Let f(u)=1
f(v_{2i-1})=5i-2, ,1≤i≤n/2
f(v_{2i+1})=5i+2 , 1≤i≤n/2
Define the weight function **f*** on the edge G as **f*=f(uv)=|f(u) - f(v)|.** Then **f*** decomposed the edge G as $\mathbf{P_2}+\sum_{i=1}^{n-3} \boldsymbol{P_{5i+1}}+\sum_{i=1}^{n-4} \boldsymbol{P_{5i+2}}$.
The Star graph decomposes into parts as shown above.
The Star graph is a labeled graph.

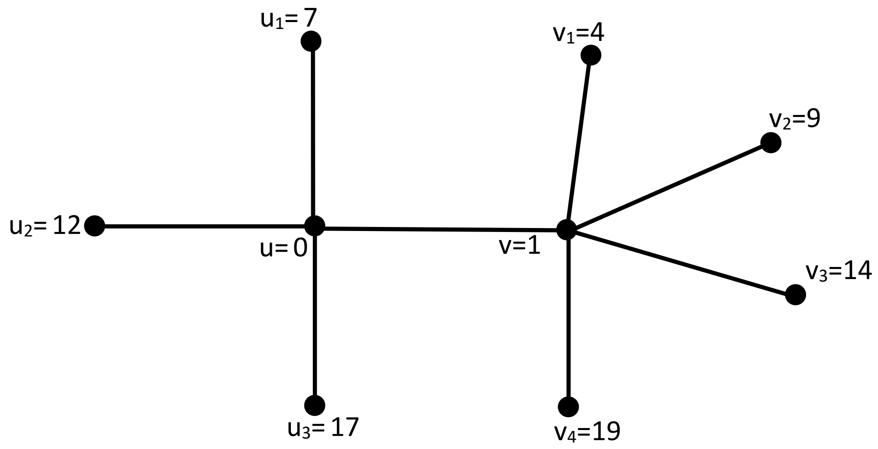
Example: $K_{(1,6)}$



Theorem: 2.5
A Bistar graph $\mathbf{B_{(m,n)}}$ is a labeled graph with common weight decomposition.

Proof:
Let G=$\mathbf{B_{(m,n)}}$ be a Bistar graph. Let V(G)={u,u_1,u_2,….u_m,v,v_1,v_2,…….v_n}.
Let E(G)={uu_i/ 1≤i≤m}U{vv_j/ 1≤j≤n}U{uv}**.**
|V(G)|=|m+n+2|
Now, define a function f for its vertices to the set of integers as follows
Let f(u) = 0
f(v) = 1
f(u_i) = 5i+2 ,i=1,2,…,m
f(v_j) = 5j-1 ,j=1,2,…,n
Define the weight function f* on the edge of G as **f*=f(uv)=|f(u) - f(v)|.** Then **f*** decomposed the edge G as $\sum_{i=1}^{m} \boldsymbol{P_{5i+2}}+ \sum_{i=1}^{n} \boldsymbol{P_{5i-2}}+\mathbf{P_1}$.
The Bistar graph decomposed by its weight function.
Therefore Bistar graph is a labeled graph.

Example: $B_{(3,4)}$



Theorem:2.6

The Subdivision of Star graph **$S(K_{1,n})$** is a labeled graph with common weight decomposition.

Proof:

Let $G = S(K_{1,n})$ be a Subdivision of Star graph.

Let $V(G)=\{u,u_1,u_2,\ldots,u_n, v_1,v_2,\ldots\ldots,v_n\}$

$E(G)=\{uv_i, 1\leq i\leq n\}U\{u_iv_i, 1\leq i\leq n\}$

$|V(G)|=2n+1$

Now, define a function $f : V(G) \longrightarrow \{1,2,\ldots,2n+1\}$

Let $f(u) = 0$

$f(u_i)=2i \quad , 1\leq i\leq n$

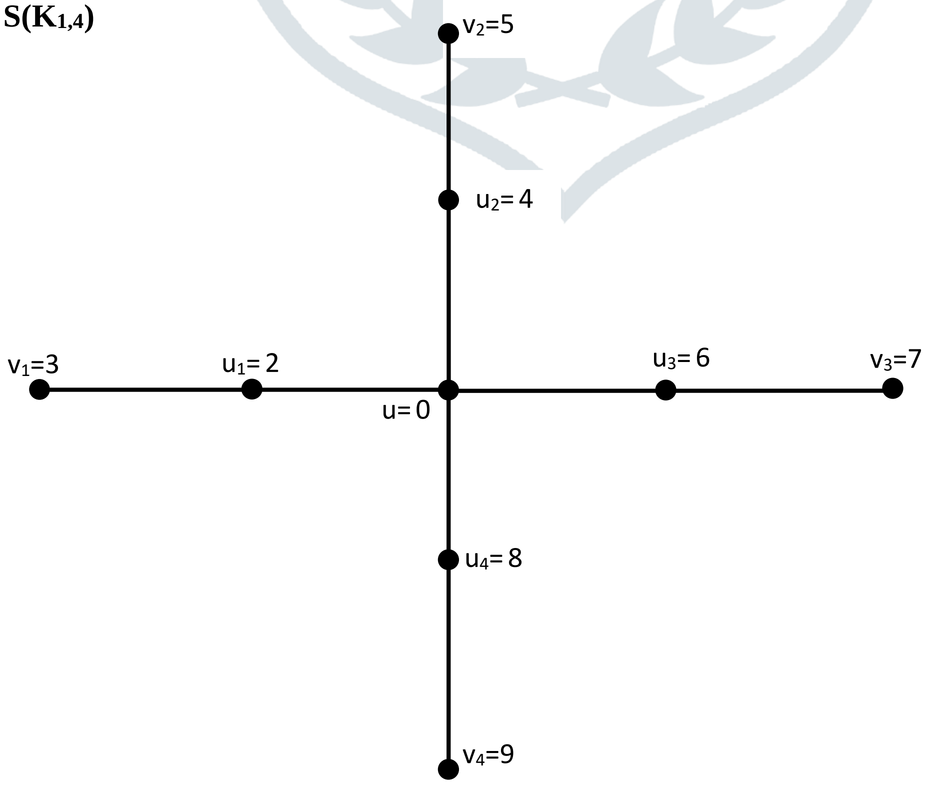
$f(v_i)=2i+1 \quad , 1\leq i\leq n$

Define the weight function **f^*** on the edge of G as **$f^*=f(uv)=|f(u) - f(v)|$.** Then **f^*** decomposed the edge G as **$nP_1+ \sum_{i=1}^{n} P_{2i}$.**

The Subdivision of Star graph decomposed by its weight function.

Therefore $S(K_{1,n})$ is a labeled graph.

Example: $S(K_{1,4})$



Theorem:2.7

The Subdivision of Wheel graph **$S(W_n)$** is a labeled graph with common weight decomposition.

Proof:

Let $G = S(W_n)$ be a Subdivision of Star graph.

Let $V(G)=\{u,u_1,u_2,\ldots,u_n, v_1,v_2,\ldots\ldots,v_{2n}\}$

$E(G)=\{uu_i, 1\leq i\leq n\}U\{u_iv_j, 1\leq i\leq n, 1\leq j\leq 2n\}U\{ v_jv_{j+1},/ 1\leq j\leq 2n-1\}U\{v_{2n}v_1\}$

$|V(G)|=3n+1$

Now, define a function $f :V(G) \longrightarrow \{1,2,\ldots.,3n+1\}$

Let $f(u) = 0$

$f(u_i)=2i+1$, $1\leq i\leq n$

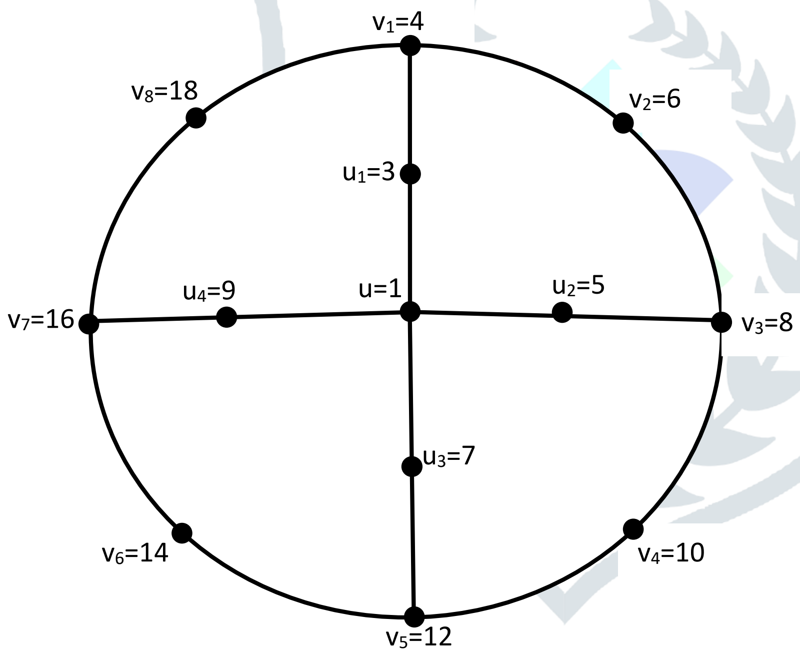
$f(v_j)=2j$, $1\leq j\leq 2n$

Define the weight function **f^*** on the edge of G as **$f^*=f(uv)=|f(u) - f(v)|$.** Then **f^*** decomposed the edge G as **$(2n-1)P_2+ \sum_{i=1}^{n} P_{2i} +P_{2(n-1)}$.**

The Subdivision of Wheel graph decomposed by its weight function.

Therefore $S(W_n)$ is a labeled graph.

Example: $S(W_4)$



Theorem:2.8

The Triangular Snake graph **T_n** is a labeled graph with common weight decomposition.

Proof:

Let $G =$ **T_n** be a Subdivision of Star graph.

Let $V(G)=\{ v_1,v_2,\ldots\ldots,v_{2n+1}\}$

$E(G)=\{v_iv_{i+1}, 1\leq i\leq 2n\}U\{v_{2i-1}v_{2i+1}, 1\leq i\leq n\}$

$|V(G)|=2n+1$

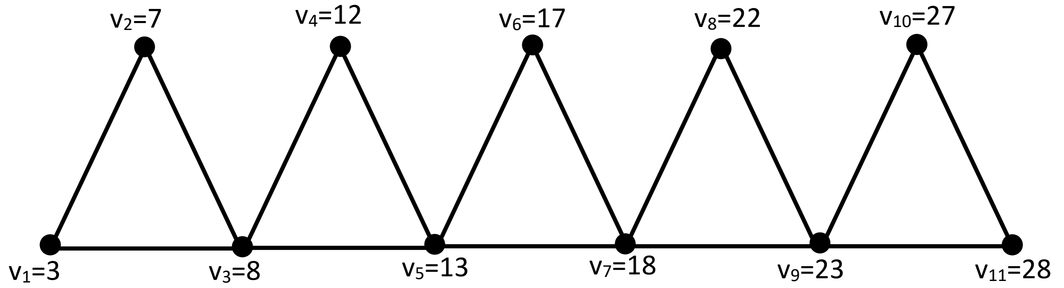
Now, define a function $f :V(G) \longrightarrow \{1,2,\ldots.,2n+1\}$

Let $f(v_{2i-1})=5i-2$, $1\leq i\leq n$

$f(v_{2i+1})=5i+2$, $1\leq i\leq n$

Define the weight function **f*** on the edge of G as **f*=f(uv)=|f(u) - f(v)|.** Then **f*** decomposed the edge G as $\mathbf{nP_1+n\sum_{i=1}^{2} P_{i+3}}$.
The Triangular Snake graph $\mathbf{T_n}$ decomposed by its weight function.
Therefore $\mathbf{T_n}$ is a labeled graph.

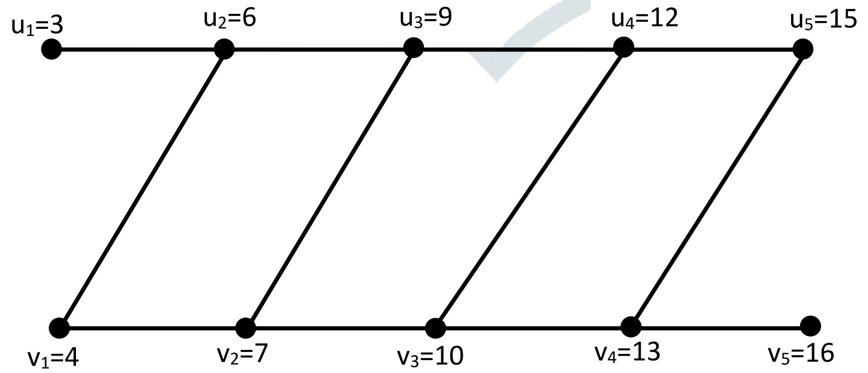
Example: $\mathbf{T_5}$



Theorem: 2.9
The Graph Z-(P_n) is a labeled graph with common weight decomposition.

Proof:
Let **G=** Z-(P_n) be a graph.
Let $V(G)=\{u_i,v_i \,/\, 1\leq i\leq n\}$
$E(G)=\{u_iu_{i+1},\ 1\leq i\leq n-1\}U\{v_iv_{i+1},\ 1\leq i\leq n\}U\{v_iu_{i+1},\ 1\leq i\leq n-1\}$
$|V(G)| =|2n|$
Now define a function f from **V(G)** to the set of all positive integers as follows,
Let $f(u_i) =3i \quad , 1\leq i\leq n$
$f(v_i) =3i+1 \ , 1\leq i\leq n$
Define the weight function **f*** on the edge of G as **f*=f(uv)=|f(u) - f(v)|.** Then **f*** decomposed the edge G as $\mathbf{2(n-1)P_3U(n-1)P_2}$
The graph Z-(P_n) decomposed into parts as shown above.
The graph Z-(P_n) is a labeled graph.

Example: Z-$\mathbf{P_5}$



III CONCLUSION

Labeled graph is the copies of current interest due to its diversified applications. Here we investigate Nine results corresponding to labeled graphs similar work can be carried out for other families also.

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