A STUDY ON INTERCONNECTION NETWORKS AND GRAPHS

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Abstract :Graph theory is a fundamental and powerful mathematical tool for designing and analyzing interconnection network is a graph. This fact has been universally accepted by computer engineers and scientists. In this paper we focus on interconnection networks. Some of its well-known topological structure, transmitting problem, fault tolerances of processors and an application to transmitting path problem in vibroacoustis.

***IndexTerms*–Interconnection network, transmittingproblem, faulttolerance, vibroacoustis.**

1.INTRODUCTION

The architecture of an interconnection network is always represented by a graph where vertices represent process and edges represent links between processors. It is almost impossible to design a network that is optimum from all aspects. One has to design a suitable network depending on its properties and requirements. Thus many graph are proposed as possible interconnection network topologies. Thus these graph can be referred to as good graphs. For this reason the theory of interconnection network is referred to as GOOD GRAPH THEORY. In the design of an interconnection network, one of the most fundamental considerations is the reliability of the network, which can be usually characterized by connectivity and edge-connectivity of the topological structure of the network. The advent of very large scale integrated circuit technology has enabled the construction of very complex and large interconnection networks. Most probably, the next generation of super-computers will achieve its gain by increasing the number of processing elements, rather than by using faster processors. The most difficult technical problem in constructing a supercomputer will be the design of the interconnection network through which the processor communicate. Selecting an appropriate and adequate topological structure of interconnection network will become a critical issue, on which many research effects have been made over the past decade.

2. INTRODUCTION TO INTERCONNECTION NETWORKS

The topological structure of an interconnection network can be modeled by a graph. Interconnection networks are designed for use at different levels within and across computer system to meet the operational demands of various application areas - high performance computing, storage I/O, cluster/workgroup/enterprise system, inter networking and so on. A system following Hayes [135], may be defined informally as a collection of objects called components, connected to formal coherent entity with a well-defined function or purpose. The function performed by the system is determined by those performed by its components and by the manner in which the components are interconnected. A connection pattern of the component in a system is called an interconnection network, or network for short, of the system. Topologically, an interconnection network can essentially depict structural feature of the system. In other words, an interconnection network of a system provides logically a specific way in which all components of the system are connected. A graph can also be consider as a topological structure of some interconnection network. Topologically, graph and interconnection network are the same thing.

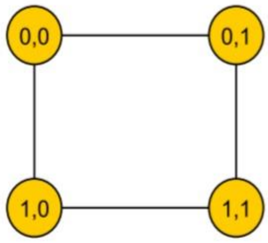
2.1. WELL-KNOWN TOPOLOGICALCAL STRUCTURE OF INTERCONNECTION NETWORKS

2.1.1HYPERCUBE NETWORKS

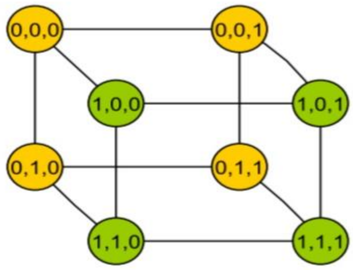
The hypercube suggested by Sullivan and Bashkow [240] is one of the most popular, versatile and efficient topological structure of interconnection networks. The hypercube has many excellent features, and thus becomes the first choice for the topological structure of parallel processing and computing systems.

TWO EQUIVAL DEFINITION:
The topological structure of a hypercube network in the n-dimensional cubes, shortly n-cubes whose graph-theoretical model is an undirected graph and denoted by Qn.
1. DEFINITION USING BINARY SEQUENCE:
The vertex set V of set Qn consists of all binary sequence of length 'n' on the set {0,1}. i.e., V= $\{x_1,x_2,\ldots..,x_n;\ x_i \in\{0,1\}$,i = 1,2,...n}. Two vertices x= $x_1x_2 \ldots \ldots x_n$and y= $y_1y_2 \ldots \ldots y_n$ are linked by an edge iff x and y differ exactly in one coordinate i.e. $\sum_{i=1}^{n}|x_i - y_i|$=1.

The first cube Q_1



The first cube Q_2



The first cube Q_3

2. DEFINITION USING CARTESIAN PRODUCT
$Q_n = Q_{n-1} \times Q_1 = k_2 \times k_2 \times \ldots\ldots \times k_2$(n times); where $Q_1 = k_2$.

NOTE:
Two definition of Qn are equivalent. In other words, the graph defined by the first definition is isomorphic to one defined by the second.

2.1.2 HAMMING GRAPH

Hamming graphs are Cartesian product of complete graph and thus generalize the concept of hypercube.
Hamming graph are natural generalization of hypercube. In fact hypercube can be characterized as bipartate hamming graph if ∃ integer 'k', $n_1, n_2, \ldots., n_k$ such that G≅ $k_{n1} \times k_{n2} \times \ldots. \times k_{nk}$.

2.2.3 GRID AND MESH NETWORK

The Cartesian product of two path is known as grid or mesh i.e, $P_l \times P_m$; where $P_l$and $P_m$are undirected paths and is denoted by G(l,m). Mesh and its variants are more simple and popular interconnection networks in the research community. The mesh network G(l,m) is a subgraph of the hypercube Qn, where n = $[\log_2 l]$ + $[\log_2 m]$. The mesh network can be embedded into the hypercube with dilation one.

2.2 BASIC PRINCIPLES OF NETWORK DESIGN

As we know that the topological structure of a network is a graph. We use the langue of graph theory to introduce these principles only as the topological structure is considered. Following Bermond and Peryrat[29] these principle can be stated as follows:

2.2.1 SMALL AND FIXED DEGREE

The degree of a graph corresponds the number of connections to each component. This number is bounded by the number of the interface available for I/O devices attached to each component in the network. An excess of any physical connection will result in replacement of the components in the network to increase the number of interfaces. The large degree, the more wiring. More wiring not only costs much money, and also is disadvantageous to implementation of VLSI (very large scale integration) layout. Thus a small or fixed maximum degree is desirable.

2.2.2 SMALL TRANSMISSION DELAY [i.e, small diameter or average distance]

Since transmission delay or signal degradation for sending a message from one vertex to another is approximately proportional to the number of times that a message has to be stored and forwarded by intermediate vertices. Thus a small average distance or diameter is desired to obtain a highly efficient interconnection network. In particular, diameter should be bounded by a given value for a real-time processing system.

2.2.3MAXIMUN FAULT TOLERENCE

The network must continue to work in case of vertex or edge failures. Different notions of fault tolerance exist, the simplest one corresponding to connectivity (or edge connectivity) of the graph, that is the minimum number of vertices (or edges) which must be deleted in order to destroy all paths between a pair of vertices. The maximum connectivity is desirable since it corresponds to not only the maximum fault tolerance of the network but also the the maximum number of internally (or edge) disjoint paths between any two distinct vertices.

2.2.4 EASY ROUTING ALGORITHM

The routing is an important function of communication networks. It specifies a fixed route which carries the message from one vertex to another. Thus the choice of an easy routing algorithm is important. A routing algorithm strongly depends on the chosen topological structure. Thus a network should be designed such that a routing algorithm can be easily obtained.

2.2.5 EMBEDDABILITY OF OTHER TOPOLOGIES

This important issue deals with the ability of a given architecture to match various algorithm that solve different types of problems. The network built would enable one to use various algorithm originally designed for another topological structure. In other words, when a graph is used as an interconnection network, it should contain certain subgraph structures, since existence of these structure has special importance for executing certain algorithm.

2.2.6 SYMMETRY

We can divide that all components behave in the same manner and that they communicate in similar ways. This implies at least some regularity and some symmetric properties on the graph. A highly symmetry network is desirable since it is advantage to construction and simulation of some algorithm.

2.2.7 EXTENDABILITY

It should be possible to build a network of any given size or at least two build arbitrary large version of the network. Furthermore, it would be easy to construct large networks from small ones. When a small network is extended some desirable properties should remained and some useful parameters should be calculated easily.

3. HYPERCUBE NETWORKS

In this section we will present desirable structural properties of hypercube networks and its various generalization and enhancements.

3.1 HYPERCUBE

As we discussed earlier the topological structure of hypercube network in the dimensional cube is an undirected graph and is denoted by Qn. $Q_n = Q_{n-1} \times Q_1 = k_2 \times k_2 \times \ldots\ldots \times k_2$(n times); where $Q_1 = k_2$.

PROPERTIES
*Qn is n-regular, has 2^n vertices and $n2^{n-1}$ edges.
*Qn is bipartate.
*Qn is Hamiltonian if n ≥ 2 and Eulerian if n is even.
*Qn has diameterd(Qn)-n.
*Qn has connectivity K(Qn)-n.
*Qn is a Cayley graph Cr(s) and hence is vertices-transitive where r = $z_2 \times \ldots.\times z_2$and s = {100...00,010....00,…..,00....0}.

3.2 GENERALIZED HYPERCUBES

Mathematically, it is quite natural to generalize hypercubes to more general forms. Here we present there such forms, which generalize the hypercube from three distinct aspects.

3.2.1 GENERALIZED HYPERCUBE

Bhuyan and Agarwal [31] generalized Qn to the n-dimensional generalized hypercube denoted by Q$(d_1, d_2, \ldots, d_n)$ where di ≥ 2 is an integer for each i = 1,2,...n. As a generalization of the first definition of Qn the vertex set of Q$(d_1, d_2, \ldots, d_n)$ is the set. V = $\{x_1, x_2, \ldots. x_n$: $x_i \in$ (1,2,…..d_{i-1}); i=1,2…n} and two vertices x = $x_1 x_2 \ldots. x_n$ and y= $y_1 y_2 \ldots. y_n\}$ are linked by a edge iff they differ exactly in one coordinate. As a generalization of the second definition of Qn, Q$(d_1, d_2, \ldots, d_n)$ can be defined as the cartesian product$kd_1 \times kd_2 \times \ldots.. \times kd_n$

PROPERTIES
*Q$(d_1, d_2, \ldots, d_n)$ is (d1 + d2 + ... + dn −n)-regular.
*The diameter of Q$(d_1, d_2, \ldots, d_n)$ is n.
*The connectivity of Q$(d_1, d_2, \ldots, d_n)$ is $d_1 + d_2 + \cdots + d_n\}$.
*Q$(d_1, d_2, \ldots, d_n)$ is a Cayley graph and hence is vertex transitive.

3.2.2 UNDIRECTED TOROIDAL MESH

The n-dimensional undirected toroidal mesh denoted by C($d_1, d_2, \dots, d_n$)is also considered as a generalization of Qn where di≥3 is an integer for each i=1,2,...n.According to the first definition of the vertex set of C($d_1, d_2, \dots, d_n$)) is the set V = {$x_1, x_2, \dots, x_n$: x_i ∈{0,1,..,d_{i-1}},i = 1,2,..,n} and two vertices x= $x_1x_2 \dots, x_n$ and y= $y_1y_2 \dots y_n$ are linked by an edge iff$\sum_{i=1}^{n} |x_i - y_i|$= 1. As generalization of second definition of Qn ;C($d_1, d_2, \dots, d_n$)can be defined as the cartesian products C$d_1 \times Cd_2 \times \dots \times Cd_n$where C$d_i$ is an undirected cycle for each i=1,2,..n.

PROPERTIES
*C($d_1, d_2, \dots, d_n$)is 2n-regular.
*C($d_1, d_2, \dots, d_n$)regular.
*The diameter of C($d_1, d_2, \dots, d_n$)is$\sum_{i=1}^{n}[\frac{1}{2} d_i]$.
*The connectivity of C($d_1, d_2, \dots, d_n$)is 2n.
*C($d_1, d_2, \dots, d_n$)is a cayley graph and hence is vertex-transitive.

3.2.3 DIRECTED TOROIDAL MESH

The n dimensional directed toroidal mesh denoted by $\vec{C}$(d1,d2,…d_n) is also thought of as a generalization of Qn where di ≥ 2 is an integer for each i=1,2,..n. According to the first definition of Qn the vertex set of *$\vec{C}$(d1,d2,…d_n) is the set V = {$x_1, x_2, \dots, x_n$: x_i ∈{0,1,..,d_{i-1}},i = 1,2,..,n} and two vertices x= $x_1x_2 \dots, x_n$and y= $y_1y_2 \dots y_n$are linked by a directed edge from x to y in $\vec{C}$(d1,d2,…d_n)iff$\sum_{i=1}^{n}(x_i - y_i)$=.1 As a generalization of the second definition of Qn, $\vec{C}$(d1,d2,…d_n) can be defined as the cartesian product C$d_1 \times Cd_2 \times \dots \times Cd_n$where Cdi is a directed cycle of length di ≥ 2 for each i=1,2..n.

PROPERTIES
*$\vec{C}$(d1,d2,…d_n) is n-regular
*The diameter of $\vec{C}$(d1,d2,…d_n) is d_1+d_2 ++…+d_n − n
*The connectivity of $\vec{C}$(d1,d2,…d_n) is n
*$\vec{C}$(d1,d2,…d_n) is a cayley graph and hence is vertex-transitive.

3.3 SOME ENHANCEMENT ON HYPERCUBES

We have seen that the hypercubes has many desirable and attractive properties. However, the hypercubes has its own intrinsic drawback, such as its diameter is large. As a result of a focused attention several enhancements of hypercubes have been proposed to improve some properties such as diameter.

3.3.1 CROSSED CUBES

Two binary x= $x_1x_2$and y= $y_1y_2$are to be pair-related, denoted by x y if and only if (x,y) ∈{(00,00),(10,10),(01,11),(11,01)}. The n-dimensional crossed cube, denoted by CQn (n ≥ 2) is such an undirected graph, its vertex set is the same as the vertex set of QnQn, two vertices x=x_1x_2…..$x_n$and y=y_1y_2 ….$y_n$are linked by an edge iff there exist j(1≤ j ≤n) such that,

a) x_n……x_{j+1}=y_n……. y_{j+1}
b) x_j≠ y_j
c) x_{j-1} = y_{j-1} if j is even
d) $x_{2i}x_{2i-1}$ ~ $y_{2i}y_{2i-1}$ for each i=1,2,…,[$\frac{1}{2}$ j]-1

PROPERTIES
*CQn is n-regular, has 2^n vertices and n2^{n-1} edges.
*CQn has diameter [$\frac{1}{2}$ (n+1)] and has connectivity n.
*CQn is vertex transitive
*CQn contains cycles with any length l(4≤l≤2^n)
*CQn contains a complete binary tree with height n-1 as its subgraph.

3.3.2 FOLDED HYPERCUBE

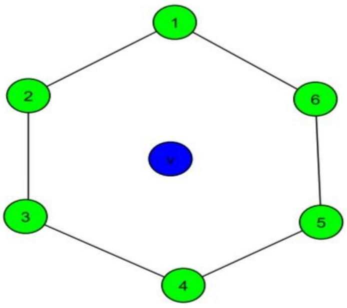
The n-dimensional folded hypercubes, denoted by FQnis an undirected graph obtained from Qn by adding all complementary edges. For two vertices x=x_1x_2…..$x_n$and y=y_1y_2 ….$y_n$of FQn x,y ∈ E(FQn) is a complementary edge iff their bites are the complement of each other i.e., y_i=$\overline{x_i}$ for each i=1,2…n

PROPERTIES
*FQn is (n+1)-regular, has 2^n vertices and $(n+1)2^{n-1}$ edges
*FQn has diameter $[\frac{n}{2}]$
*FQn has connectivity n+1.

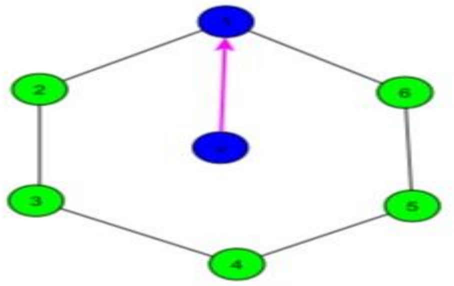
4. TRANSMITTING PROBLEM AND FAULT TOLERANCE

4.1 TRANSMITTING PROBLEM

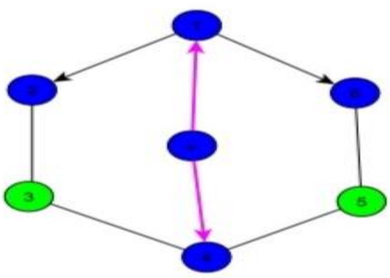
Let G= (V,E) be a graph representing the topology for a network. Let V be a special vertex outside G, called a host processor which is connected to each vertex G. The host processor is the sender of the message to be transmitted to all of the vertices in G. Each time unit. the host may send its message to any single vertex of the graph G, according to its choice. At the same time, each processor that has already received the message can send the message to all of its neighbors in one unit of time. The objective is to minimize the number of time unit such that all of the vertices in G can receive the message. The minimum number of time units for G is the optimal transmitting time for G denoted by t(G).
Consider the graph C6.
In the transmitting problem, there must be a host vertex V.



Suppose that in the first unit of time message is passed from v to 1.



In second unit of time the message is automatically passed from 1 to its neighbors 2 and 6. Then from the host processor, a message can be transmitted to all at the same time.



In third time all vertices get message.
We can consider the transmitting problem as a problem to not only determine the optimal transmitting time but also to minimize the workload of the host is defined as the number of time units in which the host send the message to processors in G.

Thus in the (t,s) transmitting scheme each processor in G receives the message either from the host or from its neighbor within t time units. While the workload host is S, t is called feasible transmitting time. The cost of a (t,s) transmitting scheme is the ordered pair (t,s).

4.1.1 TRANSMITTING SCHEME IN HYPERCUBES

For n-dimensional hypercube Qn, Alon[8] proves that t(Qn) = $[\frac{n}{2}]$+ 1.
Procedure is very simple. The host vertex will pass the message first to any arbitrary vertex and then to that vertex at hamming distance n from that vertex. Afterwards the host just waits.

4.2 MENGER'S THEOREM

Let G be a connected undirected graph or a strongly connected diagraph, x and y be two distinct vertices of G. Then

a) $\zeta(G; x, y) = K(G; x, y)\ if\ (x, y) \notin E(G)$
b) $\eta(G; x, y) = \lambda(G; x, y)$

where $\zeta(G; x, y)$ is the maximum number of internally disjoint (x,y)-paths in G.η(G;x,y), is the maximum number of edge-disjoint (x,y)-paths in G.K(G;x,y) is maximum cardinality over all (x,y)-vertex cut of G is called (x,y)-connectivity. λ(G;x,y) isthe minimum cardinality over all (x,y) edge cuts if G is called (x,y)-edge connectivity.

NOTE
Menger's theorem is one of the most fundamental results in graph theory and a foundation of topological structure design and analysis of interconnection network as well.

4.3 FAULT TOLERANCE

Fault of some components and/or communication line in a large scale system are inevitable. However the presence of faults gives rise to a large number of problems to have to be handled for some applications. Generally speaking, the solutions of these problems are difficult as the set of fault is not known in advance. Nevertheless, they have attracted considerable research interest the recent decade, and many nice results have been obtained.

4.4 FAULT TOLERANT DIAMETER

The concept of fault tolerant diameter was introduced by Krishnamoorthy M.S and Krishnamurthy.B (1987), who gave an upper bound of the faulttolerant diameter of cartesian product graph G1□G2.
i.e, D_{k1+k2}(G1□G2) ≤ D_{k1}(G1) + D_{k2}(G2)

REMARK
Xu et al. (2005) pointed out that is this bound is not correct and showed that D_{k1+k2}(G1□G2) ≤ D_{k1} (G1) + D_{k2}(G2)+ 1

5.VIBROACOUSTICS AND GRAPH THEORY

5.1 VIBROACOUSTICS

Over the last decades, vibroacoustics has become a necessary aspect to consider in many type of industries.The vibroacoustic research focusses on the simulation, analysis, monitoring and control of the vibroacoustic behavior of mechanical, mechatronic and biomechanical products and process, as typically encocentered in the transportation, the industrial machinery, the energy and the health care sector. For instance, in the automotive industry, attention is increasingly paid to improving the passengers comfort. This implies reducing the noise and vibration levels in the passenger's cabin. The same could be applied to trains, planes and ships. In the building industry, the standards regarding the admissible noise levels in dwelling have customarily been raised due to increasing social awareness. Furthermore, not only noise and vibration issues are a concern to human wellbeing but also they may affect the endurance of the operating life of structures, i.e. excessive vibrations can cause damages in the structure itself. In all of these situations, it is important to know the behavior of the mechanical systems in order to solve noise and vibration problems. The dynamic characteristics of the mechanical system can be obtained by means of a vibroacoustic analysis, which shall be conducted experimentally or numerically. The experimental approaches are all based on measurements over the built-up system. For instance, to obtain the vibration modes of a system, one may perform experimental modal analysis whereas a Transmission Path Analysis (TPA) provides information on how the energy is transmitted throughout the system.

5.2 INTRODUCTION TO MODELING IN VIBROACOUSTICS

The vibroacoustic behavior of a physical system can be modelled using several numerical methods. The most common criterium to classify them is according to the frequency range, i.e. low frequency, mid frequency and high frequency methods. As a first approximation, to determine the frequency range, the characteristic physical dimension of the system may be compared to the dominant wavelength in the dynamic response. Therefore, when the wavelength is larger or comparable to the dimensions of the system, it is considered a low frequency response. On the contrary, when the wavelength is considerably smaller, the problem is set at high frequencies. Finally, the mid frequency range corresponds to two possible situations. First, the intermediate case where the wave-length is neither comparable to the system dimensions nor small enough to be considered high frequency. Second, the case when some components show low frequency behavior and some others show high frequency behavior. Each frequency range has its own characteristics and that is the reason why each method has a limited scope of validity.

5.2.1 LOW AND HIGH FREQUENCY NUMERICAL METHOD

In the low frequency range, deterministic approaches can be used to predict the vibration modes because there are few modes and they are well-separated. The most applied techniques are the Finite Element Method (FEM) and the Boundary Element Method (BEM). These are element-based methods because they consist in dividing the problem domain or its boundary into a largest of small elements. The problem unknowns are approximated by polynomials inside every element. The size of the element is chosen to guarantee the accuracy of the approximations, typically, between 6 and 10 elements per wavelength are needed. As the frequency increases and hence the wavelength shortens, more elements increasing the matrix system dimensions become necessary. As a result, the increase of the computational cost limits the application of these methods to low frequencies.
In the high frequency range, the opposite situation is found: the modal density and the modal overlap are high. In addition, since the wavelength is short, the system behavior becomes more sensitive to smallvariations in the physical parameters. Therefore, taking into account that in a real context such variations are unavoidable, determining the response of a single system becomes meaningless and average responses shall be computed instead. Consequently, statistical approaches such as the Statistical Energy Analysis (SEA) are the most used methods. In SEA, the system is divided into a small group of subsystems and the spatially averaged energy level is computed for each of them. The number of degrees of freedom in SEA corresponds to the number of subsystems, which makes the computational cost fairly affordable. However, the assumptions that are made to allow ensemble averaging are only fulfilled at high frequencies. Consequently, statistical methods are restricted to this frequency range.

5.2.2 THE MID FREQUENCY PROBLEM

As mentioned above, there is a frequency gap where none of the above mentioned methods can be used for the correct description of the system response.The mid-frequency methods appear to fill this frequency gap. In recent years, several strategies have been followed to tackle the mid-frequency problem. They can be sorted in three groups. The first includes the methods which try to extend the applicability of low frequency methods to higher frequencies. For instance, the Wave Based Methods (WBM) which adopt exact solutions of the governing partial differential equations as basis functions, instead of the polynomial bases of finite elements.The second category is made up of approaches that try to extend the statistical methods to lower frequencies. For instance, proposing alternatives to compute the SEA parameters like the Energy Distribution Methods or attempting to relax the SEA assumptions which restrict the validity of SEA to a limited number of cases.The last class consists in combining statistical and deterministic approaches, to deal with the cases where there is a mix of dynamic behavior, as described before. In such cases, hybrid strategies are proposed.One of them is the FE/SEA approaches which uses FE for the deterministic components and SEA for the highly random components.In a nut shell, the method goes as follows. FE is used to model the deterministic components and SEA is used for the random components, modelling them as SEA subsystems.

5.3 STATISTICAL ENERGY ANALYSIS

The Statistical Energy Analysis was developed in the early 1960s partly motivated by the need to predict the vibrational response of satellite launch vehicles and their payloads. These are really huge systems formed by many components each of which has different characteristics and a high amount of vibration modes in the frequency range of study. Thus, at that time and for the reasons stated before, an alternative had to be proposed. The first works considered as the origins of SEA were made independently by R.H. Lyon and P. W. Smith, Jr [Lyon and DeJong, 1998].

5.3.1 PRINCIPLES AND VALIDITY SCOPES OF STATISTICAL ENERGY ANALYSIS

Statistical Energy Analysis is used to predict the vibroacoustic behavior of a mechanical system at high frequencies. The system consists of a group of coupled elements, normally, structures, cavities or ducts. To model it with SEA, it is divided in several subsystems which correspond to groups of vibration modes with similar characteristics.In order to make this definition of the system which consists in dividing it in subsystems and establishing the power flows, SEA lays on the following principles.

1) The power flow between two subsystems is proportional to the difference of their energy densities or modal energies. This is known as the Coupling Power Proportionality (CPP) condition.
2) The energy is stored in resonance modes. Thus, the amount of energy a subsystem is able to keep is proportional to the number of modes it has.
3) The energy always flows from the subsystem with higher energy to the one with lower energy.
4) The injected power in a subsystem is either transferred to other subsystems or dissipated in the same subsystem.
5) From a wave approach point of view, every point in a subsystem has the same average energy level, in other words, the energy density is homogeneous and isotropic all over the subsystem. This is known as the diffuse field assumption. From a modal pointof view, it is equivalent to state that modal energy equipartition exists, which means that the subsystem

energy is uniformlyshared amongst the modes. In other words, that the average modal energy is equal for all the modes in a subsystem.

In addition, to simplify the formulation and the development of the models, it is also assumed that:

6) The excitation forces are uncorrelated white noises to ensure that all the modes in the subsystem are equally excited. Typically, for the plates a rain-on-the-roof excitation is used.
7) The energy can only be input in the subsystems, never in the couplings between subsystems. Usually, the couplings will be considered conservative. However, more complex approaches deal with the inclusion of non-conservative couplings in the system.
8) All the modes in the same subsystem and the same frequency band have the same damping loss factor.
9) There is no coupling between the modes of the same subsystem.

As seen, SEA is a statistical method and it deals with average values of energy.

All in all, SEA is a useful method to work with high frequency models but the system must fulfill some conditions which in some cases may be quite restrictive.

5.3.2 FORMULATION OF STATISTICAL ENERGY ANALYSIS

The SEA model can be buit once the condition of validity of SEA are guaranteed in the system. The parameter needed at each frequency band of stand are the loss factors. There are three types of loss factors.

1. COUPLING LOSS FACTOR: The coupling loss factor (CLF). ηij is the ratio of energy transferred from, subsystem i to subsystem j in a radian cycle.
2. INTERNAL LOSS FACTOR: The internal or damping loss factor (ILF) ηid is the ratio of energy lost as heat in subsystem i in a radian cycle.
3. TOTAL LOSS FACTOR: The total loss factor (TLF) ηi is the total energy lost in subsystem i in a radian cycle,

ηi=ηid+$\sum_{j,j\neq i}^{N} \eta ij$, where N is the total number of subsystems.

5.4 ENERGY DISTRIBUTION MODEL

The energy distribution (ED) models aim at giving a general description of the dynamic behavior of a system in terms of vibroacoustic energy. To that end, the system is split into a set of subsystems and the so called Energy Influence Coefficients (EIC) are computed to characterize the energy sharing between them. The main purpose is to obtain the frequency averaged subsystem energies when some part of the system is submitted to a broadband excitation. The EICs can be computed either from theoretical modal developments, numerical approaches using the finite element method or scaling procedure or by resorting to experimental procedures relying on the power injection method.

5.5 STSTISTICAL MODEL ENERGY DISTRIBUTION ANALYSIS

Statistical modal Energy distribution Analysis (SmEdA), originally proposed in, can be envisaged as a particular case of ED method, in which SEA hypotheses are relaxed to extend its range of applicability to mid frequencies. However, SmEdA has a clear distinctive feature with respect to most ED methods in the sense that power balance equations are not established between subsystems but rather between the resonant modes of different subsystems. These modes can be extracted from the modal bases of uncoupled subsystems, which can be computed using FEM, thanks to the dual modal formulation (DMF). This offers the possibility of considering subsystems with complex geometries and varying properties. Moreover, circumventing SEA energy equipartition allows one to deal with locally excited subsystems with low modal overlap, as well as to evaluate the spatial distribution of energy density within subsystems. Recently, SmEdA has been extended to incorporate the contribution of non-resonant transmission through condensation of the DMF equations. This has resulted in the appearance of indirect coupling between modes in non-physically connected subsystems, standard non-resonant paths in SEA being recovered as a particular case

5.6 TRANSMISSION PATH ANALYSIS

Vibroacoustic analysis may be used to solve noise and vibration problems. Generally, these situations consist of a vibroacoustic source that generates an excessive energy level in another part of the system, normally termed target or receiver.Consider for instance, a car where the vibrations produced by the engine generate an uncomfortable noise in the passenger compartment. Therefore, some parts of the system will have to be modified to reduce the energy level at the target up to an acceptable value.A different way to tackle the problem is by determining how energy is transmitted from the vibroacoustic sources to the targets. In other terms, to identify the energy transmission paths. The experimental methods that traditionally have followed this approach are known as Transmission Path Analysis (TPA) techniques.

The TPA methods can be classified into 1-step processes or 2-step processes.

› The 1-step methods, only use operational data and thus, just one phase is needed.
› The 2-step methods consist of two steps of measurements. In the first step, the system is characterized in its static state whereas in the second one, the measurements are carried out under operational conditions.

The problem with 1-step methods is the separation of partially correlated sources. For this reason, in the 1980's, the 2-step TPA approaches appeared. The most relevant is a 2-step method commonly known as classical TPA. The aim of this method is knowing how the operational loads acting on a system influence the response of some selected degrees of freedom of the system.

Let us describe in detail how the contribution of an energy transmission path is computed. Consider the plate system (fig A). The SEA model is built considering only flexural waves. The source set at Plate 1 and the target at Plate 6. The energy contribution to the target subsystem of the path starting at Plate 1 followed by Plate 2 and ending at Plate 6 will be computed step by step, following the same procedure we can create the SEA diagram corresponding to this system where the path to be computed is marked in yellow. First, it is supposed that energy in subsystem 4 is only input from subsystem 1. Note that this assumption is equivalent to assume that, except for 1 and 4, all the subsystems are blocked, so their energy is null. This description coincides with the concept of direct transmissibility which has also been used to compute transmission path analysis in SEA systems.

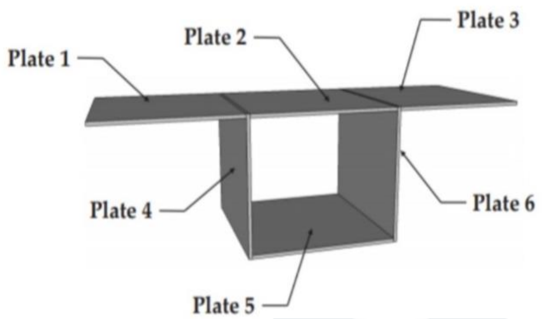


Fig A: Plate SEA system

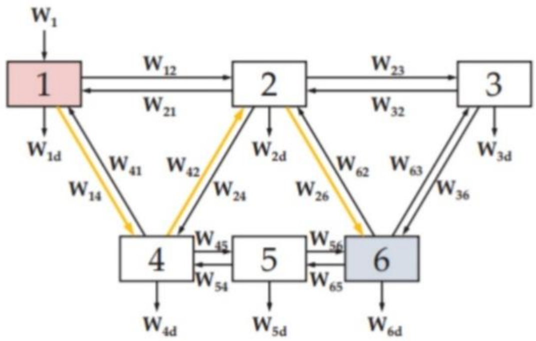


Fig B: Energy transmission path

5.7 USE OF GRAPH THEORY TO OBTAIN THE ENERGY TRANSMISSION IN A SEA MODEL

In Section 5.6, the worth of obtaining the energy transmission paths in a vibroacoustic system has been proved. Besides a review of some experimental procedures, the analytical definition of an energy transmission path in a SEA system has been given. However, a direct method to obtain such paths and to work with them is still missing.
The graph represented the SEA system is called SEA graph and is defined as follows:
A SEA graph, $G_{\mathrm{SEA}} = \{U_{\mathrm{SEA}}, E_{\mathrm{SEA}}\}$ is a simple diagraph such that every node ui in U_{SEA} corresponding to a SEA subsystem and such that directed arcs (ui,uj), (uj,ui) ∈ ESEA exist between subsystem ui and uj, whenever they are coupled in the SEA model.

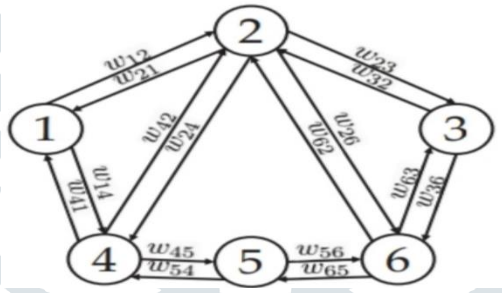


Fig: C

Thus, the SEA graph that corresponds to the SEA system in Fig:A is the one depicted in Fig:C.

6. CONCLUSION

In this paper, we had discussed about interconnection networks and graphs. Graphs are considered as the topological structure of interconnection networks. Here we discussed some well-known topological structures of interconnection networks. We also discussed transmitting problem and fault tolerences of processors. Interconnection networks has many application in real world. In this paper, an overview of the numerical methods used to solve vibroacoustic problems is given. First,the numerical methods used in vibroacoustics are reviewed. Special emphasis is given to Statistical Energy Analysis, Energy Distribution Analysis and Statistical modal Energy distribution Analysis. Then,the potential of TPA is introduced. Finally, the first link between graph theory and vibroacoustic models, in this case SEA models, is established by defining a SEA graph.

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