MHD Boundary Layer Hyperbolic Fluid flow over a Vertical Cone with Uniform Suction

G. Raju
Department of Mathematics, IIIT Ongole Campus, RGUKT-516330, Andhra Pradesh, India.

Abstract: In this work, we addressed the characteristics of two dimensional magnetic boundary layer flow of Hyperbolic Tangent fluid towards vertical cone with uniform suction condition. To develop the mathematical descriptions of Hyperbolic fluid, energy and momentum are accounted. By as set of non-similarity transformation, the proposed leading PDEs of flow phenomena are converted into non-linearly ODEs with boundary condition and then the solved numerically. The numerical solutions of the non-dimensional system equations have been illustrated. MATLAB software as well-recognized scheme is operated to solve problem for numerous values of governing parameters.

***Index Terms* - MHD Hyperbolic Fluid, Weissenberg Number, Suction, Injection, MATLAB BVP4C method.**

INTRODUCTION

A The flow that excited by a vertical cone is applicable at most of the industrial sector like, extrusion processes in plastic, at metal industries and also in manufacturing processes such as spinning of metals, glass fiber, and in wire drawing, etc. [1-4]. The tangent hyperbolic fluid is used far for different laboratory experiments. Friedman et al. [5] have used the tangent hyperbolic fluid model for large-scale magneto-rheological fluid damper coils. The number of researchers worked on non-Newtonian fluids. Examples of such fluids include coal-oil slurries, grease, shampoo, paints, custard, cosmetic products, and physiological liquids. The classical equations employed in simulating Newtonian viscous flows i.e. the Navier–Stokes equations fail to simulate a number of characteristics of non-Newtonian fluid. Recent investigations have implemented, Amanulla et al. [6] described a wide number of numerical solutions for Magneto-hydrodynamic and non- Newtonian fluid flow through a vertical cone with the presence of slip effects, using finite difference method. Cheng [7] investigated heat transfer of power law nanofluid passed above a truncated cone embedded in a porous medium. Elbashbeshy et al. [8] analyzed radiation effects non-Newtonian fluid flow past a vertical truncated cone. Amanulla et al. [9] discussed the velocity slip and temperature jump conditions on hydrodynamic fluid flow over a circular cylinder. Reddy and Pradeepa [10] studied viscous dissipation and sorret effects on natural convection flow over a truncated cone in the presence of Biot number. Several authors [11-13] investigated heat transfer of non-Newtonian fluid of various physical problems.

The present work, we inspect theoretically and also computationally the steady-state transport things in magneto-hydrodynamic non-Newtonian fluid flow from a vertical cone by means of Convective heating effects. It has been found that the Magnetic field is extremely influence the characteristics of heat transfer and velocity in the flows over curved body. Relevant examples are expressed by Bég et al. [14] (used for cylindrical geometries), Alkasasbeh et al. [15] who addressed radiative effects, and also considered the drag effects of porous medium and Kasim et al. [16] who utilized a prominent viscoelastic model. With above analysis in mind, we are interested in analytical approximation of convective flow (non-Newtonian) in vertical cone. The reduced highly nonlinear partial differential equations are solved with help of accurate Keller-Box Technique (finite difference) (5-10). The impact of various pertinent parameters, like Prandtl number (Pr), Weissenberg parameter (We), Power law index (m), Magnetic parameter (M) and Suction/Injection parameter are plotted and discussed

MATHEMATICAL FORMULATION:

Consider a steady, two dimensional free convective heat transfers along the vertical porous plate embedded in non-Newtonian Tangent Hyperbolic fluid. The x-coordinate (tangential) is measured along the surface of the porous plate from the lowest point and the y-coordinate (radial) is directed perpendicular to the surface. The equations for continuity, momentum and energy can be written as follows:

A steady, laminar, two dimensional boundary layer flows and heat transfer of an incompressible tangent hyperbolic fluid over a vertical porous plate. Both the plate and Tangent Hyperbolic fluid are maintained primarily at the equivalent temperature. straight away they are raised to a temperature $T_w > T_\infty$, the ambient temperature of the fluid which remains unchanged.

$$\frac{\partial(ur)}{\partial x}+\frac{\partial(vr)}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu(1-n)\frac{\partial^2 u}{\partial y^2}+\sqrt{2}\nu n\Gamma\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)\cos A-\frac{\sigma B_0^2}{\rho}u \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \quad (3)$$

where u and v are the velocity components in the x- and y-directions respectively, ν=μ/ρ is the kinematic viscosity of the Tangent Hyperbolic fluid, β is the coefficient of thermal expansion, α is the thermal diffusivity, β is the temperature, and ρ is the density of the fluid. The Tangent Hyperbolic fluid model therefore introduces a mixed derivative (second order, first degree) into the momentum boundary layer Eq. (2). Boundary conditions are:

$$\begin{aligned}&As \quad y=0\ :\ u=u_W,\ \ v=-v_W,\ T=T_W\\ &At \quad y\to\infty:\ u\to 0,\ \ T\to T\end{aligned} \quad (4)$$

Here T_∞ is the free stream temperature, k is the thermal conductivity, h_w is the convective heat transfer coefficient, and T_w is the convective fluid temperature. The stream function ψ is defined by $u=\frac{\partial\psi}{\partial y}$ and $v=-\frac{\partial\psi}{\partial x}$, and therefore, the continuity equation is automatically satisfied.

In order to provide the governing equations and the boundary conditions in dimensionless form, the following non-dimensional quantities are introduced.

$$\begin{aligned}&\xi=\frac{V_0 x}{\nu Gr^{1/4}},\eta=\frac{y}{x}(Gr)^{1/4},\psi=\nu(Gr)^{1/4}\left(f(\xi,\eta)+\frac{\xi}{2}\right)\\ &\theta(\xi,\eta)=\frac{T-T_\infty}{T_w-T_\infty},Gr_x=\frac{g\beta_T(T_w-T_\infty)x^3\cos\mathrm{A}}{4\nu^2},M=\frac{\sigma B_0^2x^2}{\rho\nu Gr^{1/2}}\end{aligned} \quad (5)$$

In view of the transformations defined in (5). The boundary layer equations (2)-(3) are reduced to the following nonlinear, dimensionless partial differential equations for momentum, energy for the regime.

$$(1-n)f'''-\frac{1}{2}f'^2+\frac{7}{4}ff''+nWef''f'''-Mf'+\theta=\frac{7}{4}\xi\left(f'\frac{\partial f'}{\partial\xi}-f''\frac{\partial f}{\partial\xi}\right) \quad (6)$$

$$\frac{1}{\Pr}\theta''+\frac{7}{4}f\theta'=\frac{7}{4}\xi\left(f'\frac{\partial\theta}{\partial\xi}-\theta'\frac{\partial f}{\partial\xi}\right) \quad (7)$$

The transformed dimensionless boundary conditions are as follows:

$$\begin{aligned}&At\ \ \eta=0\ \ :\ \ f=S,\ f'=0,\theta=1\\ &As\ \eta\to\infty:\ \ f'\to 0,\ \ \theta\to 0\end{aligned} \quad (8)$$

The skin friction coefficient (shear stress at the sphere surface) and Nusselt number (heat transfer rate) can be defined using the transformations described above with the following expressions.

$$\frac{1}{2}Gr^{-3/4}C_f=f''(\xi,0)+\frac{We}{2}(f''(\xi,0))^2 \quad (9)$$

$$Gr^{-1/4}Nu=-\theta'(\xi,0) \quad (10)$$

NUMERICAL SOLUTION WITH KELLER-BOX IMPLICIT METHOD

The Keller-Box implicit difference method is implemented to solve the nonlinear boundary value problem defined by eqns. (7)–(8) with boundary conditions (10).This technique despite recent developments in other numerical methods as elaborated by Keller, 1978. This method has been used widely and efficiently for over three decades in a large spectrum of nonlinear fluid mechanics problems. These include laminar transport phenomena [17-18] and Viscoelastic boundary layer flows [19-20].

RESULTS AND DISCUSSIONS

In order to obtain a physical approaching into the problem, a representative set of numerical results are presented in Figs. 1-5. The numerical problem comprises of two independent variables (ξ,η), two dependent fluid dynamic variables (f,θ) and thermo-physical and body force control parameters, viz., We, n, Pr, M, f_w. The following default parameter values i.e., We = 0.3, n = 0.1, Pr = 1.0, M = 1.0, f_w = 0.3 are discussed.

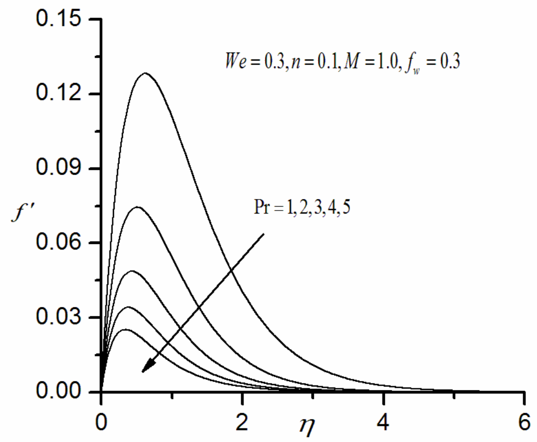

Fig. 1(a) Velocity profiles for different values of Pr

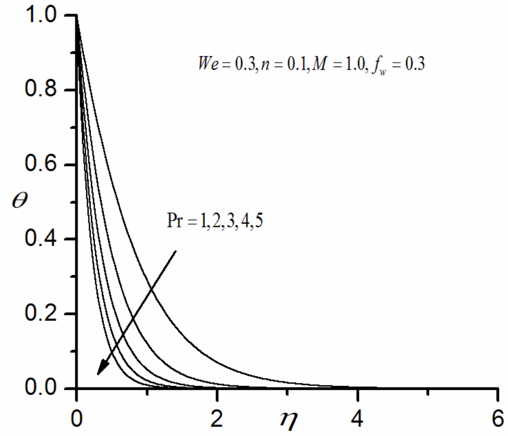

Fig. 1(b) Temperature profiles for different values of Pr

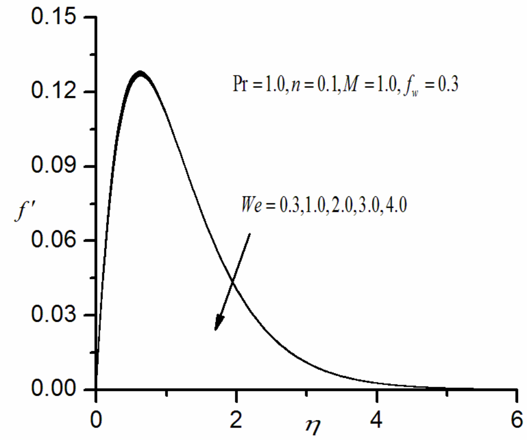

Fig. 2(a) Velocity profiles for different values of We

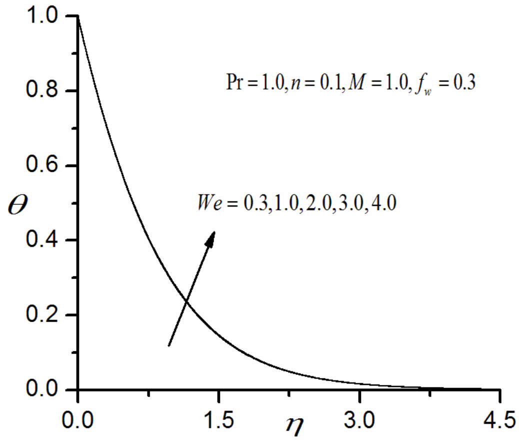

Fig. 2(b) Temperature profiles for different values of We

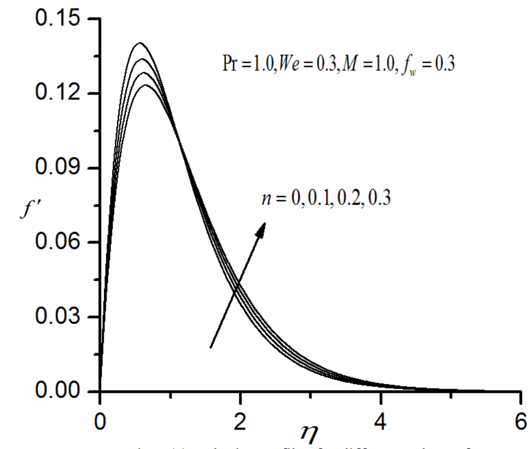

Fig. 3(a) Velocity profiles for different values of n

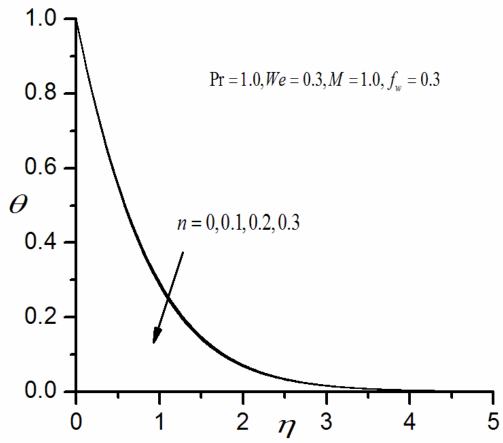

Fig. 3(b) Temperature profiles for different values of n

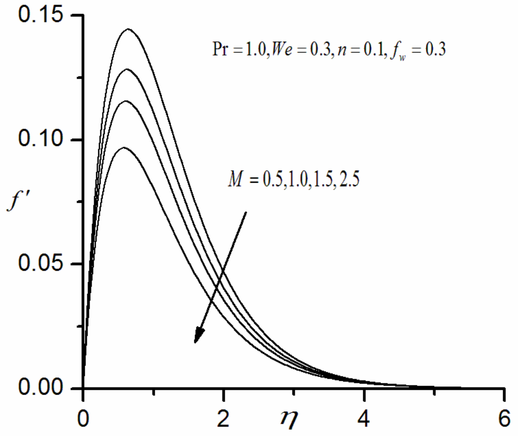

Fig. 4(a) Velocity profiles for different values of M

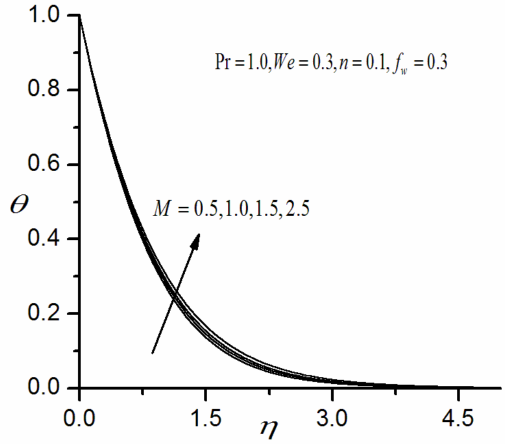

Fig. 4(b) Temperature profiles for different values of M

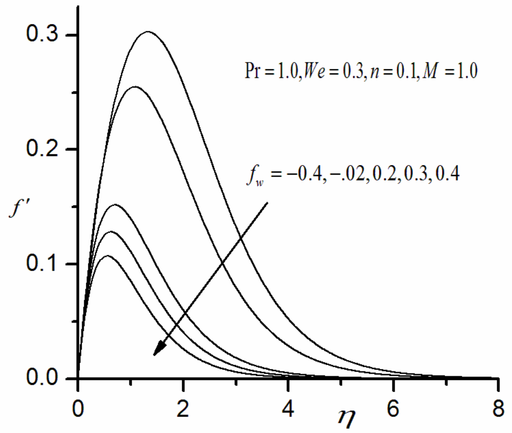

Fig. 5(a) Velocity profiles for different values of fw

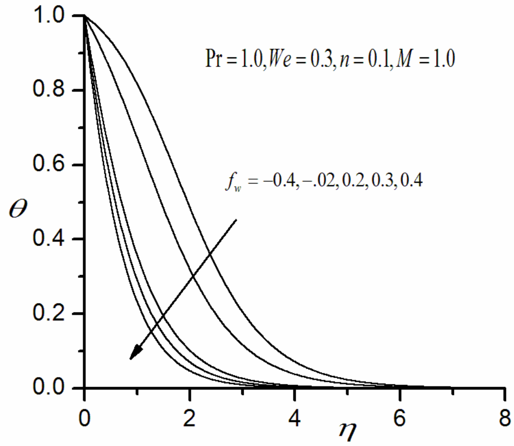

Fig. 5(b) Temperature profiles for different values of fw

Figure 1(a)-1(b) depict the velocity $f'(\eta)$, temperature $\theta(\eta)$ distributions for various values of Prandtl number Pr. It is observed that an increase in the Prandtl number significantly decelerates the flow i.e., velocity and temperature decreases. Figure 2(a)-2(b) shows the velocity $f'(\eta)$, temperature $\theta(\eta)$ distributions, the influence of Weissenberg number We, absorbed that velocity increases, temperature and concentration are decreases.

Figures 3(a) - 3(b) illustrates the effect of the power law index, *n*, on the velocity $f'(\eta)$ and temperature $\theta(\eta)$ distributions through the boundary layer regime. Increasing *n* velocity is increased. Conversely temperature is consistently reduced with increasing values of *n*.

Figures 4(a)-4(b) the dimensionless velocity $f'(\eta)$, temperature $\theta(\eta)$ for various values of magnetic parameter M are shown. Fig. 4(a) represents the velocity profile for the different values of magnetic field parameter M. It is observed that velocity of the flow decreases significantly with increasing values of magnetic parameter M. In Fig. 3(b), the temperature distribution increases with increasing magnetic values.

Figure 5(a)-5(b) this plots indicates the influence of Suction/ Injection parameter f_w. We observed that increase the Suction/ Injection f_w parameter throughout the region decreases the velocity $f'(\eta)$ and temperature $\theta(\eta)$.

CONCLUSIONS:

Mathematical solutions have been obtained for the convection heat transfer boundary layer flow through the vertical porous plate the presence of Suction/Injection effects, using the local non-similarity finite difference method.

1. Increasing Weissenberg number We, decreases velocity, whereas increases temperature.
2. Increasing power law index, n, increases velocity, whereas, decreases temperature.
3. Increasing Suction/Injection f_w, decreases velocity, temperature.
4. Increasing magnetic parameter M, increases velocity, whereas, decreases temperature.

REFERENCES

[1] Altan., T., S. Oh, H. Gegel, Metal Forming Fundamentals and Applications, American Society of Metals, Metals Park, OH, (1979).

[2] Fisher, E.G., Extrusion of Plastics, Wiley, New York, (1976).

[3] Tadmor, Z., and I. Klein, Engineering principles of plasticating extrusion, Polymer Science and Engineering Series, New York, Van Norstrand Reinhold Co., (1970).

[4] Sakiadis, B.C., Boundary-layer behavior on continuous solid surface: I. Boundary-layer equations for two-dimensional and axisymmetric flow, AIChe Journal. 7(1961) 26-28.

[5] Friedman, A.J., S.J. Dyke, B.M. Phillips, Over-driven control for large-scale MR dampers, Smart Mater. Struct. 22 (2013) 045001, 15pp.

[6]. Amanulla, C. H., N. Nagendra and M. Suryanarayana Reddy (2017). MHD Flow and Heat Transfer in a Williamson Fluid from a Vertical Permeable Cone with Thermal and Momentum Slip Effects: A Mathematical Study, Frontiers in Heat and Mass Transfer, 8, 40.

[7]. Cheng, C.Y. (2012). Free convection of non-Newtonian nanofluids about a vertical truncated cone in a porous medium, Int. Comm. Heat Mass Transf., 39(9), 1348–1353.

[8]. Elbashbeshy, E. M. A., T. G. Emam, and E. A. Sayed (2016). Effects of thermal radiation on free convection flow and heat transfer over a truncated cone in presence of pressure work and heat generation/absorption, Thermal Science, 20(2), 555-565.

[9]. Amanulla, C. H., N. Nagendra, and M. Surya Narayana Reddy, (2018). Numerical Simulation of Slip Influence on Electric Conducting Viscoelastic Fluid Past an Isothermal Cylinder, Frontiers in Heat and Mass Transfer, 10, 10.

[10]. Ram Reddy, C. H., and T. Pradeepa (2016). Non- similarity solution of micropolar fluid flow over a truncated cone with soret and viscous dissipation effects using spectral quasilinearization method, Int. J. Appl. Comput. Math., 1-15.

[11]. Amanulla, C. H., N. Nagendra and M. Surya Narayana Reddy (2017). Numerical Study of Thermal and Momentum Slip Effects on MHD Williamson Nanofluid from an Isothermal Sphere, Journal of Nanofluids, 6(6), 1111–1126.

[12]. Amanulla, C. H., N. Nagendra, and M. Surya Narayana Reddy (2017). Computational analysis of non-Newtonian boundary layer flow of nanofluid past a semi-infinite vertical plate with partial slip, Nonlinear Engineering, 1-15.

[13]. Amanulla, C.H., Nagendra, N., and Suryanarayana Reddy, M., 2017, "Multiple Slip Effects on MHD and Heat Transfer in a Jeffery Fluid over an Inclined Vertical Plate," International Journal of Pure and Applied Mathematics, 113(7), 137-145.

[14]. Bég, O. A., J. Zueco, M. Norouzi, M. Davoodi, A. A. Joneidi and A. F. Elsayed (2014). Network and Nakamura tridiagonal computational simulation of electrically-conducting biopolymer micro-morphic transport phenomena, Computers in Biology and Medicine, 44, 44–56.

[15]. Alkasasbeh, H. T., M. Z. Salleh, R. Nazar and I. Pop (2014). Numerical solutions of radiation effect on Magnetohydrodynamic free convection boundary layer flow about a solid sphere with Newtonian heating, Applied Mathematical Sciences, 8(140), 6989 – 7000.

[16]. Kasim, A. R. M., N. F. Mohammad, I. Anwar and S. Shafie (2013). MHD effect on convective boundary layer flow of a viscoelastic fluid embedded in porous medium with Newtonian heating, Recent Advances in Mathematics, 4, 182-189.

[17]. Amanulla, C.H., Nagendra, N. and Suryanarayana Reddy, M., "Numerical Simulations on Magnetohydrodynamic Non Newtonian Nanofluid Flow Over a Semi-Infinite Vertical Surface with Slip effect," Journal of Nanofluids, 2018, 7(4), 718-730.

[18]. Amanulla, C.H., Nagendra, N. and Suryanarayana Reddy, M., "Numerical Simulation of Slip Influence on the Flow of a MHD Williamson Fluid Over a Vertical Convective Surface," Nonlinear Engineering, 2018, 1-13.

[19]. Amanulla, CH, S Saleem, AR Wakif, MM AlQarni, "MHD Prandtl fluid flow past an isothermal permeable sphere with slip effects," Case Studies in Thermal Engineering, vol. 14, 100447, 2019.

[20]. Amanulla, CH., Abderrahim Wakif, Zoubair Boulahia, M. Suryanarayana Reddy, N. Nagendra, Numerical investigations on magnetic field modeling for Carreau non- Newtonian fluid flow past an isothermal sphere, Journal of the Brazilian Society of Mechanical Sciences and Engineering (2018) 40:462.