A Study on Distinct Parameters of Dominator Chromatic Number in Line Graph of a Complete Graph

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Abstract:

In this paper, we have established bounds for dominator colouring in line graph of a K_n. The distinct dominator chromatic parameters used here are dominator chromatic number $\chi_d\left(L((K_n))\right)$,total dominator chromatic number $\chi_{td}\left(L((K_n))\right)$, 2- dominator chromatic number χ_{2d} inverse dominator chromatic number $\chi_d'\left(L(G)\right)$ and also proper colouring is applied to $(L((K_n)))$, find the chromatic number $\chi(L(K_n))$of L(K_n). Using these parameters we have studied some of its properties and illustrated with examples.

Keywords:

Chromatic number $\chi(L(K_n))$, dominator chromatic number $\chi_d\left(L(K_n)\right)$, total dominator chromatic number $\chi_{td}\left(L(K_n)\right)$, 2- dominator chromatic number $\chi_{d,2}\left(L(K_n)\right)$, inverse dominator chromatic number $\chi_d'\left(L(K_n)\right)$.

1. Introduction:

In graph theory, colouring and domination are two important areas which have been extensively studied where vast number of practical problems are involved. The dominator colouring possesses the blends of graph coloring as well as domination in graphs. Dominator colouring was introduced by Ralucca Michelle Gera in 2006.(1). In this paper, we determined the distinct parameters of dominator chromatic number for line graph of a complete graph L(K_n). We start with notation and more formal definitions.

Definition 1.1: (4)

An assignment of colours to the vertices of a graph such that no two adjacent vertices get the same colour is called *proper colouring* of a graph. The minimum number of colours needed to colour a graph G is called the *chromatic number* and is denoted by $\chi(G)$.

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Definition 1.2:

A non- empty subset D of V is called a *dominating set* of G if every vertex of D is adjacent to each vertex of V-D. The *domination number* $\gamma(G)$ of a graph G is the minimum cardinality of a dominating set of G.

Definition 1.3: (1)

A proper colouring is called *dominator colouring* if each vertex of the graph dominates every vertex of some colour class. The *dominator chromatic number* $\chi_d(G)$ is the minimum number of colours required for a dominator coloring of G.

Definition 1.4: (3)

The *total dominator colouring* of a graph G is a proper colouring of G in which each vertex of the graph is adjacent to every vertex of some colour class. The *total dominator chromatic number* $\chi_{td}(L(K_n))$ of G is the minimum number of colour classes in total dominator colouring of G.

Definition 1.5: (2)

The *2- dominator colouring* of graph G with minimum degree atleast one is a proper colouring of graphs with the extra property that every vertex in the graph properly dominates a two colour class. The *2- dominator chromatic number* $\chi_{d,2}(L(K_n))$ of G is the minimum number of colour classes in 2- dominator colouring of G.

Definition 1.6:

The *inverse dominator colouring* of graph G is a proper colouring of G in which every vertex in D' properly dominates the colour class in V- D'. The *inverse dominator chromatic number* $\chi_{d'}(L(K_n))$ of G is the minimum number of colour classes in inverse dominator colouring of G.

Definition 1.7:

The graph in which any two distinct vertices are adjacent is called *complete graph* and it's denoted by K_n, where n is the order of the graph.

Definition 1.8:

Line graph L(G) of a graph G is that the vertices of L(G) are the edges of G and two vertices of L(G) are adjacent if the corresponding edges of G are adjacent.

2. Bounds on Distinct dominator chromatic number in line graph of a complete graph $L(K_n)$:

Theorem 2.1:

Let $L(K_n)$ be a line graph of a complete graph. Then $\chi(L(K_n)) = \chi_d(L(K_n)) = n \; \forall \; n > 5$

(−1)

Proof:

Given that, $L(K_n)$ is a line ~~graph of K_n and hence~~ by definition of line graph we have vertices and (−1)(−2) edges.

We prove this by induction method,

By the definition of proper colouring, $\chi(L(K_n))$ is the chromatic number of

$L(K_n)$ For n=6, we get the line graph of order 15 and size 60 we have, (ie)

$V(L(K_6)) = \{v_1, v_2, v_3, \ldots, v_{15}\}$

By the definition of proper colouring, we assigned the colors set $C_1, C_2, C_3, C_4, C_5, C_6$ to the vertices $v_1, v_2, v_3, v_4, v_5, \ldots\ldots v_{15}$ such that no two adjacent vertices get the same colour.

$\therefore \; \chi(L(K_6)) = n = 6$

For n= 7, we get the line graph of order 21 and 105 edges.

(ie) $V(L(K_7)) = \{v_1, v_2, v_3, \ldots, v_{21}\}$

By the definition of proper colouring, we assigned the colours $C_1, C_2, C_3, C_4, C_5, C_6, C_7$ for the vertices $v_1, v_2, v_3, v_4, v_5, \ldots\ldots v_{21}$ such that no two adjacent vertices get the same colour.

$\therefore \; \chi(L(K_7)) = n = 7$

Similarly, for n= 8 the line graph is of order 28 and size 168.

Therefore by proper colouring we assigned the colours $C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8$ for the vertices $\{v_1, v_2, v_3, v_4, v_5, \ldots\ldots v_{28}\}$ such that no two adjacent vertices get the same colour.

$\therefore \; \chi(L(K_8)) = n = 8$

Proceeding in this way for all n > 5, we have $\chi(L(K_n)) = n \longrightarrow (1)$

Let $\chi_d(L(K_n))$ be the dominator chromatic number for line graph of K_n.

For n= 6, we have $V(L(K_6)) = \{v_1, v_2, v_3, \ldots, v_{15}\}$

The dominating set of $L(K_6)$ is $D = \{v_3, v_6, v_{13}\}$

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By the definition of dominator colouring, we assigned colour (C_1) to the dominating set

D= {v_3, v_6, v_{13}}. Since no two of its vertices are adjacent in dominating set and the remaining vertices of V-D are colored by C_2, C_3, C_4, C_5, C_6 using proper colouring C_2= { v_1, v_9},

C_3={ v_2, v_7, v_{11}}, C_4={ v_4, v_8}, C_5={ v_5, v_{10}, v_{15}}, C_6={ v_{12}, v_{14}} So

by the definition of dominator colouring, we have

$$\chi_d(L(K_6)) = 6$$

For n= 7, we have V(L(K_7)) = {v_1, v_2, v_3,…,v_{21}} and size 105.

The dominating set of V(L(K_7)) is D= {v_1, v_{15}, v_{18}}

Since no two vertices of D are adjacent, so we required colour one (C_1) for proper colouring and the remaining vertices of V(L(K_7)) ={ v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{12}, v_{13}, v_{14}, v_{11}, v_{12}, v_{13}, v_{14}, v_{16}, v_{17}, v_{19}, v_{20}, v_{21} }are assigned by distinct colours C_2, C_3, C_4, C_5, C_6, C_7 respectively such that C_2= { v_2, v_4, v_6 }, C_3= { v_3, v_5, v_7 }, C_4={ v_8, v_{14}, v_{16} }, C_5= { v_9, v_{17}, v_{19} }, C_6={ v_{10}, v_{13}, v_{21}}, C_7= { v_{11}, v_{12}, v_{20}}

Since each vertex dominates every vertex of atleast one colour class, so the dominator chromatic number of L(K_7) is,

$$\chi_d(L(K_7)) = n = 7$$

For n= 8, we have V(L(K_8)) = {v_1, v_2, v_3,…,v_{28}} and size 168.

The dominating set of V(L(K_8)) is D= {v_1, v_{12}, v_{15}, v_{25}}

By the definition of dominator colouring, we need colour (C_1) for the dominating set

D= {v_1, v_{12}, v_{15}, v_{25}} as no two vertices are adjacent and for the remaining vertices in V(L(K_8))

colors C_2, C_3, C_4, C_5, C_6, C_7, C_8 are assigned respectively such that C_2= { v_2, v_{17}, v_{20} },

C_3= { v_3, v_8, v_{28} }, C_4={ v_4, v_9, v_{16}, v_{23}}, C_5= { v_5, v_{10}, v_{18}, v_{26} }, C_6={ v_6, v_{11}, v_{14}, v_{21}}

C_7= { v_7, v_{13}, v_{19}, v_{24}}, C_8= { v_{22}, v_{27} }

Since each vertex dominates every vertex of atleast one colour class, so the dominator chromatic number of L(K_8) is,

$$\chi_d(L(K_8)) = n = 8$$

Proceeding in this way for all n > 5, we have $\chi(L(K_n))$= n

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In general, for all n > 5 the graph $L(K_n)$ has $\underline{(\ -1)}$ vertices and $\underline{(\ -1)(\ -2)}$ edges. The vertex set of $L(K_n)$ is given by $V(L(K_n)) = \{v_1, v_2, \ldots, v_k\}$ where $k = \frac{n(n-1)}{2}$ and the domination number is given by $\gamma(L(K_n)) = \left\lfloor \frac{n}{2} \right\rfloor$ where $\gamma(L(K_n))$ is the integral part of $\frac{n}{2}$

Therefore by the definition of dominator colouring we have,

$$\chi_d(L(K_n)) = n \longrightarrow (2)$$

From equation (1) & (2) we conclude that $\chi(L(K_n)) = \chi_d(L(K_n)) = n \ \forall \ n > 5$

Example 2.1.1:

Consider the graph $L(K_6)$

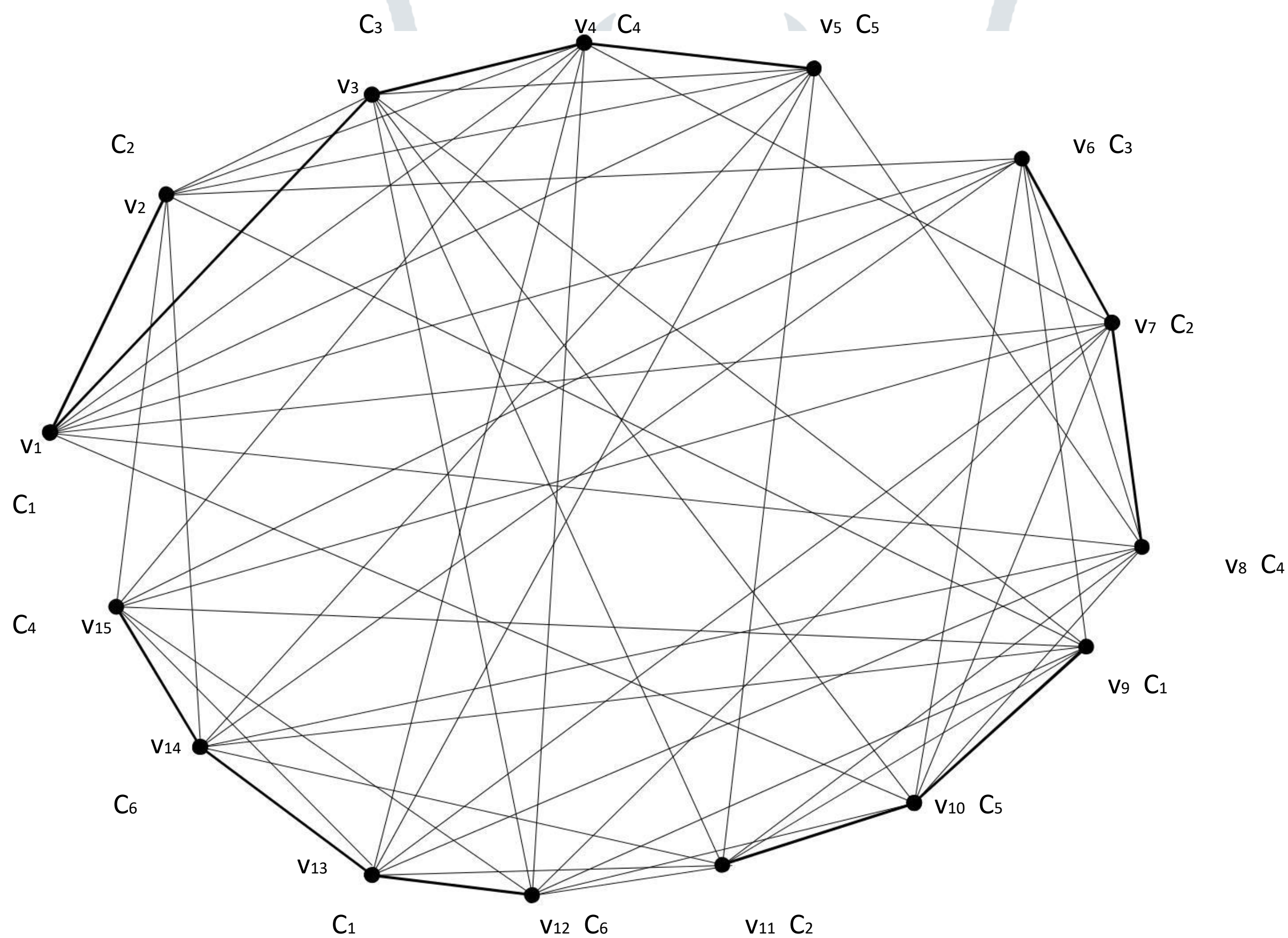

Fig: 1

By the definition of proper colouring, we assigned the colours C_1, C_2, C_3, C_4, C_5, C_6 for the vertices v_1, v_2, v_3, v_4, v_5,……v_{15} such that no two adjacent vertices get the same colour.

Hence, C_1= { v_1, v_9, v_{13} }, C_2= { v_2, v_7, v_{11} }, C_3={ v_3, v_6}, C_4= { v_4, v_8, v_{15} }, C_5={ v_5, v_{10}} C_6= { v_{12}, v_{14}}

$\therefore$ We have $\chi(L(K_n))$= 6

To find the dominator chromatic number:

Consider the graph $L(K_7)$

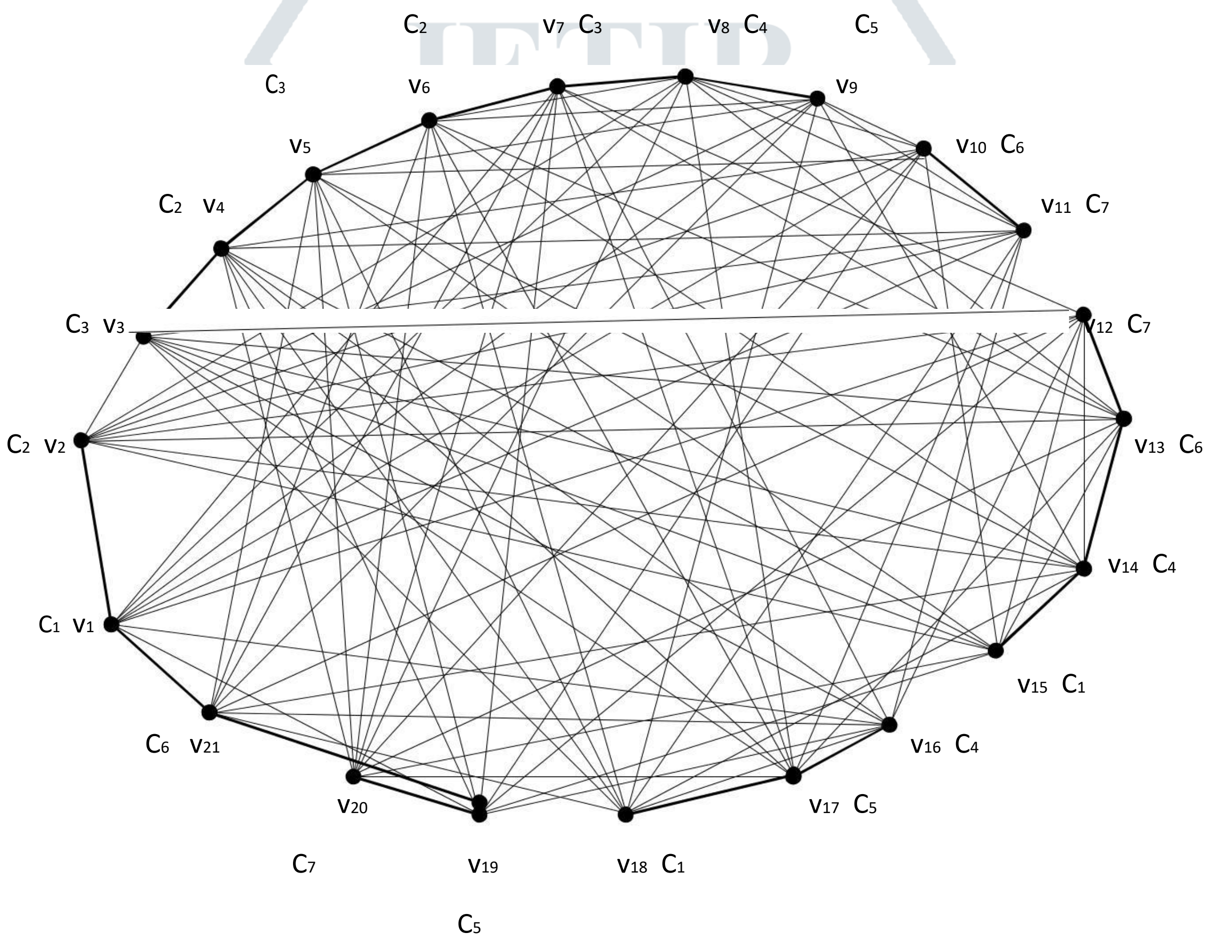

Fig: 2

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By the definition of dominator colouring, we assigned colour (C_1) to the dominating set

D={v_1, v_{15}, v_{18}} and colours C_2, C_3, C_4, C_5, C_6, C_7 for the vertices in V-D such that each vertex in L(K_7) dominates every vertex of some colour class.

(ie) C_2= { v_2, v_4, v_6 }, C_3= { v_3, v_5, v_7 }, C_4={ v_8, v_{14}, v_{16} }, C_5= { v_9, v_{17}, v_{19} }, C_6={ v_{10}, v_{13}, v_{21}} C_7= { v_{11}, v_{12}, v_{20}}

$\therefore$ We have $\chi_d\left(L(K_7)\right) = 7$

In general we have, $\chi(L(K_n)) = \chi_d\left(L(K_n)\right)$ = n for n > 5

2.2 Relation between chromatic number and dominator chromatic number:

For any line graph of a complete graph L(K_n) , $\chi(L(K_n)) \leq \chi_d\left(L(K_n)\right)$ where $\chi(L(K_n)) = \chi_d\left(L(K_n)\right)$ = n $\forall$ n > 5

Hence the result follows.

2.3 Relation between domination number, chromatic number and dominator chromatic number:

From the definition of domination number, chromatic number and dominator chromatic number, we get Max { $\chi(L(K_n)), \gamma(L(K_n))$} $\leq \chi_d(L(K_n)) \leq \chi(L(K_n)) + \gamma(L(K_n))$ where γ (L(K_n))= $\left| n \right|$

for n ≥ 3.

Theorem 2.4:

Let L(K_n) be a line graph of a complete graph, then the 2- dominator chromatic number of L(K_n) denoted by $\chi_d\ (L(K_n))\ _{,2}$ is exactly equal to (n+2) of order n >3.

(ie) $\chi_{d,2}\left(L(K_n)\right)$= (n+2) $\forall$ n >3.

Proof:

Let L(K_n) be the line graph of a complete graph. This theorem can be proved by the method of induction,

When n= 4, we've L(K_4) graph with 6 vertices and 12 edges.

Let D_2 be the 2- domination set of $L(K_4)$. We define $D_2 = \{v_1, v_3, v_5\} \subseteq V(L(K_4))$.

Now by the definition of 2- dominator colouring we required distinct the colours C_1, C_2, C_3 to the vertices in

2-dominating set $D_2 = \{v_1, v_3, v_5\}$ and the remaining vertices of V-D assigned colours C_4, C_5, C_6 such that that every vertex in the graph properly dominates a two colour classes.

$\therefore$ The 2- dominator chromatic number $\chi_{d,2}(L(K_n))$ for $L(K_4)$ is 6.

(ie) $\chi_{d,2}(L(K_4))$= (4+2) = 6

When n= 5, we've $L(K_5)$ graph with 10 vertices and 30 edges.

Let D_2 be the 2- domination set of $L(K_5)$. We define $D_2 = \{v_1, v_3, v_7, v_8, v_9\} \subseteq V(L(K_5))$.

Now by the definition of 2- dominator colouring we required distinct colors C_1, C_2, C_3 for the vertices in 2-dominating set $D_2 = \{v_1, v_3, v_7, v_8, v_9\}$ and colours C_4, C_5, C_6, C_7 to the vertices in

$V-D_2$ such that that every vertex in the graph properly dominates a two colour classes.

$\therefore$ The 2- dominator chromatic number $\chi_{d,2}(L(K_n))$ for $L(K_5)$ is 7.

(ie) $\chi_{d,2}(L(K_5))$= (5+2) = 7

When n= 6, we've $L(K_6)$ graph with 15 vertices and 60 edges.

Let D_2 be the 2- domination set of $L(K_6)$. We

define $D_2 = \{v_1, v_3, v_5, v_9, v_{10}, v_{12}, v_{14}\} \subseteq V(L(K_6))$.

Now by the definition of 2- dominator colouring we required the colours C_1, C_2, C_3, C_4 for the vertices in 2-dominating set $D_2 = \{v_1, v_3, v_5, v_9, v_{10}, v_{12}, v_{14}\}$and colours C_4, C_5, C_6, C_7, C_8 to the vertices in $V-D_2$ such that that every vertex in the graph properly dominates a two color classes.

$\therefore$ The 2- dominator chromatic number $\chi_{d,2}(L(K_n))$ for $L(K_6)$ is 8.

(ie) $\chi_{d,2}(L(K_6))$= (6+2) = 7

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Similarly for n terms we have $\chi_{d,2}\left(L\left(K_n\right)\right)$= (n+2)

In general we conclude that $\chi_{d,2}\left(L\left(K_n\right)\right)$= (n+2) for n >3.

Example 2.1.2:

Consider the graph $L(K_4)$

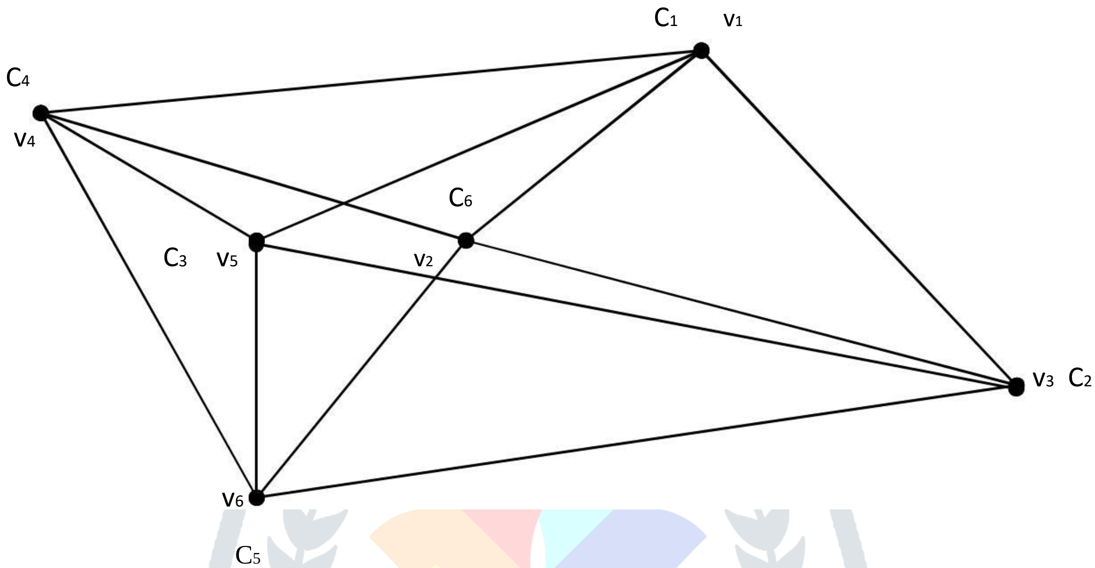

Fig: 3

By the definition of 2- dominator colouring we assigned the colors C_1, C_2, C_3 for the vertices in 2-dominating set D_2 = {v_1, v_3, v_5} and colours C_4, C_5, C_6 to the vertices in

V-D_2 = { v_2, v_4, v_6 } such that that every vertex in the graph properly dominates a two colour classes.

$\therefore$ The 2- dominator chromatic number $\chi_{d,2}\left(L\left(K_n\right)\right)$for $L(K_4)$ is 6.

(ie) $\chi_{d,2}\left(L\left(K_4\right)\right)$= (4+2) = 6

In general, we have $\chi_{d,2}\left(L\left(K_n\right)\right)$= (n+2) for n >3.

Theorem 2.5:

For any line graph of a complete graph $L(K_n)$, the inverse dominator chromatic number of $L(K_n)$ denoted by $\chi_{d'}(L(K_n))$ is equal to (n-1) for all even n >3.

(ie) $\chi_{d'}(L(K_n))$ = n-1 $\forall$ n > 3 and n is even, $n \in Z^+$

Proof:

Given that, $L(K_n)$ is the line graph of a complete graph and $\chi_{d'}(L(K_n))$is the inverse dominator chromatic number of line graph of a K_n.

For n=4, we have $V(L(K_4)) = \{v_1, v_2, v_3, \ldots, v_6\}$

The dominating set is $D = \{v_1, v_6\} \subseteq V(L(K_4))$

Since $D' = \{v_2, v_5\} \subseteq V-D$ is the inverse dominating set of $L(K_4)$, by the definition of inverse dominator colouring we assigned colour (C_1) for both the vertices in the inverse dominating set $D' = \{v_2, v_5\}$ since the vertices are not adjacent in D' and colors C_2, C_3, C_4 to the vertices in V- D' such that that every vertex in D' properly dominates some colour class in V- D' .

$\therefore$The inverse dominator chromatic number $\chi_{d'}(L(K_n))$ for $L(K_4)$ is 4.

(ie) $\chi_{d'}(L(K_4))$ = 4-1= 3

For n=6, we have $V(L(K_6)) = \{v_1, v_2, v_3, \ldots, v_{15}\}$

The dominating set is $D = \{v_3, v_6, v_{13}\}$. Since $D' = \{v_1, v_{12}, v_{14}\} \subseteq V-D$ is the inverse dominating set of $L(K_6)$, by the definition of inverse dominator coloring we assigned the colour (C_1) for the vertex in inverse dominating set $D' = \{v_1, v_{12}, v_{14}\}$ and assigning colours C_2, C_3, C_4, C_5, C_6 to the vertices in V- D' such that that every vertex in D' properly dominates the colour class in V- D' .

$\therefore$The inverse dominator chromatic number $\chi_{d'}(L(K_n))$ for $L(K_6)$ is 6.

(ie) $\chi_{d'}(L(K_6))$ = 6-1 = 5

Proceeding in this way, for n vertices we have,

$\chi_d'(L(K_n))$ = n- 1 for n > 3, for all even n $\in Z^+$

Therefore from (1) and (2) we have, $\chi_d'(L(K_n))$ is equal to (n-1) for all even n >3 .

(ie) $\chi_d'(L(K_n))$ = n-1 for n > 3 where n is even.

Example 2.1.3:

Consider the line graph of K_4

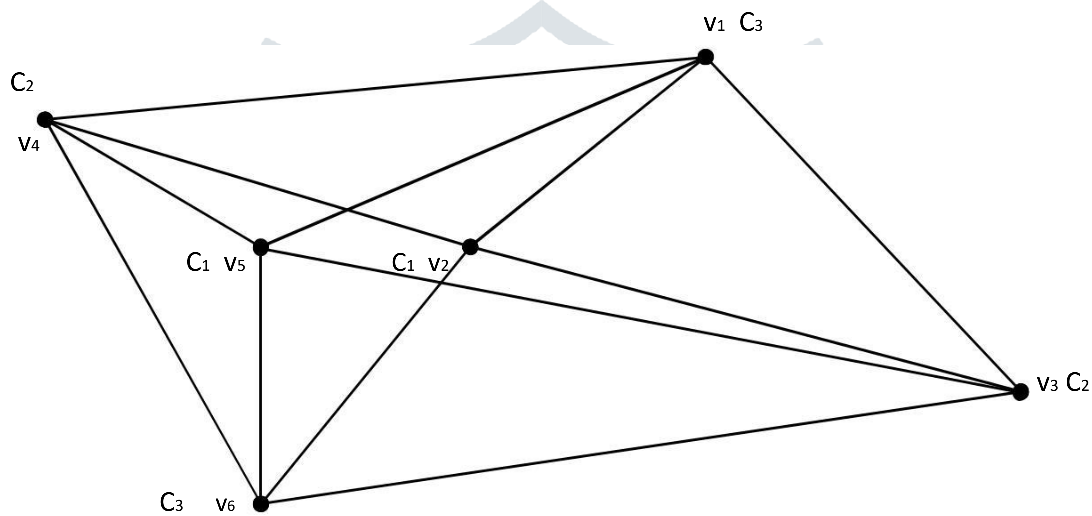

Fig: 4

For n=4, we have $V(L(K_4)) = \{v_1, v_2, v_3, \ldots, v_6\}$

The dominating set is $D = \{v_1, v_6\} \subseteq V(L(K_4))$

Since $D' = \{v_2, v_5\} \subseteq V-D$ is the inverse dominating set of $L(K_4)$, by the definition of inverse dominator colouring we required colour (C_1) for the vertices in the inverse dominating set $D' = \{v_2, v_5\}$ and colours C_2, C_3, C_4 to the vertices in V- D' such that that every vertex in D' properly dominates the colour class in V- D' .

Therefore, $\chi_d'(L(K_4))$ = 4-1= 3

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2.6 Observations:

1. $\chi(L(K_n)) = \chi_d(L(K_n)) = \chi_{d,2}(L(K_n)) = \chi_{d'}(L(K_n)) = \chi_{td}(L(K_n)) = 3$ for line graph of complete graph K_3.

2. For any line graph of a complete graph $L(K_n)$, the chromatic number $\chi(L(K_n))$ and dominator chromatic number $\chi_d(L(K_n))$ is equal to 'n' for $n > 5$.

3. Total dominator colouring $\chi_{td}(L(K_n))$ is always greater than $\chi_{d,2}(L(K_n))$ and $\chi_{d'}(L(K_n))$.

3. Conclusion:

In this paper, we have obtained exact bounds for distinct dominator chromatic number and also found the relation between domination number, chromatic number and dominator chromatic number. This paper can be extended by using distinct parameters and obtained various bounds for line graph of a complete graph $L(K_n)$.

References:

1. R.Gera, S.Horton , C.Rasmussen, *Dominator colourings and safe clique partitions ,* Congressus Numerantium 181, 19-20 (2006).

2. T.W.Hatnes, S.T.Hedetniemi and P.J.Slater, *Domination in Graphs*: Advanced Topics, Marcel Dekker, New York, (1998).

3. L.Jethruth Emelda Mary, P.Ananthi, *Dominator colouring of some special graphs*, International Journal of Innovative Research in Technology, Science & Engineering (IJIRTSE) ISSN: 2395-5619, Volume – 1, Issue – 4. June 2015

4. K.Kavitha , N.G.David, *Dominator colouring of some classes of graphs*, International journal of mathematical Archeive-3(11), p.p. 3954-3957, Nov- 2012.

5. A. Sangeetha Devi and M. M. Shanmugapriya, *Application of 2-Dominator Colouring in Graphs Using MATLAB*, Journal of Computer and Mathematical Sciences, Vol.7(4), 168-174, April 2016.

6. A. Sudhakaraiah and V. Raghava Lakshmi, *Dominator Chromatic Number of Circular-Arc Overlap Graphs* ,ISSN 2224-5804, Vol.2, No.9, 2012

7. Dr. B Tamilselvi and Kokila S, *Dominator colouring and total dominator colouring of certains graphs*, International Journal of Applied Research 2017; 3(12): 151-154