(;) Q Fuzzy Subrings

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Abstract: In this paper we introduce the notions of (;) Q Fuzzy Subrings and (;) Q Fuzzy Ideals. We give example which are (;) Q fuzzy subrings [(;) Q fuzzy ideals] but not Q-fuzzy subrings [Q-fuzzy ideals]. Finally we have characterized (;) Q f uzzy ideals of a ring.

Key words: Q-fuzzy subring, Q-fuzzy ideal, (;) Q Fuzzy Subring, (;) Q Fuzzy ideal and homomorphism.

1 Introduction

Zadeh in 1965 [12] introduced fuzzy sets, after which several researchers explored on the generalization of the notion of fuzzy sets. W. Liu [5] de ned a fuzzy subring of a ring. Bhakat and Das introduced the concepts of (2; 2 _q) -fuzzy groups [1, 2] and (2; 2 _q) -fuzzy subring [3]. Yao [10, 11] contined to research (;) fuzzy normal subgroups, (;) fuzzy quotient subgroups and (;) fuzzy subrings. Solairaju et.al. [6, 7, 8] have introduced and de ned a new algebraic structure called Q -fuzzy subring of a ring. P. Sarangapani et.al. [9] introduced Q fuzzy version of (;) fuzzy ideals of near ring. We introduce (;) Q-fuzzy subrings, (;) Q fuzzy ideals and established some results.

2 Preliminaries

De nition 2.1. Let X be a non-empty set and Q be a non-empty set. A Q -fuzzy subset A of X is a function A : X Q ! [0; 1]:

De nition 2.2. Let (R; +;) be a ring and Q be a non-empty set. A A -fuzzy subset A of R is said to be a Q -fuzzy subring of R if the following conditions are satis ed:

(i) A(x + y; q) A(x; q) ^ A(y; q)

(ii) A(x; q) A(x; q)

(iii) A(xy; q) A(x; q) ^ A(y; q); for all x; y 2 R; q 2 Q:

De nition 2.3. Let A be any Q -fuzzy subset of a ring R: For t 2 [0; 1]; the set A = fx 2 R=A(x; q) g is called level subset of A:

De nition 2.4. Let be a mapping from ring R_1 to ring R_2: For any fuzzy set B in R_2; we de ne a new fuzzy set de ned as ^{1}(B) in R_1 where ^{1}(B)(x; q) = B((x); q); 8x 2 R; q 2 Q:

For any fuzzy set A in R; we de ne (A) by

(A)(y; q) = supfA(x; q)=f(x) = yg; 8y 2 R_2

De nition 2.5. A Q -fuzzy subset A of ring R is called an (; _q) Q -fuzzy subring of R if for all x; y 2 R and q 2 Q;

(i) A(x y; q) A(x; q) ^ A(y; q) ^ 0:5

(ii) A(xy; q) A(x; q) ^ A(y; q) ^ 0:5

De nition 2.6. A Q -fuzzy subset A of ring R is called an (; _q) Q -fuzzy ideal of R if for all x; y 2 R and q 2 Q;

(i) A(x y; q) A(x; q) ^ A(y; q) ^ 0:5

(ii) A(xy; q) A(x; q) _ A(y; q) ^ 0:5

3 (;) Q -fuzzy Subrings

De nition 3.1. Let R be a ring and Q be a non-empty set. A Q -fuzzy subset A of R is said to be a (;) Q -fuzzy subring of R if the following conditions are satis ed.

(i) A(x + y; q) _ A(x; q) ^ A(y; q) ^ :

(ii) A(x; q) _A(x; q) ^ :

(iii) A(xy; q) _A(x; q) ^ A(y; q) ^ :

for all x and y in R and q in Q:

Remark 3.2. An Q-fuzzy subring is a (;) -Q-fuzzy subring with = 0 and = 1 and a (2; 2 _q) -Q-fuzzy subring is a (;) -Q-fuzzy subring with, = 0 and = 0:5:

Thus every Q-fuzzy subring and (2; 2 _q) Q-fuzzy subring of R is a (;) -Q-fuzzy subring of R: However, the converse is not necessarily true as shown in the following example.

Example 3.3. Consider the ring R = (Z_4; ;); where Z_4 = f0; 1; 2; 3g and let Q = fqg: Let the Q-fuzzy subset A be de ned by A(0; q) = 0:47; A(1; q) = A(3; q) = 0:46; A(2; q) = 0:48; Then, by routine calculation A is a (0:1; 0:4) -Q-fuzzy subring of R: But, since A(2 2; q) = A(0; q) = 0:47 A(2; q) ^ A(2; q) = 0:48 and A(2 2; q) = A(0; q) = 0:47 A(2; q) ^ A(2; q) ^ 0:5 = 0:48; A is neither an Q-fuzzy subring of R nor a (2; 2 _q) -Q-fuzzy subring of R:

Theorem 3.4. Let A be a (;) Q -fuzzy subring of a ring R: Then A(0; q) _ A(x; q) ^ for all x 2 R and q 2 Q; where 0 is the identity in R:

Proof. The proof is straight forward and omitted. □

Theorem 3.5. A is a (;) Q -fuzzy subring of a ring R if and only if A(x y; q) _ A(x; q) ^ A(y; q) ^ and A(xy; q) _ A(x; q) ^ A(y; q) ^ ; for all x; y in R and q in Q:

Proof. Let A be a (;) -Q-fuzzy subring of R; then

A(x y; q) _ = (A(x y; q) _) _ ((A(x; q) ^ A(y; q)) ^) _

= (A(x; q) _) ^(A(y; q) _) ^(_) A(x; q) ^(A(y; q) ^) ^ :

= A(x; q) ^ A(y; q) ^

A(xy; q) _ A(x; q) ^ A(y; q) ^ (* A is (;) -Q-fuzzy subring)

Conversely, suppose

(i) A(x y; q) _A(x; q) ^ A(y; q) ^ and

(ii) A(xy; q) _A(x; q) ^ A(y; q) ^ :

then A(0; q) _ A(x x; q) _ A(x; q) ^ A(x; q) ^ (by (i))

= A(x; q) ^ :

A(x; q) _ = A(0 x; q) _ = A(0 x; q) _ _ [A(0; q) ^ A(x; q) ^] _

= (A(0; q) _) ^ (A(x; q) _) ^ (_) (A(x; q) ^) ^ (A(x; q) ^)

= A(x; q) ^ :

A(x + y; q) _ = [A(x (y); q) _] _ [A(x; q) ^ A(y; q) ^] _

= f(A(x; q) _) ^ (A(y; q) _) ^ (_

)g (A(x; q)) ^ (A(y; q) ^) ^

= A(x; q) ^ A(y; q) ^

Clearly A(xy; q) _ A(x; q) ^ A(y; q) ^ :

Therefore A is a (;) -Q-fuzzy subring of R: □

Proposition 3.6. Let A be a non-empty subset of a ring R and f be a Q-fuzzy

subset de ned by f(x; q) = 8 >; if x 2 A < > ; if x 2= A; where 0 < 1 :

Then f is a (;) -Q-fuzzy subring of R if and only if A is a subring of R:

Proof. Let f be a (;) -Q-fuzzy subring of R and x; y 2 A; q 2 Q:

Then, (a) f(x; q) = f(y; q) = and by assumption f(x y; q) _ f(x; q) ^f(y; q) ^ = ; which implies that f(x y; q) = (since 0 < 1:) Hence x y 2 A:

(b) Similarly, f(xy; q) _ f(x; q) ^ f(y; q) ^ = ; which implies that f(xy; q) = (since 0 < 1:) Hence xy 2 A: Therefore A is a subring of R:

Conversely, suppose that A is a subring of R and x; y 2 R:

(a)(i)If x; y 2 A; q 2 Q; then by assumption x y 2 A; which implies that f(x y; q) = = f(x; q) ^ f(y; q): So f(x y; q) _ f(x; q) ^ f(y; q) ^ :

(ii)If x; y 2= A; (or one of x and y 2= A) q 2 Q; then f(x; q) ^ f(y; q) = and f(x y) = or ; this implies that f(x y; q) f(x; q) ^ f(y; q) and f(x y; q) _ = f(x; q) ^ f(y; q) ^

(b)(i)If x; y 2 A; q 2 Q; then by assumption xy 2 A; which implies that f(xy; q) = = f(x; q) ^ f(y; q): So f(xy; q) _ f(x; q) ^ f(y; q) ^ :

(ii)If x; y 2= A; (or one of x and y 2= A) q 2 Q; then f(x; q) ^ f(y; q) = and f(x y) = or ; this implies that f(xy; q) f(x; q) ^ f(y; q) and f(xy; q) _ f(x; q) ^ f(y; q) ^ : Therefore f is a (;) -Q-fuzzy subring of R:

□

Theorem 3.7. If A and B are two (;) Q -fuzzy subrings of a ring R; then their intersection A \ B is a (;) Q -fuzzy subring of R:

Proof. Let x; y 2 R; q; ; q2 2 Q and C = A \ B:

C(x y; q) _ = [A(x y; q) ^ B(x y; q)] _

= [A(x y; q) _] ^ [B(x y; q) _]

[A(x; q) ^ A(y; q) ^] ^ [B(x; q) ^ B(y; q) ^] = A(x; q) ^ A(y; q) ^ B(x; q) ^ B(y; q) ^

= [A(x; q) ^ B(x; q)] ^ (A(y; q) ^ B(y; q) ^)

= C(x; q) ^ C(y; q) ^

= [A(xy; q) _] ^ [B(xy; q) _]

[A(x; q) ^ A(y; q) ^] ^ [B(x; q) ^ B(y; q) ^] = [A(x; q) ^ A(y; q) ^ B(x; q) ^ B(y; q)] ^

= [A(x; q) ^ B(x; q)] ^ [A(y; q) ^ B(y; q)] ^

= C(x; q) ^ C(y; q) ^

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Hence C = A \ B is a (;) Q -fuzzy subring of R:

Proposition 3.8. Let R be a ring and C_A be the characteristic function of a subset A of R: Then C_A is a (;) -Q-fuzzy subring of R if and only if A is a subring of R:

Proof. Since $C_A(x; q)$ = 81; if x 2 A

< 0; if x = A;

Suppose C_A is a (;) - : > 2 2A; q 2 Q: Then,

(a) $C_A(x; q) = C_A(y; q) = 1$ and by assumption C_A(x y; q) _ $C_A(x; q)$ ^ $C_A(y; q)$ ^ =

; which implies that C_A(x y; q) = 6= 0) C_A(x y; q) = 1 Thus x y 2 A:

(b) Similarly, $C_A(xy; q)$ _ $C_A(x; q)$ ^ $C_A(y; q)$ ^ = 6= 0 which implies that $C_A(xy; q)$ = 1: Thus xy 2 A: Therefore A is a subring of R:

Conversely, suppose that A is a subring of R and x; y 2 R:

(a)Let x; y 2 R be such that x; y 2 A; q 2 Q; then by assumption x y 2 A; xy 2 A: Thus C_A(x y; q) = 1 $C_A(x; q)$ ^ $C_A(y; q)$ and $C_A(xy; q)$ = 1 $C_A(x; q)$ ^ $C_A(y; q)$:

(b)If x; y 2= A; q 2 Q; then C_A(x y; q) $C_A(x; q)$ ^ $C_A(y; q)$ = 0 and $C_A(xy; q)$ $C_A(x; q)$ ^ $C_{A^c}(y; q)$ = 0:

(c) If x 2 A and y 2= A or x 2= A and y 2 A then C_A(x y; q) $C_A(x; q)$ ^ $C_A(y; q)$ = 0 and $C_A(xy; q)$ $C_A(x; q)$ ^ $C_A(y; q)$ = 0:

Thus C_A is a Q-fuzzy subring of R and by Remark C_A is a (;) -Q-fuzzy subring of R: □

Theorem 3.9. Let A be a Q -fuzzy subset of a ring R: Then A is a (;) Q - fuzzy subring of R i A is a subring of R; for all 2 (;]:

Proof. Let A be a (;) Q fuzzy subring of R:

Let 2 (;] and x; y 2 A then

A(x y; q) _ A(x; q) ^ A(y; q) ^ >

So A(x y; q) and x y 2 A : Similarly xy 2 A :

Conversely, let A be a Q -fuzzy subring of R for all 2 (;]: If there exist x; y 2 R; such that A(x y; q) _ < = A(x; q) ^ A(y; q) ^ ; then x; y 2 A and x y 2= A : This is a contradiction with that A is subring.

Hence A(x y) _ A(x; q) ^ A(y; q) ^

holds for all x; y 2 R: Similarly, we have A(xy; q) _ A(x; q) ^ A(y; q) ^ :

Therefore A is a (;) Q -fuzzy subring of R: □

De nition 3.10. Let A be a Q -fuzzy subset of R: Then A is called a (;) Q -fuzzy ideal of R if

(i) A(x y; q) _A(x; q) ^ A(y; q) ^ :

(ii) A(xy; q) _[A(x; q) _ A(y; q)] ^ ; for all x; y 2 R and q 2 Q:

Remark 3.11. A Q-fuzzy ideal is a (;) -Q-fuzzy ideal with = 0 and = 1 and a (2; 2 _q) -Q-fuzzy ideal is a (;) -Q-fuzzy ideal with, = 0 and = 0:5:

Thus every Q-fuzzy ideal and (2; 2 _q) -Q-fuzzy ideal of R is a (;) -Q-fuzzy ideal of R: However, the converse is not necessarily true as shown in the following example.

Example 3.12. Consider the ring R = (Z_4; ;); where Z_4 = f0; 1; 2; 3g and let Q = fqg: Let the Q-fuzzy subset A be de ned by A(0; q) = 0:47; A(1; q) = A(3; q) = 0:45; A(2; q) = 0:48; Then, by routine calculation A is a (0:1; 0:4) Q-fuzzy ideal of R: But, since A(2 2; q) = A(0; q) = 0:47 A(2; q) ^ A(2; q) = 0:48 and A(2 2; q) = A(0; q) = 0:47 A(2; q) ^ A(2; q) ^ 0:5 = 0:48; A is neither a Q-fuzzy ideal of R nor a (2; 2 _q) -Q-fuzzy ideal of R

Theorem 3.13. Let A be a Q -fuzzy subset of R: Then A is a (;) Q -fuzzy ideal of R i A is a ideal of R for all 2 (;]:

Proof. Let A be a (;) Q -fuzzy ideal of R; then for all 2 (;]; we have that A is a subring of R from Theorem 3.4. Let x 2 A ; r 2 R then

A(x; q) and A(xr; q) _ A(x; q) ^ A(r; q) ^ > : So A(xr; q) and xr 2 A : Similarly rx 2 A : Hence A is an ideal of R:

Conversely, let A be an ideal of R for all 2 (;]: If there exist x; y 2 R such that A(xy; q) _ < < [A(x; q) _ A(y; q)] ^ : Now [A(x; q) _ A(y; q)] ^ > : Then A(x; q) ; A(y; q) and : Hence x; y 2 A and A being an ideal of R; xy 2 A and A(xy; q) _ :

It is contradiction. It follows that

A(xy; q)_ (A(x; q)_A(y; q))^ for all x; y 2 R and q 2 Q: From Theorem 3.4 we have that

A(x y; q) _ A(x; q) ^ A(y; q) ^ for all x; y 2 R; q 2 Q:

Therefore A is a (;) Q -fuzzy ideal of R: □

Theorem 3.14. Let R_1 and R_2 be two rings and f : R_1 ! R_2 be a homomorphism. If B is a (;) -Q fuzzy subring(ideal) of R_2; then the pre image f ^{1}(B) of R_2 under f is a (;) -Q fuzzy subring(ideal) of R_1:

Proof. Let B is a (;) -Q fuzzy subring of R_2:

Let x_1; x_2 2 R_1 and q; q_1; q_2 2 Q:

f ^{1}(B)(x_1 x_2; q) _ = B(f(x_1 x_2); q) _

= B(f(x; q) f(x_2; q)) _

B(f(x; q)) ^ B(f(x_2; q)) ^

= f ^{1}(B)(x_1; q) ^ f ^{1}(B)(x_2; q) ^ :

f ^{1}(B)(x_1x_2; q) _ = B(f(x_1x_2); q) _

= B(f(x; q)f(x_2; q)) _

= B(f(x; q)) ^ B(f(x_2; q)) ^

= f ^{1}(B)(x_1; q) ^ f ^{1}(B)(x_2; q) ^ :

f ^{1}(B)(x; q_1q_2) _ = B(f(x); q_1q_2) _

B(f(x); q_1) ^ B(f(x); q_2) ^

= f ^{1}(B)(x; q_1) ^ f ^{1}(B)(x; q_2) ^ :

Hence f ^{1}(B) is a (;) -Q fuzzy subring. □

Theorem 3.15. Let f : R1 ! R2 be a homomorphism of rings and let

A be a (;) -Q fuzzy subring(ideal) of R1: Then f(A) is a (;) -Q fuzzy subring(ideal) of R2:

Proof. For all y1; y2 2 R2: We have

f(A)(y1 y2; q) _ = supfA(x1 x2; q)=f(x1 x2) = y1 y2g _

= supfA(x1 x2; q) _ =f(x1 x2) = y1 y2g

supfA(x1; q) ^ A(x2; q) ^ =f(x1) = y1; f(x2) = y2g

= supfA(x1; q)=f(x1) = y1g ^ supfA(x2; q)=f(x2) = y2g ^

= f(A)(y1; q); f(A)(y2; q) ^ :

f(A)(y1y2; q) _ = supfA(x1x2; q)=f(x1x2) = y1 y2g _

= supfA(x1x2; q) _ =f(x1x2) = y1 y2g

supfA(x1; q) ^ A(x2; q) ^ =f(x1) = y1; f(x2) = y2g

= supfA(x1; q)=f(x1) = y1g ^ supfA(x2; q)=f(x2) = y2g ^

= f(A)(y1; q); f(A)(y2; q) ^ :

f(A)(x1; q1q2) _ = supfA(x1; q1q2)=f(x1) = y1g _

= supfA(x1; q1q2) _ =f(x1) = y1g

supfA(x1; q1) ^ A(x1; q2) ^ =f(x1) = y1g

= supfA(x1; q1)=f(x1) = y1g ^ supfA(x1; q2)=f(x1) = y1g ^

= f(A)(x; q1) ^ f(A)(x; q2) ^ :

Hence f(A) is a (;) - Q fuzzy subring. □

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