Further Results of Intuitionistic Fuzzy Bi-Implication Operator ↔ for IFMs

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Abstract: In this paper, we study boundaries, regularities, first place anti-tonicity, second place isotonicity and distributive properties of ↔ operator over
$\wedge$ and $\vee$.

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Intuitionistic Fuzzy Matrix(IFM), Intuitionistic Fuzzy Set(IFS), Intuitionistic Fuzzy implication operator (IFIO), Intuitionistic Fuzzy bi-implication operator(IFBIO).

1 Introduction

After the introduction of fuzzy set theory by Zadeh[13] in1965, fuzzy concepts evolved in almost all fields. Hiroshi Hashimoto[2,3] used implicatin operator in
fuzzy matrix theory and obtained results in sub-inverse of fuzzy matrix using 1
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fuzzy relation equation.After the generalization of fuzzy theory by Atanassov[1] as Intuitionistic Fuzzy set theory Im et. al[4] extended it to Intuitionistic Fuzzy Matrix. Sriram and Murugadas developed this implication operator to Intuitionistic fuzzy set and extended it to Intuitionistic fuzzy matrix.Yunhua xiao et.,al.[11] defined adjoint pair and residuated lattice using a new kind of IFIO. The authors in [6,7] introduced hook implication $\rightharpoondown$ for IFS as well as IFM,discussed the relation with $\rightarrow$ implication operator and obtained maximum solution (minimum solution) for the inequality $A \times X \times B \leq C$, ($A \diamond X \diamond B \geq C$) max-min (min-max) product. Further the authors in [5,6,8] defined bi implication operator for IFS, extended it to Intuitionistic

Fuzzy Matrix, in this paper we studied Some properties of it using bi implication operator.

2 Preliminaries

Definition 2.1.. [1] An Intuitionistic Fuzzy Set(IFS) A in E (universal set) is defined as an object of the following form $A = \{ x, {}_A(x), {}_A(x) / x \in E\}$, where the functions: ${}_A(x) : E \rightarrow [0, 1]$ and ${}_A(x) : E \rightarrow [0, 1]$ define the membership and non-membership functions of the element $x \in E$ respectively and for every $x \in E : 0 \leq {}_A(x) + {}_A(x) \leq 1$.

For simplicity we consider the pair x, x' as membership and non-membership function of an IFS with $x + x' \leq 1$.

Definition 2.2.. For $x, x', y, y' \in$ IF S, define $x, x' \vee y, y' = \max\{x, y\}, \min\{x', y'\}$. $x, x' \wedge y, y' = \min \{x, y\} , \max \{x', y'\}$.

x, x' and y, y' are comparable, that is $x, x' \geq y, y'$, if $x \geq y$ and $x' \leq y'$.
$x, x'\ ^c = x', x$.

Definition 2.3.. Let $X = \{x_1, x_2, ...x_m\}$ be a set of alternatives and $Y = \{y_1, y_2, ...y_n\}$ be the attribute set of each element of X. An Intuitionistic Fuzzy Matrix (IFM) is defined by $A = ((x_i, y_j), {}_A(x_i, y_j), {}_A(x_i, y_j))$ for $i = 1, 2...m$ and $j = 1, 2, ...n$, where ${}_A : X \times Y \rightarrow [0, 1]$ and ${}_A : X \times Y \rightarrow [0, 1]$ satisfy

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the condition $0 \leq {}_A(x_i, y_j) + {}_A(x_i, y_j) \leq 1$. For simplicity we denote an intuitionistic fuzzy matrix (IFM) as matrix of pairs $A = (a_{ij} , a'_{ij})$ of non negative real numbers satisfying $a_{ij} + a'_{ij} \leq 1$ for all i, j .

Definition 2.4.. For any two comparable elements $x, x' , y, y' \in$ IF S , define

$$x, x' \leftarrow y, y' = \begin{cases} 1, 0 & \text{if } x, x' \geq y, y' \\ x, x' & \text{if } x, x' < y, y' \end{cases}$$

Definition 2.5.. For $a, a' , b, b' \in$ IF S, define
$a, a' \leftrightarrow b, b' = (a, a' \leftarrow b, b') \wedge (a, a' \rightarrow b, b')$, that is

b, b' if $a, a' > b, b'$

$\langle a, a' \rangle \leftrightarrow \langle b, b' \rangle = \begin{cases} \langle 1, 0 \rangle & \text{if } \langle a, a' \rangle = \langle b, b' \rangle \\ \langle a, a' \rangle & \text{if } \langle a, a' \rangle < \langle b, b' \rangle \end{cases}$

Easily $\langle a, a' \rangle \leftrightarrow \langle b, b' \rangle = \langle b, b' \rangle \leftrightarrow \langle a, a' \rangle$.

Definition 2.6.. Let $X = \{x_1, x_2, ...x_m\}$ be a set of alternatives and $Y = \{y_1, y_2, ...y_n\}$ be the attribute set of each element of X. An Intuitionistic Fuzzy Matrix (IFM) is defined by $A = (\langle (x_i, y_j), \mu_A(x_i, y_j), \nu_A(x_i, y_j) \rangle)$ for i = 1, 2...m and j = 1, 2, ...n, where $\mu_A : X \times Y \to [0, 1]$ and $\nu_A : X \times Y \to [0, 1]$ satisfy the condition $0 \le \mu_A(x_i, y_j) + \nu_A(x_i, y_j) \le 1$. For simplicity we denote an intuitionistic fuzzy matrix (IFM) as matrix of pairs $A = (\langle a_{ij}, a'_{ij} \rangle)$ of non negative real numbers satisfying $a_{ij} + a'_{ij} \le 1$ for all i, j .

For any two elements $A = (\langle a_{ij}, a'_{ij} \rangle)$, $B = (\langle b_{ij}, b'_{ij} \rangle) \in F_{mn}$ and $C \in F_{np}$, define

1. $A \oplus B = (\langle \max\{a_{ij}, b_{ij}\}, \min\{a'_{ij}, b'_{ij}\} \rangle)$, (component wise addition)
2. $A \odot B = (\langle \min\{a_{ij}, b_{ij}\}, \max\{a'_{ij}, b'_{ij}\} \rangle)$ (component wise multiplication) for all $1 \le i \le m$ and $1 \le j \le n$.
3. $J = (\langle 1, 0 \rangle)$ the Universal matrix(matrix in which all entries are $\langle 1, 0 \rangle$)
4. $I = (\langle \delta_{ij}, \delta'_{ij} \rangle)$ (Identity Matrix) where $\langle \delta_{ij}, \delta'_{ij} \rangle = \begin{cases} \langle 1, 0 \rangle & \text{if } i = j \\ \langle 0, 1 \rangle & \text{if } i \neq j \end{cases}$.
   The Zero matrix O is the matrix in which all the entries are $\langle 0, 1 \rangle$.
5. $A \ge B$ if $a_{ij} \ge b_{ij}$ and $a'_{ij} \le b'_{ij}$ for all i, j and $A > B$ if $a_{ij} > b_{ij}$ or $a'_{ij} < b'_{ij}$ for all i, j , in which case A and B are comparable.
6. $A^C = (\langle a'_{ij}, a_{ij} \rangle)$ (complement of A).
7. $AC = (\langle \max^n_{k=1}\min\{a_{ik}, c_{kj}\}, \min^n_{k=1}\max\{a'_{ik}, c'_{kj}\} \rangle)$
   $= (\langle \sum^n_{k=1} a_{ik}c_{kj}, \prod^n_{k=1}(a'_{ik} + c'_{kj}) \rangle)$.

Here $a_{ik}c_{kj} = \min\{a_{ik}, c_{kj}\}$ and $a_{ik} + c_{kj} = \max\{a_{ik}, c_{kj}\}$. Some times we can use $\bigvee^n_{k=1}$ for $\prod^n_{k=1}$ and $\bigwedge^n_{k=1}$ for $\sum^n_{k=1}$.

Definition 2.7.. For IF M s $A = (\langle a_{ij}, a'_{ij} \rangle) \in F_{mn}$, $B = (\langle b_{ij}, b'_{ij} \rangle) \in F_{mn}$ and $C = (\langle c_{ij}, c'_{ij} \rangle) \in F_{np}$, define

$A \vee B = (\langle a_{ij}, a'_{ij} \rangle \vee \langle b_{ij}, b'_{ij} \rangle)$, (which is equivalent to $A \oplus B$)

$A \wedge B = (\langle a_{ij}, a'_{ij} \rangle \wedge \langle b_{ij}, b'_{ij} \rangle)$, (which is equivalent to $A \odot B$)

3 Results using IFBI($\leftrightarrow$) Operator

Theorem 3.1.. IFBI operator $\leftrightarrow$ satisfies boundaries

1. $0, 1 \leftrightarrow 0, 1 = 1, 0$
2. $0, 1 \leftrightarrow 1, 0 = 0, 1$
3. $1, 0 \leftrightarrow 0, 1 = 0, 1$
4. $1, 0 \leftrightarrow 1, 0 = 1, 0$

The following theorem shows that dominance of truth of consequent and dominance of falsity of antecedent fails.

Theorem 3.2.. Suppose $a, a' \in$ A is an arbitrary element in IFS A, then the IFBI operator $\leftrightarrow$ has the following regularities

1. $a, a' \leftrightarrow 1, 0 = a, a'$
2. $1, 0 \leftrightarrow a, a' = a, a'$
3. $0, 1 \leftrightarrow a, a' = 0, 1$

Proof:

Proofs of the above two Theorems are trivial from the Definition of IFBI operator.

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Lemma 3.3.. If a, a', b, b', $c, c' \in$ IF S, then the IFBI operator satisfies
$[a, a' \vee b, b'] \leftrightarrow c, c' \leq (a, a' \leftrightarrow c, c') \vee (b, b' \leftrightarrow c, c')$

Proof:

Case 1. Assume $a, a' < b, b'$

sub case 1. If $b, b' < c, c'$, then $a, a' < c, c'$, $b, b' \leftrightarrow c, c' = a, a' \vee b, b', b, b' = b, b'$

sub case 2. If $a, a' > c, c'$, then $b, b' > c, c'$, $b, b' \leftrightarrow c, c' = c, c' \vee c, c', c, c = c, c'$

sub case 3. If $a, a' < c, c'$ and $b, b' > c, c'$, $b, b' \leftrightarrow c, c' = a, a' \vee c, c', c, c' = c, c'$

sub case 4. If $c, c' \neq a, a'$ and $c, c' = b, b'$, $b, b' \leftrightarrow c, c' = a, a' \vee 1, 0, b, b' \leftrightarrow b, b' = 1, 0, 1, 0 = 1, 0$

sub case 5. If $c, c' = a, a'$ and $c, c' \neq b, b'$, $b, b' \leftrightarrow c, c' = 1, 0 \vee a, a', a, a' < 1, 0$

Case 2. Assume $a, a' > b, b'$

sub case 1. If $b, b' < c, c'$, then $a, a' < c, c'$, $a, a' \leftrightarrow c, c' = a, a' \vee b, b', a, a' = a, a'$

sub case 2. If $a, a' > c, c'$ and $b, b' < c, c'$, $a, a' \leftrightarrow c, c' = c, c' \vee b, b', c, c' = c, c'$

sub case 3. If $a, a' > c, c'$ and $b, b' > c, c'$, $a, a' \leftrightarrow c, c' = c, c' \vee c, c', c, c' = c, c'$

sub case 4. If $c, c' \neq a, a'$ and $c, c' = b, b'$, $a, a' \leftrightarrow c, c' = b, b' \vee 1, 0, b, b' < 1, 0$

sub case 5. If $c, c' = a, a'$ and $c, c' \neq b, b'$, $a, a' \leftrightarrow c, c' = 1, 0 \vee b, b', 1, 0 = 1, 0$

Therefore the Lemma holds.

Remark 3.4.. The above Lemma can be generalized as $\overset{n}{\underset{=1}{}} a_i, a_i' \leftrightarrow c, c' \leq \overset{k}{\underset{i=1}{}} (a_i, a_i' \leftrightarrow c, c')$

The following Theorem proves that the IFBI operator satisfies the distributive inequality over $\wedge$.

Lemma 3.5.. If a, a', b, b', c, c' are arbitrary IFS, then the IFBI operator 6

$\leftrightarrow$ satisfies
$a, a' \leftrightarrow [b, b' \wedge c, c'] \geq (a, a' \leftrightarrow b, b') \wedge (a, a' \leftrightarrow c, c')$

Proof:

Case 1. Assume $a, a' < b, b'$

sub case 1. If $b, b' < c, c'$, then $a, a' < c, c'$, $a, a' \leftrightarrow b, b' = a, a' \wedge a, a', a, a' = a, a'$

sub case 2. If $a, a' > c, c'$, then $b, b' > c, c'$, $a, a' \leftrightarrow c, c' = a, a' \wedge c, c', c, c' = c, c'$

sub case 3. If $a, a' < c, c'$ and $b, b' > c, c'$, $a, a' \leftrightarrow c, c' = a, a' \wedge a, a', a, a' = a, a'$

sub case 4. If $c, c' \neq a, a'$ and $c, c' = b, b'$, $a, a' \leftrightarrow b, b' = a, a' \wedge a, a', a, a' = a, a'$

sub case 5. If $c, c' = a, a'$ and $c, c' \neq b, b'$, $a, a' \leftrightarrow a, a' = a, a' \wedge 1, 0, 1, 0 \geq a, a'$

Case 2. Assume $a, a' > b, b'$

sub case 1.If $b, b' < c, c'$, then $a, a' < c, c'$, $a, a' \leftrightarrow b, b' = b, b' \wedge a, a', b, b' = b, b'$

sub case 2. If $a, a' > c, c'$ and $b, b' < c, c'$, $a, a' \leftrightarrow b, b' = b, b' \wedge c, c', b, b' = b, b'$

sub case 3. If $a, a' > c, c'$ and $b, b' > c, c'$, $a, a' \leftrightarrow c, c' = b, b' \wedge c, c', c, c' = c, c'$

sub case 4. If $c, c' \neq a, a'$ and $c, c' = b, b'$, $a, a' \leftrightarrow b, b' = b, b' \wedge b, b', b, b' = b, b'$

sub case 5. If $\langle c, c' \rangle = \langle a, a' \rangle$ and $\langle c, c' \rangle \neq \langle b, b' \rangle$, $\langle a, a' \rangle \leftrightarrow \langle b, b' \rangle = \langle b, b' \rangle \wedge \langle 1, 0 \rangle$, $\langle b, b' \rangle = \langle b, b' \rangle$

Remark 3.6.. The above Lemma can be generalized as $\langle a, a' \rangle \leftrightarrow \bigvee_i^n \bigvee_i^n$

Lemma 3.7.. If $\langle a, a' \rangle$, $\langle b, b' \rangle$, $\langle c, c' \rangle \in$ IF S, then the IFBI operator $\leftrightarrow$ satisfies
$\langle b, b' \rangle \geq (\langle a, a' \rangle \leftrightarrow \langle b, b' \rangle)$ if $\langle a, a' \rangle \neq \langle b, b' \rangle$)

Proof:

The Proof is trivial from the definition of IFBI operator.

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Lemma 3.8.. If $\langle a, a' \rangle$, $\langle b, b' \rangle$, $\langle c, c' \rangle \in$ IF S, then the IFBI operator $\leftrightarrow$ satisfies $(\langle a, a' \rangle \wedge \langle b, b' \rangle) = \langle a, a' \rangle \wedge (\langle a, a' \rangle \leftrightarrow \langle b, b' \rangle)$

Proof:

If $\langle a, a' \rangle < \langle b, b' \rangle$, $\langle a, a' \rangle \leq \langle a, a' \rangle \wedge \langle a, a' \rangle$, $\langle a, a' \rangle = \langle a, a' \rangle$ If $\langle a, a' \rangle > \langle b, b' \rangle$, $\langle b, b' \rangle \leq \langle a, a' \rangle \wedge \langle b, b' \rangle$, $\langle b, b' \rangle = \langle b, b' \rangle$ If $\langle a, a' \rangle = \langle b, b' \rangle$, $\langle a, a' \rangle = \langle a, a' \rangle \wedge \langle 1, 0 \rangle$, $\langle a, a' \rangle = \langle a, a' \rangle$.

Now we shall prove the first place anti-tonicity and second place isotonicity fails in the case of IFBI operator $\leftrightarrow$.

Remark 3.9.. If $\langle a, a' \rangle \leq \langle b, b' \rangle$, then for any $\langle c, c' \rangle$ with $\langle a, a' \rangle \leq \langle c, c' \rangle \leq \langle b, b' \rangle$, we have
$\langle b, b' \rangle \leftrightarrow \langle c, c' \rangle = \langle c, c' \rangle$, and
$\langle a, a' \rangle \leftrightarrow \langle c, c' \rangle = \langle a, a' \rangle$, therefore
$\langle b, b' \rangle \leftrightarrow \langle c, c' \rangle \nleq \langle a, a' \rangle \leftrightarrow \langle c, c' \rangle$.
Again consider $\langle c, c' \rangle = \langle a, a' \rangle < \langle b, b' \rangle$, we get
$\langle c, c' \rangle \leftrightarrow \langle a, a' \rangle = \langle 1, 0 \rangle$, and $\langle c, c' \rangle \leftrightarrow \langle b, b' \rangle = \langle c, c' \rangle$.
Therefore $\langle c, c' \rangle \leftrightarrow \langle a, a' \rangle \nleq \langle c, c' \rangle \leftrightarrow \langle b, b' \rangle$.

Theorem 3.10.. For IFM's $A \in F_{mn}$, $B \in F_{mp}$ and $C \in F_{qn}$ the following results hold

1. $A \leftrightarrow [B \wedge C] \geq (A \leftrightarrow B) \wedge (A \leftrightarrow C)$
2. $A \leftrightarrow [B \vee C] \leq (A \leftrightarrow B) \vee (A \leftrightarrow C)$

Proof:

1. $A \leftrightarrow (B \wedge C) = (A \rightarrow (B \wedge C)) \wedge (A \leftarrow (B \wedge C)) =$
$((A \rightarrow B) \wedge (A \rightarrow C)) \wedge ((A \leftarrow B) \vee (A \leftarrow C))$

$\geq ((A \rightarrow B) \wedge (A \rightarrow C)) \wedge ((A \leftarrow B) \wedge (A \leftarrow C)) =$
$((A \rightarrow B) \wedge (A \leftarrow B)) \wedge ((A \rightarrow C) \wedge (A \leftarrow C)) = (A \leftrightarrow B) \wedge (A \leftrightarrow C)$

2. $A \leftrightarrow (B \vee C) = (A \rightarrow (B \vee C)) \wedge (A \leftarrow (B \vee C))$
$= ((A \rightarrow B) \vee (A \rightarrow C)) \wedge ((A \leftarrow B) \wedge (A \leftarrow C))$

$= ((A \rightarrow B) \wedge (A \leftarrow B) \wedge (A \leftarrow C)) \vee ((A \rightarrow C) \wedge (A \rightarrow B) \wedge (A \leftarrow C)) =$
$((A \leftrightarrow B) \wedge (A \leftarrow C)) \vee ((A \leftrightarrow C) \wedge (A \leftarrow B))$
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$\leq (A \leftrightarrow B) \vee (A \leftrightarrow C).$

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