DOMINATOR CHROMATIC NUMBER OF A BINARY TREE AND ITS RELATED PARAMETERS

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Abstract:

In this paper, we established some parameters such as domination number γ (BT), 2-domination number γ_2 (BT), Chromatic number χ(BT) and dominator chromatic number χ_d (BT) of Rooted binary tree and Perfect binary tree. Also studied about some of its properties and illustrated with examples.

Keywords: *Dominator chromatic number, 2-Domination, Colouring,*

Binary tree, rooted binary tree and Perfect binary tree

1. INTRODUCTION:

In Mathematics, graph theory is the study of binary trees. A tree in this content is made up of vertices, nodes or points which are connected by edges, sizes and lines. This binary tree was introduced by Steven Pivnik on 1993**.**

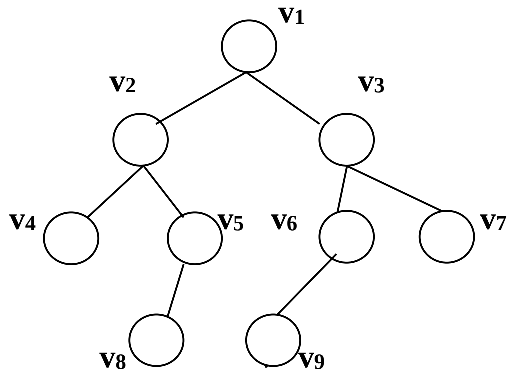
In this paper, we studied the concept of the dominator chromatic number, 2-domination number, Chromatic number of the different types of binary trees such as rooted binary tree and perfect binary tree. In these binary trees, we established some parameters they are domination number γ(BT), chromatic number χ(BT) and dominator chromatic number χ_d (BT) . The aim of this paper is to analysis the relationship between perfect binary tree and rooted binary tree in the domination, 2-domination, chromatic number and dominator chromatic number.

2. BASIC DEFINITIONS:

2.1 BINARY TREE:

A Binary tree is defined as a tree in which each node has at most two, one, zero children.

2.1 EXAMPLE:



2.1.1 TYPES OF BINARY TREES:

There are several types of binary trees:

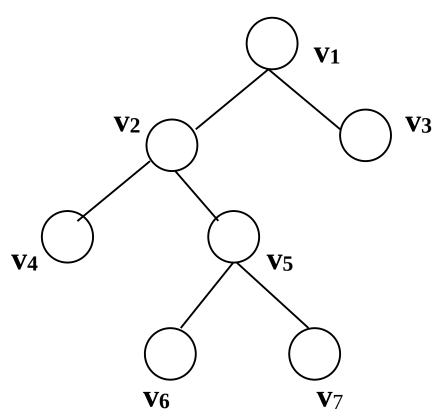
- Full binary tree
- Complete binary tree
- Rooted binary tree
- Prefect binary tree
- Infinite complete binary tree
- Balanced binary tree
- Degenerate tree

In this paper, we have chosen two binary trees namely rooted binary tree and perfect binary tree**.**

2.2 ROOTED BINARY TREE:

A Rooted binary tree has a root node and every node has at most two children.

2.2 EXAMPLE:

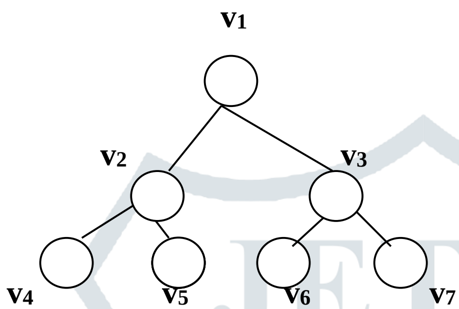


G= (RBT (4))

2.3 PREFECT BINARY TREE:

A Prefect binary tree in which all nodes have the same depth or same level.

2.3EXAMPLE:



G= (PBT (3))

2.4 DOMINATING SET:

A non-empty subset D of V (G) is said to be a dominating set of G if every vertex in D is adjacent to at least one vertex in (V-D).

2.5 DOMINATION NUMBER:

The Domination number γ (G) of a graph G is the minimum cardinality of dominating set D in G.

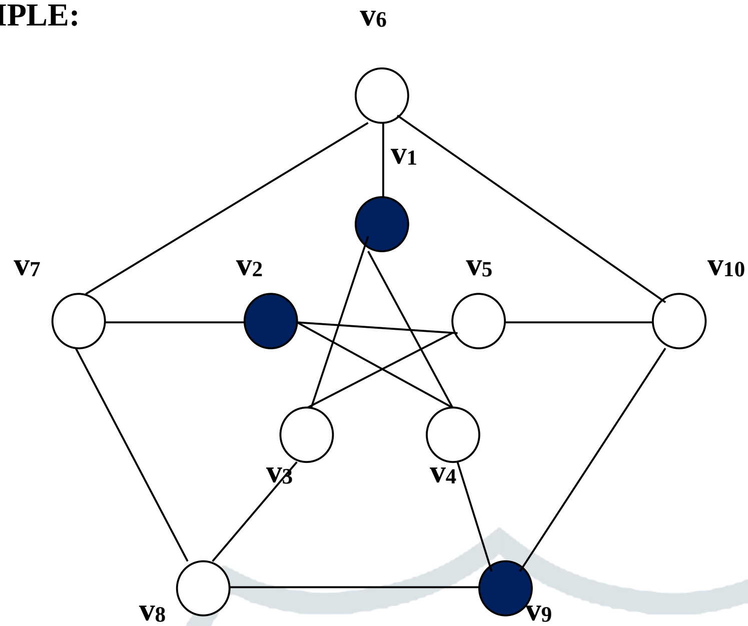
2.6 PROPER COLOURING:

A Proper colouring is defined as a vertex or edge colouring from a set of colours such that no two adjacent vertices or edges share a common colour.

2.7 DOMINATOR COLOURING:

A Dominator colouring of graph G is a proper colouring such that every colour class of V-D dominates at least one colour class of D.

2.2 EXAMPLE:



In this graph, we say that the subset S = {v_1, v_2, v_9} is the dominating set.

2.8 CHROMATIC NUMBER:

The Chromatic number of graph G is the minimum number of colour needed to colour the vertices of G so that no two adjacent vertices share the same colour. The chromatic number of a graph G is most commonly denoted by $\chi(G)$.

3. PROPERTIES OF BINARY TREES:

In this section, we studied some properties of perfect binary tree and rooted binary tree.

3.1 PROPERTIES OF PREFECT BINARY TREE:

Property 1:

The Number of nodes at depth d in a perfect binary tree is 2^d

Property 2:

A Perfect binary tree of height h has $2^{h+1}-1$ nodes.

Nodes=$2^0+2^1+\ldots\ldots\ldots..+2^h=2^{h+1}-1$

Property 3:

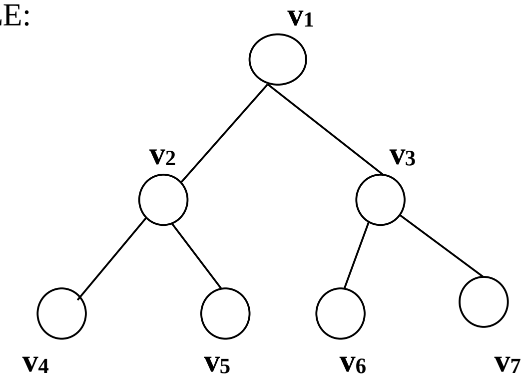
Number of leaf nodes in a perfect binary tree of height is 2^h

Property 4:

Number of internal nodes in a perfect binary tree of height h=2^h-1.

$(2^{h+1}-1)-2^h = 2^h-1$.

3.1 EXAMPLE:



In above graph, the vertex v_1 having depth=0 then, 2^0nodes.

The vertices v_4, v_5, v_6 and v_7 are having depth=2 then, 2^2 nodes. Similarly, the height is possible. The vertex v_1 having height=1then 2^1-1nodes. The vertices v_2, and v_3 are having height =2 then 2^2-1nodes.

3.2 PROPERTIES OF ROOTED BINARY TREE:

Property 1:

The depth or level of a vertex v is its distance from the root. The length of the unique path from the root to v. Thus the root has depth 0.

Property 2:

The height of a rooted tree is the length of a longest path from the root.

Property 3:

Vertices having the same parent are called siblings.

Property 4:

Any nodes which is higher than the parent is called ancestor.

Property 5:

A Vertex w is called descendant of a vertex v. Here v is a parent of w and w is a child of v.

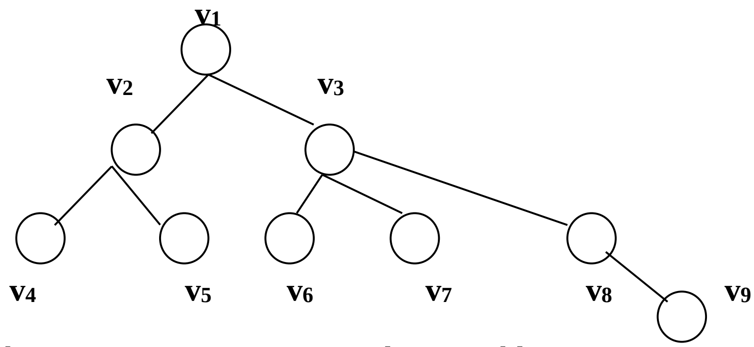
Property 6:

A Leaf node in a rooted tree is any vertex having no children.

Property 7:

A Internal nodes in a rooted tree is any vertex that has at least one child

3.2 EXAMPLE:



In the above graph, here the vertices v_2, v_3, v_4, v_5, v_6 and v_7 are sibling.

The ancestor is vertex v_1.The descendant is the vertex v_1 is a parent of v_1 and v_3 similarly, v_2 and v_3 are the child of v_1.The leaf nodes are v_4, v_5, v_6, v_7 and v_9.The internal nodes are v_2, $v_3$and v_8.

Theorem: 4.1

Let G= R(BT(n)) be a rooted binary tree of level n with order 2n-1 and size 2n-2 then the domination γ(BT(R)) and dominator chromatic number χd(BT(R)) and chromatic number χ(BT(R)) and 2-domination number are given by

i. $\gamma(BT(R))=n-1$
ii. $\chi_d(BT(R))=n$
iii. $\chi(BT(R))=2$ for all n>1

Proof:

Let G=R (BT (n)) be a rooted binary tree of level n with order 2n-1 and size 2n-2.

We prove this by the induction method.

Case 1:

When n=1,

Let RBT (1) be a Rooted Binary Tree of level 1 with one vertex and no edges. The dominating set D= $\{v_1\}$ dominating by itself. Hence domination number is one.

(i.e.) γ (RBT (1)) =1.

By definition of proper colouring and dominator colouring

$\chi_d(G)=\chi(G)=1$.

Case 2:

When n=2,

Let RBT (2) be a Rooted Binary Tree of level 2 with 3 vertices and 2 edges and

V (RBT (2))={ v_1, v_2, v_3} . By definition of domination, D= {v_1} and |D|=1 (i.e.) γ(RBT(2))=1. By definition of dominator colouring, we assigned colour 1 to the dominating set D and the remaining vertices are properly coloured by using colour 2 to the vertices v_2, v_3. Since they are non adjacent.

χ_d (RBT(2))= χ (RBT(2)=2.

Case 3:

When n=3,

Let RBT (3) be a Rooted Binary Tree of level 3 with 5 vertices and 4 edges and

V (RBT (3)) = {v_1, v_2, v_3 , v_4, v_5}. By definition of domination, D= {v_2, v_5} and |D|=2 (i.e.) γ (RBT (3)) =2. By definition of dominator colouring, we assigned colour 1 to the dominating set D and the remaining vertices are properly coloured by using colour 2 to the vertices v_1, v_3, and v_4.

χ_d (RBT(3))= χ (RBT(3)=2.

By continuing this process up to n terms.

Let RBT(n) be a Rooted Binary Tree of level n with 2n+1 vertices and 2n-2 edges and

V (RBT (n)) ={ v_1, v_2, v_3, ……., v_{2n+1} } . By definition of domination, D= {v_{n-1}} and |D|=n-1 (i.e.) γ (RBT (n)) =n-1. By definition of dominator colouring, we assigned colour 1 to the dominating set D and the remaining vertices are properly coloured by using colour 2 to the vertices.

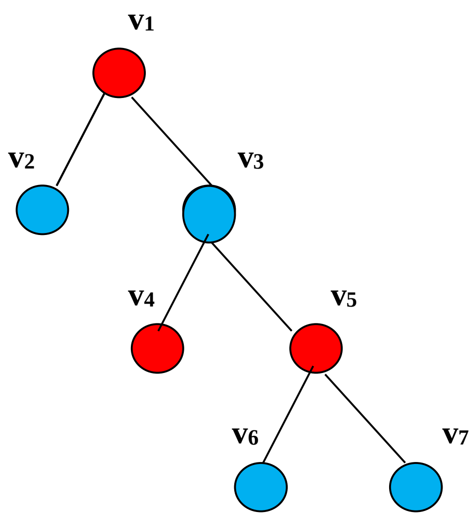
χ_d (RBT(n))=n-1.

4.1 OBSERVATION:

Since no vertices of dominating set has 2 neighbours in V-D and so the concept of 2-domination fails. Hence we conclude that every Rooted Binary Tree does not have 2-domination number.

4.1.1EXAMPLE:

G= (RBT (4)) for n=even



In the above graph G(RB(4)) is v(RBT(4))={v_1,v_2,v_3,v_4,v_5,v_6,v_7}. The dominating set is D= {v_1, v_4, v_5} and |D|=3, then γ (RBT (4)) =3.

We assigned colour (RED) to C_1= {v_1, v_4, v_5} these vertices are non-adjacent. So we assigned the same colour.

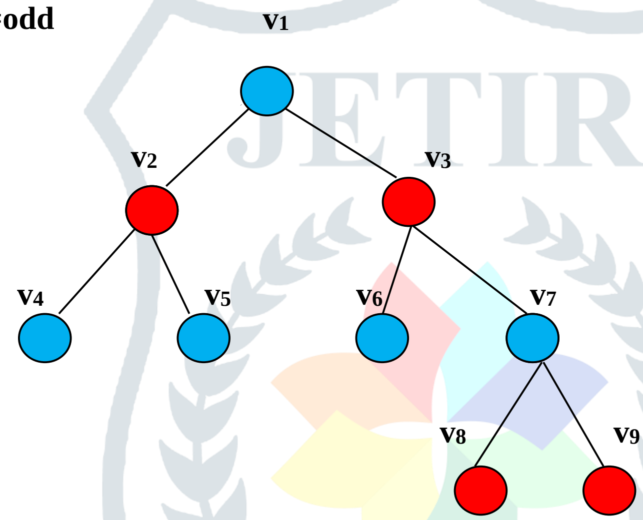
We assigned colour (BLUE) to C_2= {v_2, v_3, v_6, v_7} these vertices are properly coloured by using colouring. By using definition of dominator colouring, we assigned colour C_1 (RED) to the dominating set D and the remaining vertices are properly coloured by using colour C_2(BLUE).

The dominator chromatic number is 2.

χ_d (RBT(4))=2.

4.1.2EXAMPLE:

G= (RBT(5) for n=odd



In the above graph G(RB(5)) is v(RBT(5))={v_1,v_2,v_3,v_4,v_5,v_6,v_7,v_8,v_9}. The dominating set is D= {v_2, v_3, v_8, v_9} and |D|=3, then γ (RBT (5)) = 4.

We assigned colour (RED) to C_1= {v_2, v_3, v_8, v_9} these vertices are non-adjacent. Since we assigned same colour.

We assigned colour (BLUE) to C_2={v_1,v_4,v_5,v_6,v_7} these vertices are properly coloured by using colouring. By using definition of dominator colouring, we assigned colour C_1 (RED) to the dominating set D and the remaining vertices are properly coloured by using colour C_2(BLUE).

The dominator chromatic number is 2.

χ_d (RBT(5))=2.

Theorem 4.2:

Let G=P(BT(n)) be a perfect binary tree of level n with 2^h -1 and 2^h -2 edges the domination γ(BT(P)), and dominator chromatic number χ (BT(P)), chromatic number χ (BT(P)) and 2- domination number is given by

i. γ (BT(P))=2^{n-1} -n-1
ii. χ_d (BT(P))=2^{n-1} -n-2

iii. χ (BT(P))= 2 for all n>1

Proof:

Let G=PBT(n) be a perfect binary tree of level up to 5 with 2^h -1 vertices and 2^h -2 edges .

We prove this theorem by method of induction.

Case 1:

When n=1,

Let PBT (1) be a Perfect Binary Tree of level 1 with one vertex and no edges. The dominating set is D= {v_1} dominating by itself. Hence domination number is one (i.e.) γ (PBT (1)) =1.

By definition of proper colouring and dominator coloring

χ_d (PBT(1))=χ (PBT(1))=1

Case 2:

When n=2,

Let PBT (2) be a Perfect Binary Tree of level 2 with 3 vertices and 2 edges and

V (PBT (2)) = {v_1, v_2, v_3}. By definition of domination, D= {v_1} and |D|=1 (i.e.) γ (PBT (2)) =1. By definition of dominator colouring, we assigned colour 1 to the dominating vertex v_1 and the remaining vertices are properly coloured by using colour 2 to the vertices v_2, v_3.

χ_d (PBT(2))= χ (PBT(2))=2.

Case 3:

When n=3,

Let PBT(3) be a Perfect Binary Tree of level 3 with 5 vertices and 4 edges and

V (PBT (3)) = { v_1, v_2, v_3, v_4, v_5, v_6, v_7} . By definition of domination, D= {v_2, v_7} and |D|=2 (i.e) γ (PBT (3)) =2. By definition of dominator colouring, we assigned colour 1 to the dominating vertex v_2, v_7 and the remaining vertices are properly coloured by using colour 2 to the vertices v_1, v_3, v_4, v_5, v_6. Since they are non adjacent.

χ_d(PBT(3))= χ (PBT(3)=2.

By continuing this process up to n terms.

Let PBT(n) be a Perfect Binary Tree of level n with 2^h -1 vertices and 2^h -2 edges and

V (PBT (3)) ={ v_1, v_2, v_3, ……., v_{2^h-1} } . By definition of domination, D= {$v_{2^{n-1}-n-1}$} and |D|=2^{n-1}-n-1 (i.e) γ (RBT (n)) = 2^{n-1}-n-1. By definition of dominator colouring, we assigned colour 1 to the dominating set D, and the remaining vertices are properly coloured by using colour 2 to the vertices.

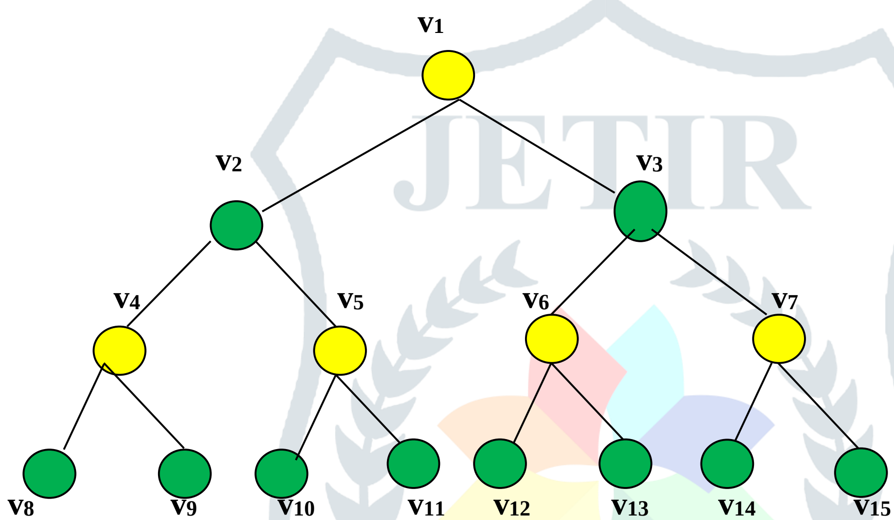
χ_d (G)= 2^{n-1} -n-2.

4.2.2OBSERVATION:

Since no vertices of dominating set has 2 neighbours in V-D and so the concept of 2-domination fails. Hence we conclude that every Perfect Binary Tree does not have 2-domination number.

4.2.1 EXAMPLE:

G= (PBT (3)) for n=even



In the above graph G(PBT(4)) is V(PBT(4))={$v_1,v_2,v_3,v_4,v_5,v_6,v_7,v_8,v_9,v_{10},v_{11},v_{12},v_{13},v_{14},v_{15}$}. The dominating set is D= {v_1, v_4, v_5, v_6, v_7} and |D|=5, then γ(PBT(3))=5.

We assigned colour (YELLOW) to C_1={v_1,v_4,v_5,v_6,v_7} these vertices are non-adjacent. Since we assigned same colour.

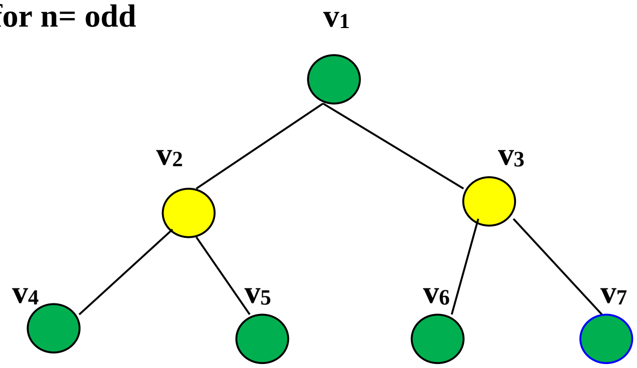
We assigned color (GREEN) to C_2={$v_2,v_3,v_8,v_9,v_{10},v_{11},v_{12},v_{13},v_{14},v_{15}$} these vertices are properly coloured by using colouring. By using definition of dominator colouring, we assigned colour C_1 (YELLOW) to the dominating set D and the remaining vertices are properly coloured by using colour C_2(GREEN).

The dominator chromatic number is 2.

χ_d (PBT(3))=2.

4.2.2EXAMPLE:

G=(PBT(3)) for n= odd



In the above graph G(PBT(3)) is v(PBT(3))={v_1,v_2,v_3,v_4,v_5,v_6,v_7}. The dominating set is D= {v_2, v_3} and |D|=2, then γ(PBT(3))=2.

We assigned color (YELLOW) to C_1={v_2,v_3} these vertices non-adjacent. since we assigned same colour.

We assigned to color (GREEN) to C_2={v_1,v_4,v_5,v_6,v_7}these vertices are properly coloured by using colouring. By using definition of dominator colouring, we assigned colour C_1 (YELLOW) to the dominating set D and the remaining vertices are properly coloured by using colour C_2 (GREEN).

The dominator chromatic number is 2.

χ_d (PBT(3))=2.

CONCLUSION:

In this paper, we have discussed about the properties of Rooted binary tree and Perfect binary trees. We established the bounds for domination, proper colouring, dominator colouring and 2-domination of perfect and rooted binary trees and we found that the bound are sharp.

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