On Super mean Labeling for Splitting graph of path and cycle

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Abstract:

Let G(V,E) be a graph with the vertex set V and the edge set E respectively. By a graph G=(V,E). We mean a finite undirected with neither loops nor multiple edges. The number of vertices of G is called order of G and it is denoted by p. Let G be a (p,q) graph. A super mean graph on G is an injection f: V $\longrightarrow$ {1,2,3,……..p+q} such that, for each edge e=uv in E labeled by $f^*(e)$=[Γ (f(u)+f(v))|2]. The set f(v) ∪ {$f^{'}$(e): e ∈ E}forms {1,2,3,……p+q}. A graph which admits super mean labelling is called super mean graph. The splitting graph $S^{'}(G)$ of a graph G is obtained by adding a new vertex $V^{'}$ corresponding to each vertex V of G such that N(V)=N($V^{'}$). We have showed that graphs $S^{'}(P_n)$ and $S^{'}(C_n)$ are super mean. Where P_n is a path on n vertices and C_n is a cycle on n vertices.

Keywords: Super mean labelling, path graph, cycle graph, splitting graph.

Introduction

Throughout this paper, by a graph we mean a finite, undirected, simple graph with neither loops nor multiple edges. Let G(V,E) be a graph with p vertices and q edges. The number of edges of g is called size of G and it is denoted by q. Terms and notations not defined here are used in the sense of Harary [1].

In 2003, Somasundaram and Ponraj [2] have introduced the notation of mean labelings of graphs. Let G be a (p,q) graph. A graph G is called a mean graph if there is an injective function f from the vertices of G to {0,1,2,….q} such that when each edge e=uv is labeled with $f^*(e)$ = (f(u) + f(v) +1)|2 if f(u)+f(v) is even and $f^*(e)$ = (f(u) + f(v) +1)|2 if f(u)+f(v) is odd, then the resulting edge labels are distinct. Furthermore, the concept of super mean labeling was introduced by Ponraj and Ramya [3]. Let f: V $\longrightarrow$ {1,2,3……p+q} be an injection on G. For each edge e=uv and an integer m≥2.

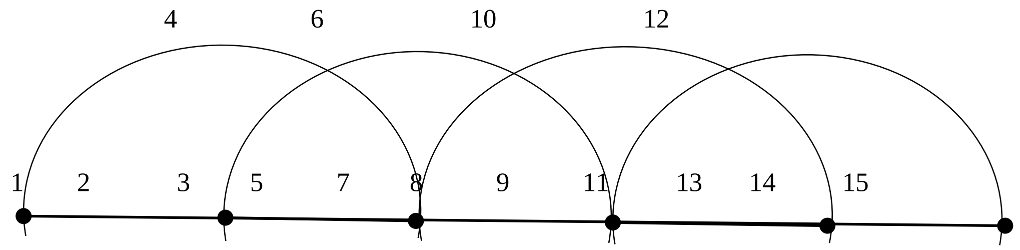
A vertex labelling of G is an assignment f: V(G) $\longrightarrow$ {1,2,3…..p+q} be an injection for a vertex labeling f, the induced smarandachely edge m – labeling f_s^* for an edge e=uv, an integer m≥2 is defined by

$$f_s^*(e)=\left\lceil \frac{f(u)+f(v)}{m} \right\rceil$$

then f is called a smarandachely super m-mean labelling if f(V(G)) ∪ {f^*(e) : e∈ E(G)}={1,2,3………………p+q}. Particularly, in this case of m=2, we know that

$$f^*(e)=\begin{cases} \frac{f(u)+f(v)}{2} & if \quad f(u)+f(v) \quad is \quad even \\ \frac{f(u)+f(v)+1}{2} & if \quad f(u)+f(v) \quad is \quad odd \end{cases}$$

such a labelling is usually called super mean labeling. A graph that admits a smarandachely super mean m-labeling is called smarandachely super mean graph if m=2. A super mean labelling of the graph P_2^6 is shown in Figure 1.1



Furthermore, discussions of mean and super mean labelings for some families of graph are provided in [4-10] and Gallian [11]. The Splitting graph S'(G) of G is obtained by adding a new vertex v' corresponding to each vertex v of G such that N(v)=N(v'). For instance, when G=pn, Splitting graph of path S'(p_n) is provided in figure 1. Since the problem on super mean labeling for graph variations are still open, the new our contributions are stated in the following sections.

Definitions:

Mean graph :A graph G is called a mean graph if there is an injective function f from the vertices of G to {0,1,2,….q} such that when each edge e=uv is labeled with $f^*(e) = (f(u) + f(v) +1)|2$ if f(u)+f(v) is even and $f^*(e) = (f(u) + f(v) +1)|2$ if f(u)+f(v) is odd, then the resulting edge labels are distinct.

Smarandachely edge m-labeling :A vertex labelling of G is an assignment f: V(G) $\rightarrow$ {1,2,3…..p+q} be an injection for a vertex labeling f, the induced smarandachely edge m – labeling f_s^* for an edge e=uv, an integer m≥2 is defined by

$$f_s^*(e)=\left\lceil \frac{f(u)+f(v)}{m} \right\rceil$$

Super mean labelling :A super mean graph on G is an injection f: V $\rightarrow$ {1,2,3,……..p+q} such that, for each edge e=uv in E labelled by $f^*(e)=[\Gamma\,(f(u)+f(v))|2]$. The set $f(v) \cup \{f^*(e): e \in E\}$forms {1,2,3,……p+q}.

Super mean graph :A graph which admits super mean labelling is called super mean graph.

Splitting graph : The splitting graph $S'(G)$ of a graph G is obtained by adding a new vertex V' corresponding to each vertex V of G such that $N(V)=N(V')$.

Path graph : The path graph P_n, is a graph with n vertices that can be enumerated such that two vertices are adjacent if and only if they are consecutive in the enumeration.

Cycle graph : The cycle graph C_n, is a graph with n vertices that can be enumerated such that two vertices are adjacent if and only if they are consecutive in the enumeration or are the first and last vertex in the enumeration.

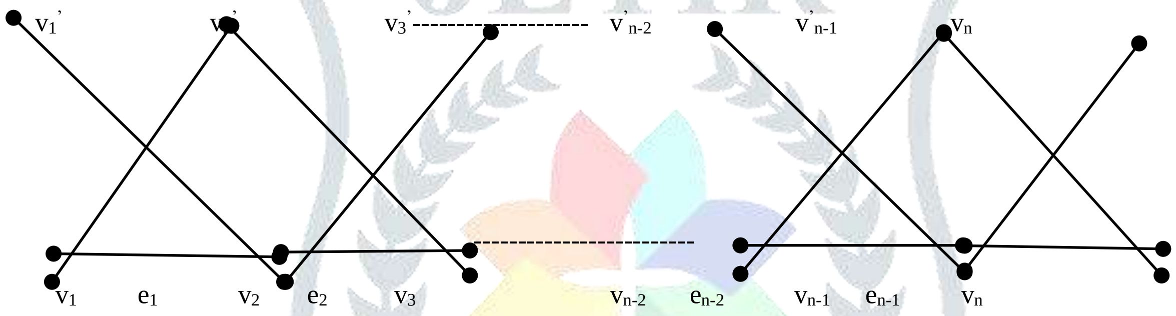


Figure 1: The splitting graph of path on n vertices

2. On Super Mean Labeling for Splitting graph of path

The theorem proposed in this section deals with the super mean labelling for splitting graph on n vertices $S'(p_n)$.

Theorem 1.

The splitting graph of path on n vertices $S'(p_n)$ is a super mean graph for all n≥3.

Proof:

Let $V(S'(p_n)=\{v_i : 1\le i\le n\} \cup \{v_i' : 1\le i\le n\}$ and $E(S'(p_n)=\{e_i,e_i',e_i'' : 1\le i\le n-1\}$ with $e_i=v_iv_{i+1}$, $e_i'=v_iv'_{i+1}$, $e_i''=v'_iv_{i+1}$ for $1\le i\le n-1$, Immediately we have that the cardinality of the vertex set and the edge set of $S'(p_n)$ are p=2n and q=4n-8 respectively, and so p+q=6n-8. Define an injection $f : v(S'(p_n)) \rightarrow \{1,2,3……6n-8\}$ for n≥3 as follows. $f(v_i)=2i-1$ for i=1,2,3….n. $f(u_i)= i+6n-13$ for i=1,2,3.......n.

And so we have

$f^*(e) =2i$ for i=1,2,3……n-1. $f^*(e')=2n+2i-2$ for i=1,2,3….n-1.

$f^*(e'')=2n+2i-1$ for i=1,2,3,….n-1.

Next we consider the following sets:

$A_1=\{f(v_i)=2i-1 : i=1,2,3….n\}$., $A_2=\{f(v_i)=2i-1 : i=1,2,3….n\}$., $A_3=\{ f^*(e) =2i : i=1,2,3……n-1\}$.,

$A_4 =\{f(e')=2n+2i-2 : i=1,2,3….n-1\}$., $A_5=\{f^*(e'')=2n+2i-1: i=1,2,3,….n-1\}$., It can be verified that $f(v(S'(p_n))) \cup f^*(E(S'(p_n))) = \bigcup_{i=1}^{5} A_i = \{1,2,3\ldots\ldots 6n-8\}$ and so f is a super mean labeling of S'(pn). Hence S'(pn) is a super mean labelling for splitting graph of path on five vertices is provided in figure 2.

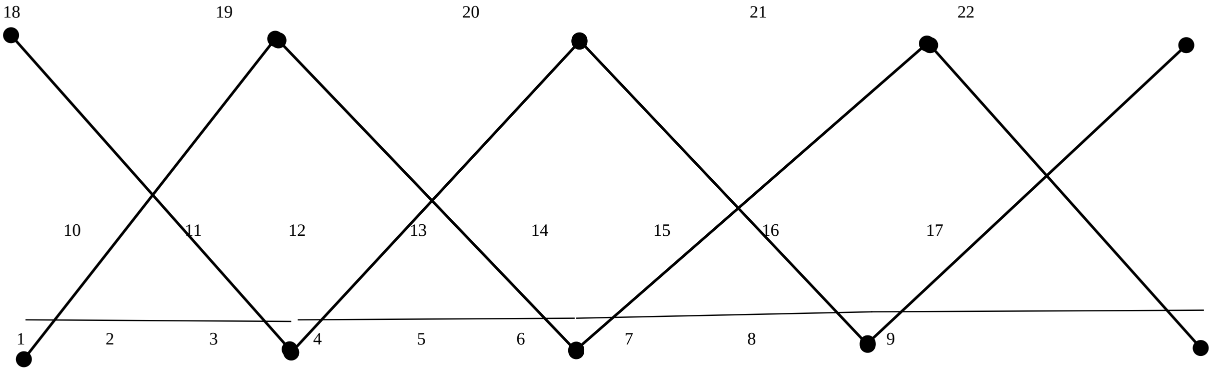


Figure 2: The super mean labelling for splitting graph of path on 5 vertices $S'(P_5)$

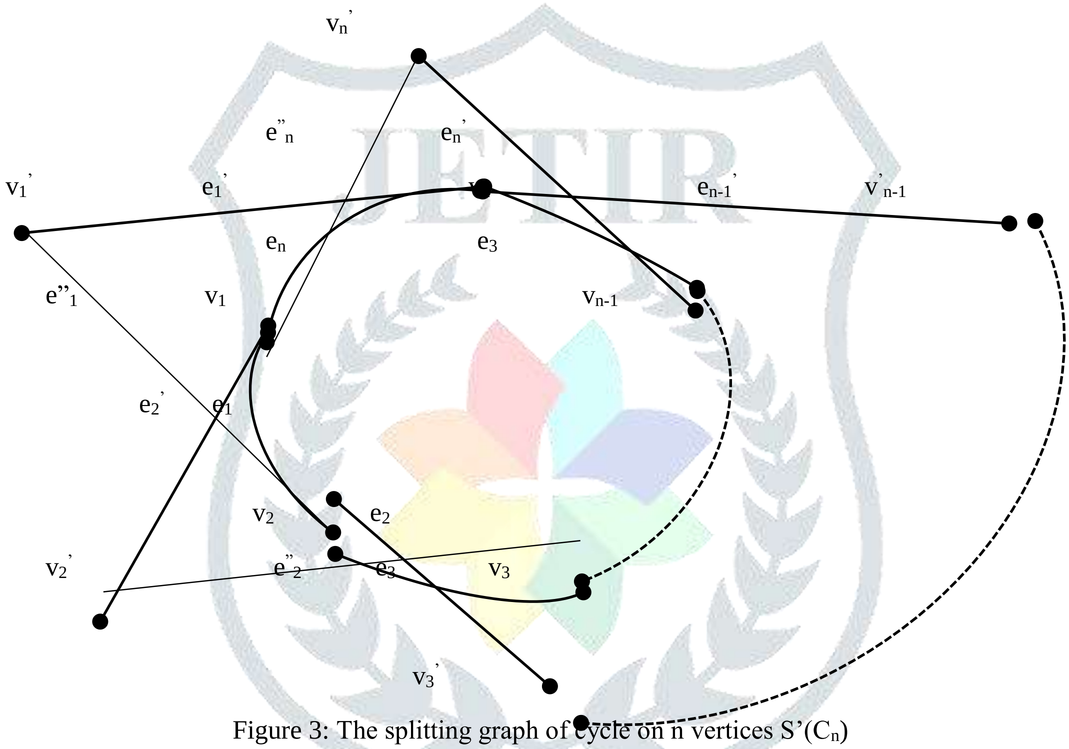


Figure 3: The splitting graph of cycle on n vertices $S'(C_n)$

3. On Super mean labeling for splitting graph of cycle.

The theorem proposed in this section deals with the super mean labeling for splitting graph of cycle on vertices.

The splitting graph of cycle on n vertices, $S'(C_n)$ is a super mean graph if either n is odd and odd $n\geq 3$ or n is even and $n\geq 6$.

Proof :

Let $V(S'(C_n))=\{ v_i : 1\leq i\leq n\} \cup \{v_i' : 1\leq i\leq n\}$ and and $E(S'(C_n))=\{e_i,e_i',e_i'': 1\leq i\leq n\}$. Where

$$e_i = \begin{cases} v_i v_{i+1} & 1\leq i\leq n-1 \\ v_1 v_i & i=n \end{cases} \quad e_i' = \begin{cases} v_i v_{i+1}' & 1\leq i\leq n-1; \\ v_i v_1' & i=n. \end{cases} \quad e_i'' = \begin{cases} v_i' v_{i+1} & 1\leq i\leq n-1; \\ v_1 v_i' & i=n. \end{cases}$$

Immediately, we have that the cardinality of the vertex set and the edge set of S'(Cn) are p=2n and q=3n respectively, and so p+q=5n. define an injection $f : v(S'(Cn)) \rightarrow \{1,2,\ldots\ldots\ldots 5n\}$ for odd $n\geq 3$ as follows.

$$f(v_i) = \begin{cases} 2i-1 & i=1,2,3.....\left\lceil \frac{n}{2} \right\rceil; \\ 2i & i=\left\lceil \frac{n}{2} \right\rceil+1, \left\lceil \frac{n}{2} \right\rceil+2.........n. \end{cases} \quad f(v_i') = \begin{cases} 4n+2i-1 & i=1,2,3.....\left\lceil \frac{n}{2} \right\rceil; \\ 4n+2i & i=\left\lceil \frac{n}{2} \right\rceil+1, \left\lceil \frac{n}{2} \right\rceil+2.......n. \end{cases}$$

And so we have

$$f^*(e_i)=\begin{cases} 2i, & i=1,2,3.....\left\lceil \frac{n}{2}\right\rceil-1; \\ n+2, & i=\left\lceil \frac{n}{2}\right\rceil; \\ 2i+1, & i=\left\lceil \frac{n}{2}\right\rceil+1,\left\lceil \frac{n}{2}\right\rceil+2.....n-1; \\ n+1, & i=n. \end{cases}$$

$$f^*(e_i^{'})=\begin{cases} 2n+2i-1, & i=1,2,3.....\left\lceil \frac{n}{2}\right\rceil; \\ 2n+2i, & i=\left\lceil \frac{n}{2}\right\rceil+1,\left\lceil \frac{n}{2}\right\rceil+2........n. \end{cases}$$

$$f^*(e_i^{''})=\begin{cases} 2n+2i, & i=1,2,3....\left\lceil \frac{n}{2}\right\rceil-1; \\ 3n+2, & i=\left\lceil \frac{n}{2}\right\rceil; \\ 2n+2i+1, & i=\left\lceil \frac{n}{2}\right\rceil+1,\left\lceil \frac{n}{2}\right\rceil+2.......n-1; \\ 3n+1, & i=n. \end{cases}$$

Next, we consider the following sets,

$A_1=\{f(v_i)=2i-1$ for $i=1,2.....\left\lceil \frac{n}{2}\right\rceil$;

$A_2=\{f(v_i)= 2i-1$ for $i=1,2,3.....\left\lceil \frac{n}{2}\right\rceil+1, \left\lceil \frac{n}{2}\right\rceil+2.......n.$

$A_3= \{f(v_i')=4n+2i-1$ for $i=1,2,3.....\left\lceil \frac{n}{2}\right\rceil$;

$A_4=\{ f(v_i')=4n+2i-8$ for $i=1,2,3.....\left\lceil \frac{n}{2}\right\rceil+1, \left\lceil \frac{n}{2}\right\rceil+2.......n.$

$A_5= \{f^*(e_i) =2i$ for $i=1,2,3.......\left\lceil \frac{n}{2}\right\rceil-1;$

$A_6= \{f^*(e_i)=n+2$ for $i=\left\lceil \frac{n}{2}\right\rceil$;

$A_7=\{ f^*(e_i)=2i+1$ for $i=1,2,3.....\left\lceil \frac{n}{2}\right\rceil+1, \left\lceil \frac{n}{2}\right\rceil+2.......n-1;$

$A_8=\{ f^*(e_i)=n+1$ for $i=n.$

$A_9 =\{f^*(e')=2n+2i-1$ for $i=1,2,3.....\left\lceil \frac{n}{2}\right\rceil$;

$A_{10}=\{ f^*(e')=2n+2i$ for $i=1,2,3.....\left\lceil \frac{n}{2}\right\rceil+1, \left\lceil \frac{n}{2}\right\rceil+2.......n.$

$A_{11}=\{ f^*(e'')=2n+2i$ for $i=1,2,3.....\left\lceil \frac{n}{2}\right\rceil-1;$

$A_{12}=\{ f^*(e'')=3n+2$ for $i=\left\lceil \frac{n}{2}\right\rceil$;

$A_{13}=\{ f^*(e'')=2n+1+2i$ for $i=\left\lceil \frac{n}{2}\right\rceil+1, \left\lceil \frac{n}{2}\right\rceil+2,.......n-1;$

$A_{14}= \{f^*(e'')=3n+1$ for $i=n.$

It can be verified that $f(v(S'(C_n).))) \bigcup f^*(E(S'(C_n).)))=\bigcup_{i=1}^{5} A_i=\{1,2,3......6n-8\}$ and so f is a super mean labelling of $S'(C_n)$. Hence $S'(C_n)$ is a super mean graph for odd $n\geq3$.

Now define an injection $f_{1}: V(S^{'}(C_n)) \rightarrow\{1,2,3......5n\}$ for even $n\geq6$ as follows

$$f_1(v_i)=\begin{cases} i, & i=1 \\ 3i-3, & i=2,3 \\ 4i-7, & i=4,5,6......\frac{n}{2}+1 \\ 4n-4i+8, & i=\frac{n}{2}+2,\frac{n}{2}+3....n-1 \\ 7i & i=n \end{cases} \qquad f_1(v_i^{'})=\begin{cases} 4n+1+i & i=0, \\ 4n+i & i=2,3 \\ 4n+2i-4 & i=4,5....\frac{n}{2}+1; \\ 3n+2i+1 & i=\frac{n}{2}+2,\frac{n}{2}+3....n-1; \\ 4n+8 & i=n. \end{cases}$$

And so we have

$$f_1^*(e_i)=\begin{cases} 2i & i=1 \\ 3i-1 & i=2,3 \\ 4i-5 & i=4,5,6....\frac{n}{2}+1; \\ 4n-4i+6 & i=\frac{n}{2}+2,\frac{n}{2}+3....n-1 \\ 4i & i=n \end{cases} \qquad f^*(e_i^{'})=\begin{cases} 2n+1, & i=1 \\ 2n+3i-3, & i=2,3 \\ 2n+4i-7, & i=4,5,6...\frac{n}{2}+1, \\ 6n-4i+8 & i=\frac{n}{2}+2,\frac{n}{2}+3...n-1 \\ 2n+7, & i=1; \end{cases}$$

$$f_1^*(e_i^{''}) = \begin{cases} 2n+2i & i=1 \\ 2n+3i-1 & i=2,3 \\ 2n+4i-5 & i=4,5,6.....\frac{n}{2}+1; \\ 6n-4i+6, & i=\frac{n}{2}+2, \frac{n}{2}+3...n-1 \\ 2n+4 & i=n. \end{cases}$$

It can be verified that f1(v(S'(Cn).))) ∪ f1*(E(S'(Cn).)))={1,2,3…..5n} and so f_1 is a super mean labeling of S'(Cn).

Hence S'(Cn) is a super mean graph for even n≥6 for illustration, a super mean labeling for splitting graph of cycle on 8 vertices is provided in figure 4.

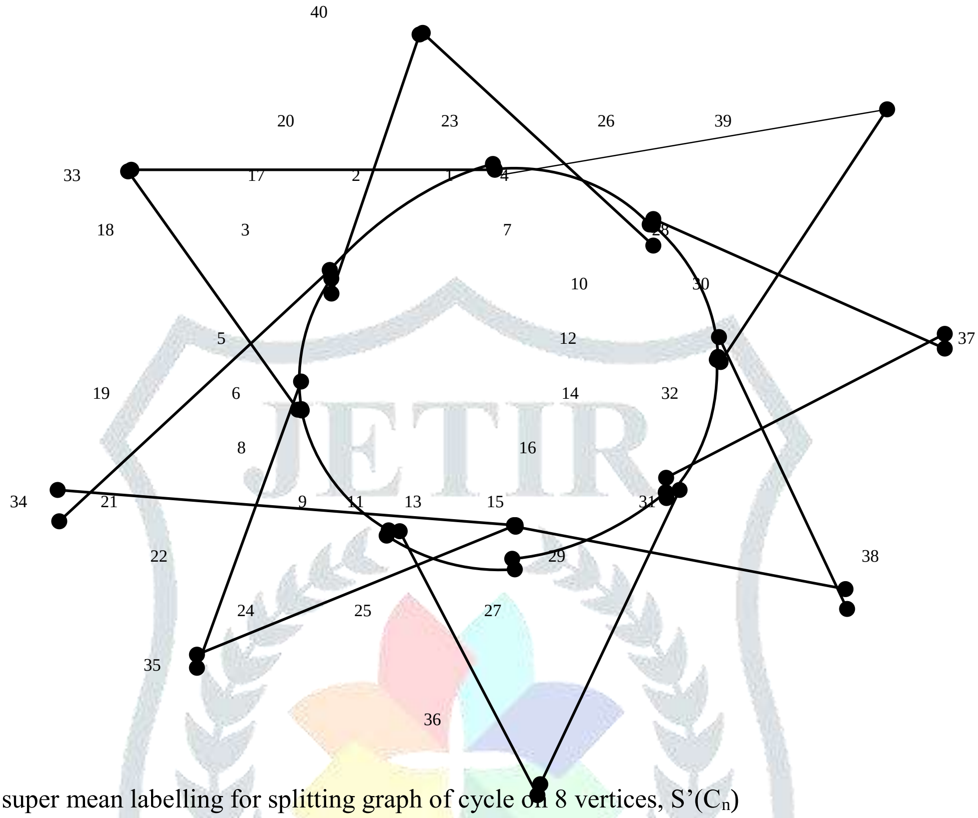


Figure 4: The super mean labelling for splitting graph of cycle on 8 vertices, S'(C_n)

Conclusion:

Here we propose new results corresponding to super mean labeling for splitting graph of path and cycle. This work is an effort to relate Smarandachely super m-mean labeling and its dual for m≥2. All results reported here are in splitting graph of path and cycle S'(P_n) and S'(C_n). In future, it is not only possible to investigate some more results corresponding to other graph families but also Smarandachely super m-mean labeling in general as well.

References:

[1] F. Harary, "Recent results on generalized Ramsey theory for graphs," in graph theory and applications, vol. 303 of Lecture Notes in Mathematics, pp. 125-138, Springer Berlin Heidelberg, Berlin, Heidelberg, 1972.
[2] S. Somasundaram and R. Ponraj, "Mean labelings of graphs," National Academy of science Letters, vol. 26, 7-8 pp. 210-213, 2003.
[3] R. Ponraj and D. Ramya, "On super mean graphs of order," Bulletin of pure and Applied Sciences, vol. 25, no.1,pp. 143-148, 2006.
[4] S.Somasundaram and R. Ponraj, "Non-existence of mean labeling for a wheel," Bulletin of pure and Applied Sciences -Mathematics and Statistics, vol. 22E, pp. 103-111, 2003.
[5] 103S.Somasundaram and R. Ponraj, "Some results on mean graphs," Pure and Applied Mathematics Journal, vol. 58, pp. 29-35, 2003.
[6] P. Jeyanthi and D. Ramya, " Super mean labelling of some classes of graphs," International Journal of Mathematical Combinatorics, vol. 1, pp. 83-91, 2012.
[7] P. Jeyanthi, D. Ramya, and P. Thangavelu, "On super mean graphs," AKCE International Journal of Mathematical Combinatorics, vol. 6, no. 1, pp. 103-112, 2009.
[8] P. Jeyanthi and D. Ramya, and P. Thangavelu, "On super mean labeling of some graphs," SUT Journal of Mathematics, vol. 46, no.1, pp. 53-66, 2010.
[9] P. Jeyanthi and D. Ramya, "Super mean graphs," Utilitas Mathematica, vol. 96, pp. 101-109, 2015.
[10] D. Ramya, R. Ponraj, and P. Jeyanthi, " Super mean labeling of graphs," Ars Combinatoria, vol. 112, pp. 65-72, 2013.
[11] J. A. Gallian, " A dynamic survey of graph labellings," The Electronic Journal of Combinatories, 2016.
[12] R. Balakrishnan and N. Renganathan, A Text Book on Graph Theory, Springer Verlag, 2000.