DOMINATION NUMBER OF DUTCH WINDMILL GRAPH

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ABSTRACT:

In this paper we present dominating sets, minimum diameter spanning tree for dutch windmill graph and some special graph, further discuss the simple connected graphs and above which have a minimum diameter spanning tree such that both have same domination number.

Introduction

Cockayne, E.J. and Hedetniemi, S.T.[1] introduced the concept dominating set. Frank Harary, Robert Z.Norman and Dorwin Cartwright [2] explained an interesting application in voting situations using the concept of dominate on. C.L. Liu [3] also discussed the application of dominance to communication network, where a dominating set represents a set of cities which acting as transmitting stations, can transmit messages to every city in the network. V.T Chandrasekaran, and N.Rajasri we discussed minimum diameter spanning tree domination in ladder and pan graph [9]. Dutch windmill graph: the dutch windmill graph $D_m^{(n)}$ is the graph obtained by taking n copies of the cycle C_m with vertex in common. The minimum cardinality of a dominating set in G is called the domination number of G and it is denoted by $\gamma\,(D_m^{(n)})$ In this paper , we discuss few simple connected graphs for which the dominating numbers of the graph and that of its minimum diameter spanning tree are the same.

Key words:

Domination, Diameter, Spanning tree, Helm graph, tadpole graph, crown graph, windmill graph, dutch windmill, flower graph.

1. Some definitions

Definition: Let $G = (V, E)$ be a graph. A subset S of V is called dominating set if every vertex in $V - S$ is adjacent to a vertex in S. The minimum cardinality of a dominating set in G is called the domination number of G and it is denoted by $\gamma(G)$.

Definition: Dutch windmill graph: the dutch windmill graph $D_m^{(n)}$ is the graph obtained by taking n copies of the cycle C_m with vertex in common. The minimum cardinality of a dominating set in G is called the domination number of G and it is denoted by $\gamma\,(D_m^{(n)})$.

Definition: The spanning tree T of the simple connected graph G is said to be a minimum diameter spanning tree. if there is no other spanning tree T' of G such that $d(T') < d(T)$

Definition: The diameter of a graph is length of the shortest path between the most distance node and it is denoted by d

2. Some results

2.1 Theorem

If G is dutch windmill graph, then $\gamma\ (D_m^{(n)}) = n\gamma\ (c_m)\ -\ (n-1)$

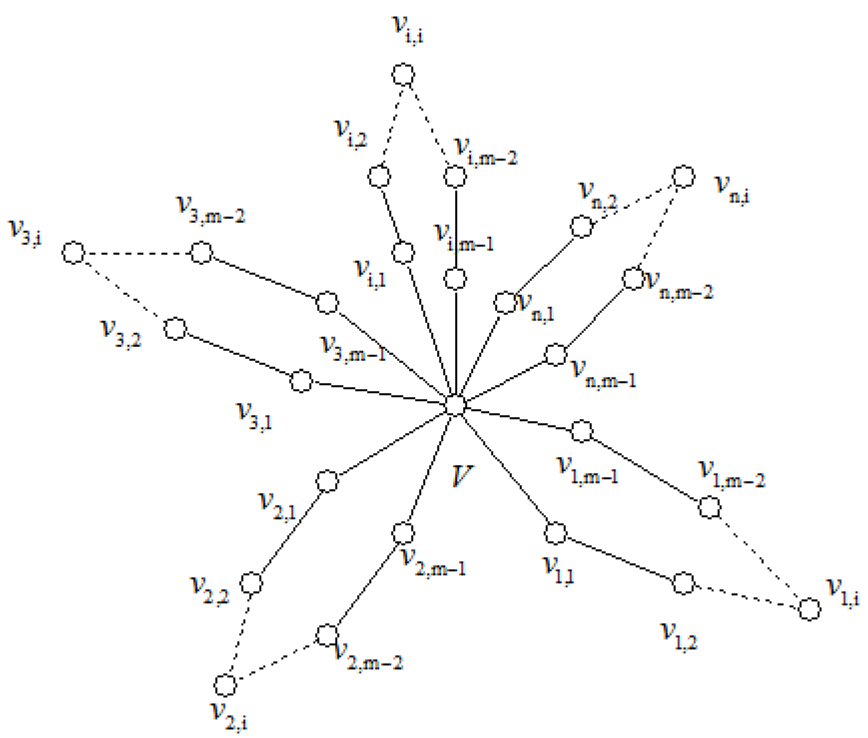

Figure 1: Dutch windmill Graph $D_m^{(n)}$

Let G be a Dutch windmill graph with m vertices and n cycle.

Let $\Delta = degv$, so must be in the minimal dominating set. $D_m^{(n)} - \{v\}$ becomes a disconnected graph of nP_{m-1}. That is $D_m^{(n)} - \{v\} = P_{m-1} \cup P_{m-1} \cup .... \cup P_{m-1}$ in the graph $D_m^{(n)}$, v is adjacent to every pendent vertices of P_{m-1} in the graph $D_m^{(n)} - \{v\}$. Hence it is enough to dominate the remaining nP_{m-3} paths.

$$\gamma(P_{m-3}) = \left\lceil \frac{m-3}{3} \right\rceil$$

$$n\gamma(P_{m-3}) = n\left\lceil \frac{m-3}{3} \right\rceil$$

$$\gamma\left(D_m^{(n)}\right) = 1 + n\gamma\left(P_{m-3}\right)$$

$$= 1 + n\left\lceil \frac{m-3}{3} \right\rceil$$

$$= 1 + n\left\lceil \frac{m}{3} \right\rceil - n$$

$$\gamma\left(D_m^{(n)}\right) = n\gamma\left(c_m\right) - (n-1)$$

Hence proved

2.2 THEOREM

A Dutch windmill graph $D_m^{(n)}$ has a minimum diameter spanning tree T for which $\gamma\left(D_m^{(n)}\right) = \gamma(T)$ if $m \not\equiv 0 \text{ or } 5 \ (mod\ 6)$.

Proof

Consider $D_m^{(n)}$ where $m \geq 5$ and $n \geq 2$ to obtain a spanning tree of $D_m^{(n)}$, we must remove an edge from each cycle to obtain a minimum diameter spanning tree T.

Case 1: $m \equiv 0 \ (mod 6)$

Let $m = 6k$ where k is some positive integer, let u be the vertex which has maximum degree the removal of vertex u must be separated into $2n$ paths. The length of n paths is p_{3k-2} and the length of another n paths is p_{3k-1}.

Now,

$$\gamma\ (T) = n\ \gamma\ (p_{3k-2}) + n\ \gamma\ (p_{3k-1}) + 1 \ldots\ldots\ldots\ldots(A)$$

here $\gamma\ (p_{3k-2}) = \left\lceil \frac{p_{3k-2}}{3} \right\rceil = k$ and $\gamma\ (p_{3k-1}) = \left\lceil \frac{p_{3k-1}}{3} \right\rceil = k$

then $\gamma\ (T) = 2nk + 1$ but $\gamma\ \left(D_{6k}^{(n)}\right) = 2nk - n + 1$.

Case 2: $m \equiv 1 \ (mod 6)$

Let $m = 6k + 1$ where k is some positive integer, let u be the vertex which has maximum degree the removal of vertex u must be separated into $2n$ paths. Can be separated into $2n$ equal paths is p_{3k-1}.

Now,

$$\gamma\ (T) = 2\ n\ \gamma\ (p_{3k-1}) + 1 \ldots\ldots\ldots\ldots(B)$$

here $\gamma\ (p_{3k-1})\ = \left\lceil \frac{p_{3k-1}}{3} \right\rceil = k$

then $\gamma\ (T)\ =\ 2nk+1$ Also $\gamma(D_{6k+1}{}^{(n)}) = 2nk+1$

Case 3: $m \equiv 2\ (mod 6)$

Let $m = 6k+2$ where k is some positive integer, let u be the vertex which has maximum degree the removal of vertex u must be separated into $2n$ paths. The length of n paths is p_{3k-1} and the length of another n paths is p_{3k} .

Now,

$$\gamma\ (T)\ =\ n\ \gamma\ (p_{3k-1})\ +\ n\ \gamma\ (p_{3k})\ +1 \ldots\ldots\ldots\ldots(C)$$

here $\gamma\ (p_{3k-1})\ = \left\lceil \frac{p_{3k-1}}{3} \right\rceil = k\ and \gamma\ (p_{3k})\ = \left\lceil \frac{p_{3k}}{3} \right\rceil = k$

then $\gamma\ (T)\ =\ 2nk+1$ Also $\gamma\ \left(D_{6k+2}{}^{(n)}\right)\ =\ 2nk+1$

Case 4: $m \equiv 3\ (mod 6)$

Let $m = 6k+3$ where k is some positive integer, let u be the vertex which has maximum degree the removal of vertex u must be separated into $2n$ paths. Can be separated into $2n$ equal paths is p_{3k} .

Now,

$$\gamma\ (T)\ =\ 2\ n\ \gamma\ (p_{3k})\ +1 \ldots\ldots\ldots\ldots (D)$$

here $\gamma\ (p_{3k})\ = \left\lceil \frac{p_{3k}}{3} \right\rceil = k$

then $\gamma\ (T)\ =\ 2nk+1$ Also $\gamma\ \left(D_{6k+3}{}^{(n)}\right)\ =\ 2nk+1$.

Case 5: $m \equiv 4(mod 6)$

Let $m = 6k+4$ where k is some positive integer, let u be the vertex which has maximum degree the removal of vertex u must be separated into $2n$ paths. The length of n paths is p_{3k+1} and the length of another n paths is p_{3k} .

Now,

$$\gamma\ (T)\ =\ n\ \gamma\ (p_{3k+1})\ +\ n\ \gamma\ (p_{3k})\ +1 \ldots\ldots\ldots\ldots(E)$$

here $\gamma\ (p_{3k+1})\ = \left\lceil \frac{p_{3k+1}}{3} \right\rceil = k+1$ *and* $\gamma\ (p_{3k})\ = \left\lceil \frac{p_{3k}}{3} \right\rceil = k$

then $\gamma\ (T)\ =\ 2nk+n+1$ Also $\gamma\ \left(D_{6k+4}{}^{(n)}\right)\ =\ 2nk\ +n+1$

Case 6: $m \equiv 5\ (mod 6)$

Let $m = 6k + 5$ where k is some positive integer, let u be the vertex which has maximum degree the removal of vertex u must be separated into $2n$ paths. Can be separated into $2n$ equal paths is p_{3k+1}.

Now,

$\gamma\ (T)\ =\ 2\ n\ \gamma\ (p_{3k+1})\ +1$ …………..(F)

here $\gamma\ (p_{3k-1})\ = \left\lceil \frac{p_{3k-1}}{3} \right\rceil = k$, then

$\gamma\ (T)\ =\ 2nk+2n+1$ *but* $\gamma\ \left(D_{6k+5}{}^{(n)}\right)\ =\ 2nk+n+1$

Hence Proved

3. Check whether $\gamma\ (G)\ = \gamma\ (T)$ for some standard graphs

- The windmill graph $K_n^{(m)}$ or $Wd(n,m)$ is the graph obtained by taking m copies of the complete graph Kn with a vertex in common. For any Wind mill graph $\gamma\ (G)\ = \gamma\ (T)\ =\ 1$.
- Crown graph is obtained by joining a pendant edge to each vertex of C_n .For any Crown graph $\gamma\ (G)\ = \gamma\ (T)\ =$ n .
- The (m,n) tadpole graph also called as dragon graph, is the graph obtained by joining a cycle graph C_m to a path graph P_n. For any tadpole graph $\gamma\ (G) \neq \gamma\ (T)$
  [by theorem 2.2]
- The helm graph H_n is the graph obtained from a wheel W_n by attaching a pendant edge at each vertex of the n cycle. For any Helm graph $\gamma\ (H_n)\ = \gamma\ (T)\ =\ n.$
- Flower graph is obtained from helm graph H_n by joining each vertex to the central vertex of the helm graph. For any flower graph $\gamma\ (Fl_n)\ = \gamma\ (T)\ =\ 1$.
- Gear graph G_n is obtained from wheel graph W_n by adding a vertex between every pair of adjacent vertices of the n - cycle. For any Gear graph $\gamma\ (G)\ = \gamma\ (T)$.
- A prism graph also known as circular ladder graph, denoted by Y_n. For any prism graph $\gamma\ (G) \neq \gamma\ (T)$.

4. Conclusion

In this article, we have discussed few graphs for which the domination number is the same that of its minimum diameter spanning tree. Further research can be done in exploring various graphs with the same property. The condition for which a graph does not posses such spanning tree may also be explored.

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