OPERATIONS ON BIPOLAR ANTI FUZZY GRAPHS

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Abstract: In this paper, we introduce the concept of bipolar anti-fuzzy graphs and operations on bipolar anti fuzzy graphs such as anti-union, anti-join, anti-Cartesian product and anti-composition. We applied such operations on some types of bipolar anti-fuzzy graphs and obtained the results on them.

Keywords: Bipolar anti fuzzy graph, bipolar fuzzy operations

I. INTRODUCTION

The concept of fuzzy graph was first introduced by Kaufmann [1] from the fuzzy relation introduced by Zedah [2]. R. Muthuraj and A. Sasireka [3] introduced the concept of anti fuzzy graphs. R. Seethalakshmi and R.B. Gnanajothi [4] discussed the concept of operations on anti fuzzy graphs but this concept was extended by R. Muthuraj and A. Sasireka [5]. In 1994, Zhang [6] initiated the concept of bipolar fuzzy sets as a generalization of fuzzy sets. Muhammad Akram [7] introduced the concept of bipolar fuzzy graphs. In this paper, we introduce the concept of operations on bipolar anti fuzzy graphs such as anti union, anti join, anti cartesian product and anti composition. We derived some results on them.

II PRIMILINARIES

Definition 2.1: A fuzzy graph G = (σ, μ) is said to be an anti fuzzy graph with a pair of functions σ : V→[0,1] and μ : V×V→[0,1], where for all u ,v ∈ V, we have μ(u ,v) ≥ σ(u) ∨ σ(v) where ∨ denotes maximum and it is denoted by $G_A(\sigma, \mu)$.

Notation: Without loss of generality let us simply use the letter G_A to denote an anti fuzzy graph.

Note: μ is considered as reflexive and symmetric. In all examples σ is chosen suitably. That is undirected anti fuzzy graphs are only considered.

Definition 2.2: The underlying anti crisp graph of an anti fuzzy graph $G_A = (\sigma, \mu)$ is denoted by $G_A^* = (\sigma^*, \mu^*)$, where $\sigma^* = \{u \in V / \sigma(u) > 0\}$ and $\mu^* = \{ (u, v) \in V\times V / \mu(u, v) > 0\}$.

Definition 2.2: Let X be a non empty set. A bipolar fuzzy set B in X is an object having the form
B = $\{(x, \mu^P(x), \mu^N(x) / x \in X\}$, where μ^P : X→ [0,1] and μ^N : X→ [-1,0]

We use the positive membership degree $\mu^P(x)$ to denote the satisfaction degree of an element x to the property corresponding to a bipolar fuzzy set B, and the negative membership degree $\mu^N(x)$ denote the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar fuzzy set B. If $\mu^P(x) \neq 0$ and $\mu^P(x) = 0$, it is the situation that x is regarded as having only positive satisfaction for B. If $\mu^P(x) = 0$ and $\mu^N(x) \neq 0$, it is the situation that x does not satisfy the property of B but somewhat satisfies the counter property of B. It is possible for an element x to be such that $\mu^P(x) \neq 0$ and $\mu^N(x) \neq 0$ when the membership function of the property overlaps that of its counter property over some portion of X.

For the sake of simplicity, we shall use the symbol B = (μ^P, μ^N) for the bipolar fuzzy set B =$\{(x, \mu^P(x), \mu^N(x) / x \in X\}$.

Definition 2.3: For every two bipolar fuzzy sets A = (μ_A^P, μ_A^N) and B = (μ_B^P, μ_B^N) in X, we define

- (A∩B)(x) = (min $(\mu_A^P(x), \mu_B^P(x))$, max$(\mu_A^N(x), \mu_B^N(x))$).
- (A∪B)(x) = (max $(\mu_A^P(x), \mu_B^P(x))$, min$(\mu_A^N(x), \mu_B^N(x))$).

Definition 2.4: Let X be a nonempty set. Then we call a mapping A = $(\mu_A^P, \mu_A^N) : X \times X \to [0,1] \times [-1,0]$ a bipolar fuzzy relation on X such that $\mu_A^P(x, y) \in [0,1]$ and $\mu_A^N(x, y) \in [-1,0]$.

Definition 2.5: Let A = (μ_A^P, μ_A^N) and B = (μ_B^P, μ_B^N) be a bipolar fuzzy sets on a set X. If A = (μ_A^P, μ_A^N) is a bipolar fuzzy relation on a set X, then A = (μ_A^P, μ_A^N) is called a bipolar fuzzy relation on B = (μ_B^P, μ_B^N) if $\mu_A^P(x, y) \leq$ min $(\mu_B^P(x), \mu_B^P(y))$ and $\mu_A^N(x, y) \geq$ max $(\mu_B^N(x), \mu_B^N(y))$ for all x, y ∈ X. A bipolar fuzzy relation A on X is called symmetric if $\mu_A^P(x, y) = \mu_A^P(y, x)$ and $\mu_A^N(x, y) = \mu_A^N(y, x)$ for all x, y ∈ X.

Note: Throughout this paper, G^* will be a crisp graph, G will be a bipolar fuzzy graph, G_A^* will be an anti crisp graph and G_A will be a bipolar anti fuzzy graph.

Definition : A bipolar fuzzy graph with an underlying set V is defined to be a pair G = (A, B) where A = (μ_A^P, μ_A^N) is a bipolar fuzzy set in V and B = (μ_B^P, μ_B^N) is a bipolar fuzzy set in E ⊆ V × V such that

$$\mu_B^P(\{x,y\}) \le \min(\mu_A^P(x), \mu_A^P(y)) \quad \text{and} \quad \mu_B^N(\{x,y\}) \ge \max(\mu_A^N(x), \mu_A^N(y))$$

for all {x, y} ∈ E. We call A the bipolar fuzzy vertex set of V, B the bipolar fuzzy edge set of E, respectively. Note that B is a symmetric bipolar fuzzy relation on A. We use the notation xy for an element of E. Thus, G = (A, B) is a bipolar graph of G^* = (V, E) if

$$\mu_B^P(xy) \le \min(\mu_A^P(x), \mu_A^P(y)) \quad \text{and} \quad \mu_B^N(xy) \ge \max(\mu_A^N(x), \mu_A^N(y)) \text{ for all } xy \in E.$$

Example: Consider the graph G^* = (V, E) such that V = {a, b, c}, E = {ab, bc, ca}. Let A = (μ_A^P, μ_A^N) is a bipolar fuzzy subset of V and let B = (μ_B^P, μ_B^N) be a bipolar fuzzy subset of E ⊆ V × V

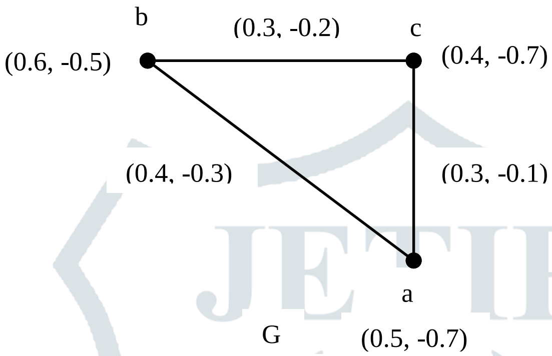


Figure 1

III OPERATIONS ON BIPOLAR ANTI FUZZY GRAPHS

Definition 3.1: A bipolar fuzzy graph is said to be bipolar anti fuzzy graph with an underlying set V is defined to be a pair G = (A, B) where A = (μ_A^P, μ_A^N) is a bipolar fuzzy set in V and B = (μ_B^P, μ_B^N) is a bipolar fuzzy set in E ⊆ V × V such that

$$\mu_B^P(\{x,y\}) \ge \max(\mu_A^P(x), \mu_A^P(y)) \quad \text{and} \quad \mu_B^N(\{x,y\}) \le \min(\mu_A^N(x), \mu_A^N(y))$$

for all {x, y} ∈ E. We call A the bipolar fuzzy vertex set of V, B the bipolar fuzzy edge set of E, respectively. Note that B is a symmetric bipolar anti fuzzy relation on A. We use the notation xy for an element of E. Thus, G_A = (A, B) is a bipolar anti fuzzy graph of G_A^* = (V, E) if

$$\mu_B^P(xy) \ge \max(\mu_A^P(x), \mu_A^P(y)) \quad \text{and} \quad \mu_B^N(xy) \le \min(\mu_A^N(x), \mu_A^N(y)) \text{ for all } xy \in E.$$

Example 3.2: Consider the graph G_A^* = (V, E) such that V = {a, b, c}, E = {ab, bc, ca}. Let A = (μ_A^P, μ_A^N) is a bipolar fuzzy subset of V and let B = (μ_B^P, μ_B^N) be a bipolar fuzzy subset of E ⊆ V × V

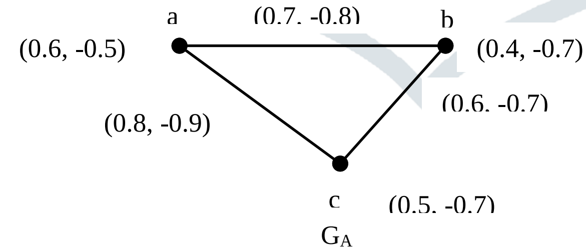


Figure 1

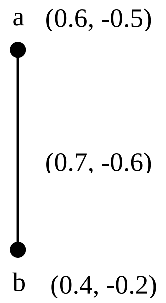
Definition 3.3: Let $A_1 = (\mu_{A_1}^P, \mu_{A_1}^N)$ and $A_2 = (\mu_{A_2}^P, \mu_{A_2}^N)$ be bipolar fuzzy subsets of V_1 and V_2 and $B_1 = (\mu_{B_1}^P, \mu_{B_1}^N)$ and $B_2 = (\mu_{B_2}^P, \mu_{B_2}^N)$ be bipolar fuzzy subsets of E_1 and E_2 respectively. Then, we denote the anti Cartesian product of two bipolar anti fuzzy graphs G_{A_1} and G_{A_2} of the graphs $G_{A_1}^*$ and $G_{A_2}^*$ by $G_{A_1} \times G_{A_2} = (A_1 \times A_2, B_1 \times B_2)$, and defined as follows:

(i) $(\mu_{A_1}^P \times \mu_{A_2}^P)(x_1, x_2) = \max(\mu_{A_1}^P(x_1), \mu_{A_2}^P(x_2))$
$(\mu_{A_1}^N \times \mu_{A_2}^N)(x_1, x_2) = \min(\mu_{A_1}^N(x_1), \mu_{A_2}^N(x_2))$ for all $(x_1, x_2) \in V$,

(ii) $(\mu_{B_1}^P \times \mu_{B_2}^P)((x, x_2), (x, y_2)) = \max(\mu_{A_1}^P(x), \mu_{B_2}^P(x_2 y_2))$
$(\mu_{B_1}^N \times \mu_{B_2}^N)((x, x_2), (x, y_2)) = \min(\mu_{A_1}^N(x), \mu_{B_2}^N(x_2 y_2))$ for all $x \in V_1$, for all $x_2 y_2 \in E_2$,

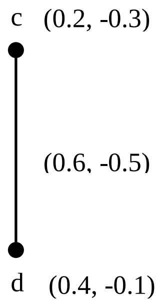
(iii) $(\mu^P_{B_1} \times \mu^P_{B_2})$ ((x₁, z), (y₁, z)) = max $(\mu^P_{B_1}(x_1y_1), \mu^P_{A_2}(z))$
$(\mu^N_{B_1} \times \mu^N_{B_2})$ ((x₁, z), (y₁, z)) = min $(\mu^N_{B_1}(x_1y_1), \mu^N_{A_2}(z))$ for all z ∈ V_2, for all x_1y_1 ∈ E_1.

Example 3.4: Consider the bipolar anti fuzzy graphs.



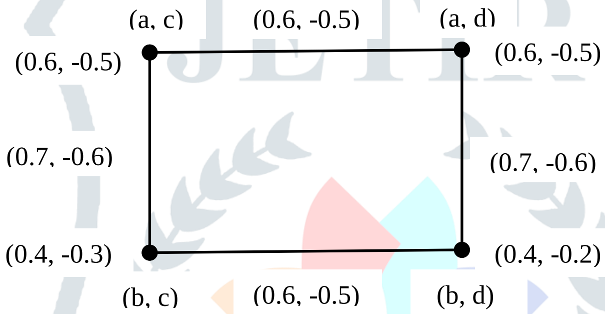
G_{A_1} -Bipolar anti fuzzy graph

Figure 2



G_{A_2} -Bipolar anti fuzzy graph

Figure 3



$G_{A_1} \times G_{A_2}$

Figure 4

Proposition 3.5: If G_{A_1} and G_{A_2} are the bipolar anti fuzzy graphs, then $G_{A_1} \times G_{A_2}$ is a bipolar anti fuzzy graph.
Proof: Let x ∈ V_1, x_2y_2 ∈ E_2. Then we have

$(\mu^P_{B_1} \times \mu^P_{B_2})$ ((x, x₂), (x, y₂)) = $\max(\mu^P_{A_1}(x), \mu^P_{B_2}(x_2y_2)) \geq \max(\mu^P_{A_1}(x), \max(\mu^P_{A_2}(x_2), \mu^P_{A_2}(y_2)))$
$= \max(\max(\mu^P_{A_1}(x), \mu^P_{A_2}(x_2)), \max(\mu^P_{A_1}(x), \mu^P_{A_2}(y_2)))$
$= \max((\mu^P_{A_1} \times \mu^P_{A_2})(x, x_2)), (\mu^P_{A_1} \times \mu^P_{A_2})(x, y_2)),$

$(\mu^N_{B_1} \times \mu^N_{B_2})$ ((x, x₂), (x, y₂)) = $\min(\mu^N_{A_1}(x), \mu^N_{B_2}(x_2y_2)) \leq \min(\mu^N_{A_1}(x), \min(\mu^N_{A_2}(x_2), \mu^N_{A_2}(y_2)))$
$= \min(\min(\mu^N_{A_1}(x), \mu^N_{A_2}(x_2)), \min(\mu^N_{A_1}(x), \mu^N_{A_2}(y_2)))$
$= \min((\mu^N_{A_1} \times \mu^N_{A_2})(x, x_2)), (\mu^N_{A_1} \times \mu^N_{A_2})(x, y_2)).$

Let z ∈ V_2, x_1y_1 ∈ E_1. Then we have

$(\mu^P_{B_1} \times \mu^P_{B_2})$ ((x₁, z), (y₁, z)) = $\max(\mu^P_{B_1}(x_1y_1), \mu^P_{A_2}(z)) \geq \max(\max(\mu^P_{A_1}(x_1), \mu^P_{A_1}(y_1)), (\mu^P_{A_2}(z))$
$= \max(\max(\mu^P_{A_1}(x_1), \mu^P_{A_2}(z)), \max(\mu^P_{A_1}(y_1), \mu^P_{A_2}(z)))$
$= \max((\mu^P_{A_1} \times \mu^P_{A_2})(x_1, z)), (\mu^P_{A_1} \times \mu^P_{A_2})(y_1, z)),$

$(\mu^N_{B_1} \times \mu^N_{B_2})$ ((x₁, z), (y₁, z)) = $\min(\mu^N_{B_1}(x_1y_1), \mu^N_{A_2}(z)) \leq \min(\min(\mu^N_{A_2}(x_2), \mu^N_{A_2}(y_2)), \mu^N_{A_2}(z))$
$= \min(\min(\mu^N_{A_1}(x_1), \mu^N_{A_2}(z)), \min(\mu^N_{A_1}(y_1), \mu^N_{A_2}(z)))$
$= \min((\mu^N_{A_1} \times \mu^N_{A_2})(x_1, z)), (\mu^N_{A_1} \times \mu^N_{A_2})(y_1, z)).$

This completes the proof.

Definition 3.6: Let A₁ = $(\mu^P_{A_1}, \mu^N_{A_1})$ and A₂ = $(\mu^P_{A_2}, \mu^N_{A_2})$ be bipolar fuzzy subsets of V_1 and V_2 and let $B_1 = (\mu^P_{B_1}, \mu^N_{B_1})$ and $B_2 = (\mu^P_{B_2}, \mu^N_{B_2})$ be bipolar fuzzy subsets of E_1 and E_2 respectively. Then, we denote the anti composition of two bipolar anti fuzzy graphs G_{A_1} and G_{A_2} of the graphs $G^*_{A_1}$ and $G^*_{A_2}$ by $G_{A_1}[G_{A_2}] = (A_1 o A_2, B_1 o B_2)$, and defined as follows:

(i) $(\mu^P_{A_1} o\, \mu^P_{A_2})$ (x_1, x_2) = max $(\mu^P_{A_1}(x_1), \mu^P_{A_2}(x_2))$
$(\mu^N_{A_1} o\, \mu^N_{A_2})$ (x_1, x_2) = min $(\mu^N_{A_1}(x_1), \mu^N_{A_2}(x_2))$ for all $(x_1, x_2) \in V$,

(ii) $(\mu^P_{B_1} o\, \mu^P_{B_2})$ $((x, x_2), (x, y_2))$ = max $(\mu^P_{A_1}(x), \mu^P_{B_2}(x_2y_2))$
$(\mu^N_{B_1} o\, \mu^N_{B_2})$ $((x, x_2), (x, y_2))$ = min $(\mu^N_{A_1}(x), \mu^N_{B_2}(x_2y_2))$ for all $x \in V_1$, for all $x_2y_2 \in E_2$,

(iii) $(\mu^P_{B_1} o\, \mu^P_{B_2})$ $((x_1, z), (y_1, z))$ = max $(\mu^P_{B_1}(x_1y_1), \mu^P_{A_2}(z))$
$(\mu^N_{B_1} o\, \mu^N_{B_2})$ $((x_1, z), (y_1, z))$ = min $(\mu^N_{B_1}(x_1y_1), \mu^N_{A_2}(z))$ for all $z \in V_2$, for all $x_1y_1 \in E_1$,

(iv) $(\mu^P_{B_1} o\, \mu^P_{B_2})$ $((x_1, x_2), (x_1, y_2))$ = max $(\mu^P_{A_2}(x_2), \mu^P_{A_2}(y_2), \mu^P_{B_1}(x_1y_1))$
$(\mu^N_{B_1} o\, \mu^N_{B_2})$ $((x_1, x_2), (x_1, y_2))$ = min $(\mu^N_{A_2}(x_2), \mu^N_{A_2}(y_2), \mu^N_{B_1}(x_1y_1))$ for all $(x_1, x_2), (x_1, y_2) \in E^0 - E$.

Example 3.7: The following Figure 5 represents anti composition of two bipolar anti fuzzy graphs of Figure 2 and Figure 3.

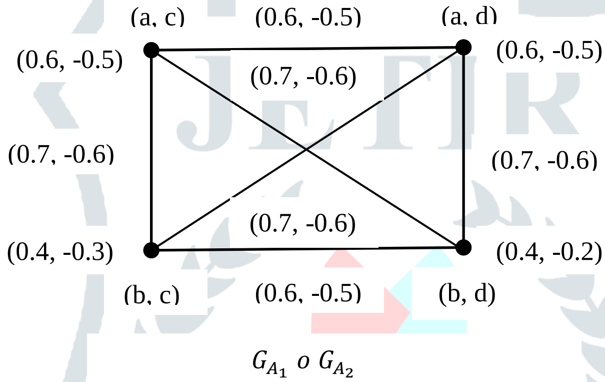


Figure 5 Anti composition of bipolar anti fuzzy graphs G_{A_1} and G_{A_2}

Proposition 3.8: If G_{A_1} and G_{A_2} are the bipolar anti fuzzy graphs, then $G_{A_1}[G_{A_2}]$ is a bipolar anti fuzzy graph.
Proof: Let $x \in V_1$ and $x_2y_2 \in E_2$. Then we have

$(\mu^P_{B_1} o \mu^P_{B_2})$ $((x, x_2), (x, y_2))$ = $\max(\mu^P_{A_1}(x), \mu^P_{B_2}(x_2y_2)) \geq \max(\mu^P_{A_1}(x), \max(\mu^P_{A_2}(x_2), \mu^P_{A_2}(y_2)))$
$= \max(\max(\mu^P_{A_1}(x), \mu^P_{A_2}(x_2)), \max(\mu^P_{A_1}(x), \mu^P_{A_2}(y_2)))$
$= \max((\mu^P_{A_1} o \mu^P_{A_2})(x, x_2)), (\mu^P_{A_1} o \mu^P_{A_2})(x, y_2)),$

$(\mu^N_{B_1} o \mu^N_{B_2})$ $((x, x_2), (x, y_2))$ = $\min(\mu^N_{A_1}(x), \mu^N_{B_2}(x_2y_2)) \leq \min(\mu^N_{A_1}(x), \min(\mu^N_{A_2}(x_2), \mu^N_{A_2}(y_2)))$
$= \min(\min(\mu^N_{A_1}(x), \mu^N_{A_2}(x_2)), \min(\mu^N_{A_1}(x), \mu^N_{A_2}(y_2)))$
$= \min((\mu^N_{A_1} o \mu^N_{A_2})(x, x_2)), (\mu^N_{A_1} o \mu^N_{A_2})(x, y_2)).$

Let $z \in V_2$ and $x_1y_1 \in E_1$. Then we have

$(\mu^P_{B_1} o \mu^P_{B_2})$ $((x_1, z), (y_1, z))$ = $\max(\mu^P_{B_1}(x_1y_1), \mu^P_{A_2}(z)) \geq \max(\max(\mu^P_{A_1}(x_1), \mu^P_{A_1}(y_1)), (\mu^P_{A_2}(z))$
$= \max(\max(\mu^P_{A_1}(x_1), \mu^P_{A_2}(z)), \max(\mu^P_{A_1}(y_1), \mu^P_{A_2}(z)))$
$= \max((\mu^P_{A_1} o \mu^P_{A_2})(x_1, z)), (\mu^P_{A_1} o \mu^P_{A_2})(y_1, z)),$

$(\mu^N_{B_1} o \mu^N_{B_2})$ $((x_1, z), (y_1, z))$ = $\min(\mu^N_{B_1}(x_1y_1), \mu^N_{A_2}(z)) \leq \min(\min(\mu^N_{A_1}(x_2), \mu^N_{A_1}(y_2)), \mu^N_{A_2}(z))$
$= \min(\min(\mu^N_{A_1}(x_1), \mu^N_{A_2}(z)), \min(\mu^N_{A_1}(y_1), \mu^N_{A_2}(z)))$
$= \min((\mu^N_{A_1} o \mu^N_{A_2})(x_1, z)), (\mu^N_{A_1} o \mu^N_{A_2})(y_1, z)).$

Let $(x_1, x_2), (x_1, y_2) \in E^0 - E$ so $x_1y_1 \in E_1$, $x_2 \neq y_2$. Then we have

$(\mu^P_{B_1} o \mu^P_{B_2})$ $((x_1, x_2), (y_1, y_2))$ = $\max(\mu^P_{A_2}(x_2), \mu^P_{A_2}(y_2), \mu^P_{B_1}(x_1y_1)) \geq \max(\mu^P_{A_2}(x_2), \mu^P_{A_2}(y_2), \max(\mu^P_{A_1}(x_1), \mu^P_{A_1}(y_1)))$
$= \max(\max(\mu^P_{A_1}(x_1), \mu^P_{A_2}(x_2)), \max(\mu^P_{A_1}(y_1), \mu^P_{A_2}(y_2)))$
$= \max((\mu^P_{A_1} o \mu^P_{A_2})(x_1, x_2)), (\mu^P_{A_1} o \mu^P_{A_2})(y_1, y_2)),$

$(\mu_{B_1}^N o \mu_{B_2}^N)$ ((x₁, x₂), (y₁, y₂)) $= \min(\mu_{A_2}^N(x_2), \mu_{A_2}^N(y_2), \mu_{B_1}^N(x_1 y_1)) \leq \min(\mu_{A_2}^N(x_2), \mu_{A_2}^N(y_2), \min(\mu_{A_1}^N(x_1), \mu_{A_1}^N(y_1)))$

$= \min(\min(\mu_{A_1}^N(x_1), \mu_{A_2}^N(x_2)), \min(\mu_{A_1}^N(y_1), \mu_{A_2}^N(y_2)))$

$= \min((\mu_{A_1}^N o \mu_{A_2}^N)(x_1, x_2)), (\mu_{A_1}^N o \mu_{A_2}^N)(y_1, y_2)),$

This completes the proof.

Definition 3.9: Let $A_1 = (\mu_{A_1}^P, \mu_{A_1}^N)$ and $A_2 = (\mu_{A_2}^P, \mu_{A_2}^N)$ be bipolar fuzzy subsets of V_1 and V_2 and $B_1 = (\mu_{B_1}^P, \mu_{B_1}^N)$ and $B_2 = (\mu_{B_2}^P, \mu_{B_2}^N)$ be bipolar fuzzy subsets of E_1 and E_2 respectively. Then, we denote the anti union of two bipolar anti fuzzy graphs G_{A_1} and G_{A_2} of the graphs $G_{A_1}^*$ and $G_{A_2}^*$ by $G_{A_1} \cup G_{A_2} = (A_1 \cup A_2, B_1 \cup B_2)$, and defined as follows:

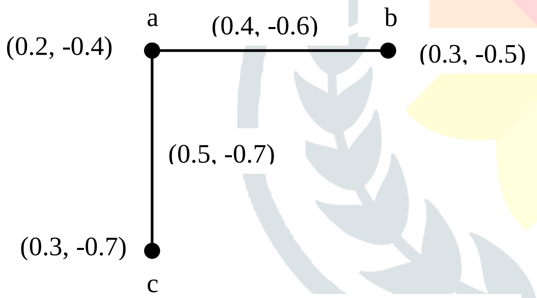
(i) $(\mu_{A_1}^P \cup \mu_{A_2}^P)(x) = \mu_{A_1}^P(x)$ if $x \in V_1 \cap \overline{V_2}$,
$(\mu_{A_1}^P \cup \mu_{A_2}^P)(x) = \mu_{A_2}^P(x)$ if $x \in V_2 \cap \overline{V_1}$,
$(\mu_{A_1}^P \cup \mu_{A_2}^P)(x) = \min(\mu_{A_1}^P(x), \mu_{A_2}^P(x))$ if $x \in V_1 \cap V_2$.

(ii) $(\mu_{A_1}^N \cap \mu_{A_2}^N)(x) = \mu_{A_1}^N(x)$ if $x \in V_1 \cap \overline{V_2}$,
$(\mu_{A_1}^N \cap \mu_{A_2}^N)(x) = \mu_{A_2}^N(x)$ if $x \in V_2 \cap \overline{V_1}$,
$(\mu_{A_1}^N \cap \mu_{A_2}^N)(x) = \max(\mu_{A_1}^N(x), \mu_{A_2}^N(x))$ if $x \in V_1 \cap V_2$.

(iii) $(\mu_{B_1}^P \cup \mu_{B_2}^P)$ (xy) $= \mu_{B_1}^P(xy)$ if xy $\in E_1 \cap \overline{E_2}$,
$(\mu_{B_1}^P \cup \mu_{B_2}^P)$ (xy) $= \mu_{B_2}^P(xy)$ if xy $\in E_2 \cap \overline{E_1}$,
$(\mu_{B_1}^P \cup \mu_{B_2}^P)$ (xy) $= \min(\mu_{B_1}^P(xy), \mu_{B_2}^P(xy))$ if xy $\in E_1 \cap E_2$.

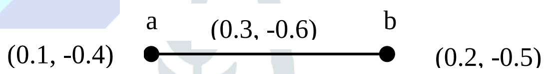
(iv) $(\mu_{B_1}^N \cap \mu_{B_2}^N)$ (xy) $= \mu_{B_1}^N(xy)$ if xy $\in E_1 \cap \overline{E_2}$,
$(\mu_{B_1}^N \cap \mu_{B_2}^N)$ (xy) $= \mu_{B_2}^N(xy)$ if xy $\in E_2 \cap \overline{E_1}$,
$(\mu_{B_1}^N \cap \mu_{B_2}^N)$ (xy) $= \max(\mu_{B_1}^N(xy), \mu_{B_2}^N(xy))$ if xy $\in E_1 \cap E_2$.

Example 3.10: Consider the bipolar fuzzy graphs.



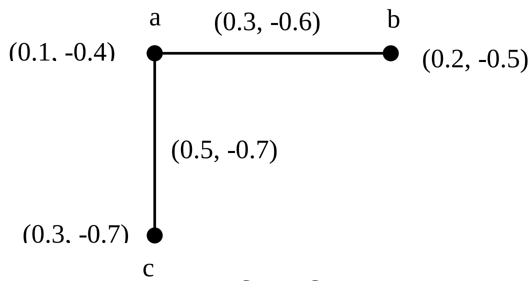
G_{A_1} -Bipolar anti fuzzy graph

Figure 6



G_{A_2} -Bipolar anti fuzzy graph

Figure 7



$G_{A_1} \cup G_{A_2}$

Figure 8

Proposition 3.11: If G_{A_1} and G_{A_2} are bipolar anti fuzzy graphs, then $G_{A_1} \cup G_{A_2}$ is a bipolar anti fuzzy graph

Proof: Let xy $\in E_1 \cap E_2$. Then

$(\mu_{B_1}^P \cup \mu_{B_2}^P)$ (xy) $= \min(\mu_{B_1}^P(xy), \mu_{B_2}^P(xy)) \geq \min(\max(\mu_{A_1}^P(x), \mu_{A_1}^P(y)), \max(\mu_{A_2}^P(x), \mu_{A_2}^P(y)))$

$= \max(\min(\mu_{A_1}^P(x), \mu_{A_2}^P(x)), \min(\mu_{A_1}^P(y), \mu_{A_2}^P(y))) = \max((\mu_{A_1}^P \cup \mu_{A_2}^P)(x), (\mu_{A_1}^P \cup \mu_{A_2}^P)(y)),$

$(\mu^N_{B_1} \cap \mu^N_{B_2})$ (xy) =max($\mu^N_{B_1}(xy), \mu^N_{B_2}(xy)) \leq$ max (min$(\mu^N_{A_1}(x), \mu^N_{A_1}(y))$,min$(\mu^N_{A_2}(x), \mu^N_{A_2}(y)))$
= min (max$(\mu^N_{A_1}(x), \mu^N_{A_2}(x))$, max$(\mu^N_{A_1}(y), \mu^N_{A_2}(y)))$ = min$((\mu^N_{A_1} \cap \mu^N_{A_2})(x), (\mu^N_{A_1} \cap \mu^N_{A_2})(y))$.

Similarly, we can show that if xy ∈ $E_1 \cap \overline{E_2}$, then

$(\mu^P_{B_1} \cup \mu^P_{B_2})$ (xy) ≥ max$((\mu^P_{A_1} \cup \mu^P_{A_2})(x), (\mu^P_{A_1} \cup \mu^P_{A_2})(y))$,

$(\mu^N_{B_1} \cap \mu^N_{B_2})$ (xy) ≤ min$((\mu^N_{A_1} \cap \mu^N_{A_2})(x), (\mu^N_{A_1} \cap \mu^N_{A_2})(y))$.

If xy ∈ $E_2 \cap \overline{E_1}$, then

$(\mu^P_{B_1} \cup \mu^P_{B_2})$ (xy) ≥ max$((\mu^P_{A_1} \cup \mu^P_{A_2})(x), (\mu^P_{A_1} \cup \mu^P_{A_2})(y))$,

$(\mu^N_{B_1} \cap \mu^N_{B_2})$ (xy) ≤ min$((\mu^N_{A_1} \cap \mu^N_{A_2})(x), (\mu^N_{A_1} \cap \mu^N_{A_2})(y))$.

This completes the proof.

Proposition 3.12: Let $\{G_{A_i}$: i ∈ ∧} be a family of bipolar fuzzy graphs with the underlying set V. Then $\cap\, G_{A_i}$ is a bipolar fuzzy graph.
Proof: For any x, y ∈ V, we have

$$\cap\, \mu^P_B(xy) = \inf_{i \in \wedge} \mu^P_B(xy) \geq \inf_{i \in \wedge} \max\{\mu^P_{A_i}(x), \mu^P_{A_i}(y)\} = \max\left\{\inf_{i \in \wedge} \mu^P_{A_i}(x), \inf_{i \in \wedge} \mu^P_{A_i}(x)\right\} = \max\{\cap\, \mu^P_{A_i}(x), \cap\, \mu^P_{A_i}(y)\},$$

$$\cap\, \mu^N_B(xy) = \sup_{i \in \wedge} \mu^N_B(xy) \leq \sup_{i \in \wedge} \min\{\mu^N_{A_i}(x), \mu^N_{A_i}(y)\} = \min\left\{\sup_{i \in \wedge} \mu^N_{A_i}(x), \sup_{i \in \wedge} \mu^N_{A_i}(x)\right\} = \min\{\cap\, \mu^N_{A_i}(x), \cap\, \mu^N_{A_i}(y)\}.$$

Hence $\cap\, G_{A_i}$ is a bipolar fuzzy graph.

Definition 3.13: Let A₁ = $(\mu^P_{A_1}, \mu^N_{A_1})$ and A₂ = $(\mu^P_{A_2}, \mu^N_{A_2})$ be bipolar fuzzy subsets of V_1 and V_2 and B_1 = $(\mu^P_{B_1}, \mu^N_{B_1})$ and B_2 = $(\mu^P_{B_2}, \mu^N_{B_2})$ be bipolar fuzzy subsets of E_1 and E_2 respectively. Then, we denote the anti join of two bipolar anti fuzzy graphs G_{A_1} and G_{A_2} of the graphs $G^*_{A_1}$ and $G^*_{A_2}$ by $G_{A_1} + G_{A_2}$ = (A_1 +A_2, B_1 + B_2), and defined as follows:

(i) $(\mu^P_{A_1} + \mu^P_{A_2})(x)$ = $(\mu^P_{A_1} \cup \mu^P_{A_2})(x)$,
$(\mu^N_{A_1} + \mu^N_{A_2})(x)$ = $(\mu^N_{A_1} \cap \mu^N_{A_2})(x)$) if $x \in V_1 \cup V_2$.

(ii) $(\mu^P_{B_1} + \mu^P_{B_2})(xy)$ = $(\mu^P_{B_1} \cup \mu^P_{B_2})(xy) = \mu^N_{B_1}(xy)$,
$(\mu^N_{B_1} + \mu^N_{B_2})(xy)$ = $(\mu^N_{B_1} \cap \mu^N_{B_2})(xy) = \mu^N_{B_1}(xy)$ if $xy \in E_1 \cap E_2$,

(iii) $(\mu^P_{B_1} + \mu^P_{B_2})(xy)$ = min $(\mu^P_{A_1}(x), (\mu^P_{A_2}(y))$
$(\mu^N_{B_1} + \mu^N_{B_2})(xy)$ = max $(\mu^N_{A_1}(x), (\mu^N_{A_2}(y))$

If xy ∈ E', where E' is the set of all edges joining the nodes of V_1 and V_2.

Example 3.14: The following Figure 9 represents anti join of two bipolar anti fuzzy graphs of Figure 2 and Figure 3.

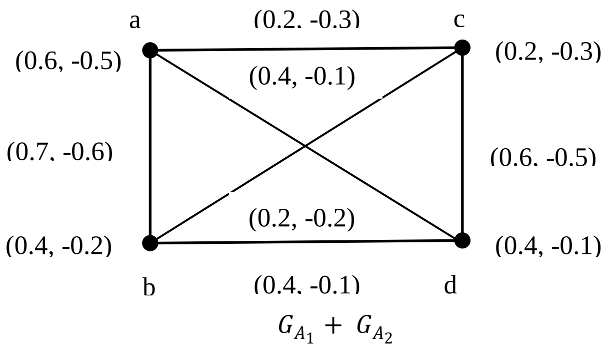


Figure 9 Anti join of bipolar anti fuzzy graphs G_{A_1} and G_{A_2}

Example 3.15: If G_{A_1} and G_{A_2} are the bipolar anti fuzzy graphs, then $G_{A_1}+G_{A_2}$ is a bipolar anti fuzzy graph
Proof: Let xy ∈ E'. Then

$$(\mu^P_{B_1} + \mu^P_{B_2})\ (xy) = \min(\mu^P_{A_1}(x), \mu^P_{A_2}(y)) \geq \min((\mu^P_{A_1} \cup \mu^P_{A_2})(x), (\mu^P_{A_1} \cup \mu^P_{A_2})(y)),$$
$$= \min((\mu^P_{A_1} + \mu^P_{A_2})(x), (\mu^P_{A_1} + \mu^P_{A_2})(y)).$$

$$(\mu^N_{B_1} + \mu^n_{B_2})\ (xy) = \max(\mu^N_{A_1}(x), \mu^n_{A_2}(y)) \leq \max((\mu^N_{A_1} \cap \mu^N_{A_2})(x), (\mu^N_{A_1} \cap \mu^N_{A_2})(y)),$$
$$= \max((\mu^N_{A_1} + \mu^N_{A_2})(x), (\mu^N_{A_1} + \mu^N_{A_2})(y)).$$

Let xy ∈ $E_1 \cup E_2$. Then the results follow proposition 3.11. This completes the proof.

We formulate the following characterizations.

Proposition 3.17: Let $G^*_{A_1}$= (V_1, E_1) and $G^*_{A_2}$ = (V_2, E_2) be anti crisp graphs and let $V_1 \cap V_2 = \emptyset$. Let A_1, A_2, B_1 and B_2 be bipolar fuzzy subsets of V_1, V_2, E_1 and E_2 respectively. Then $G_{A_1} \cup G_{A_2}$ = ($A_1 \cup A_2$, $B_1 \cup B_2$) is a bipolar anti fuzzy graph of G^*_A if and only if G_{A_1} = (A_1, B_1) and G_{A_2} = (A_2, B_2) are bipolar anti fuzzy graphs $G^*_{A_1}$ and$G^*_{A_2}$, respectively.
Proof: Suppose that $G_{A_1} \cup G_{A_2}$ is a bipolar anti fuzzy graph. Let xy ∈ E_1. Then xy ∉ E_2 and x,y ∈ $V_1 - V_2$. Thus

$$\mu^P_{B_1}(xy) = (\mu^P_{B_1} \cup \mu^P_{B_2})(xy) \geq \max((\mu^P_{A_1} \cup \mu^P_{A_2})(x), (\mu^P_{A_1} \cup \mu^P_{A_2})(y)) = \max\left(\mu^P_{A_1}(x), \mu^P_{A_2}(y)\right),$$

$$\mu^N_{B_1}(xy) = (\mu^N_{B_1} \cap \mu^N_{B_2})(xy) \leq \min((\mu^N_{A_1} \cap \mu^N_{A_2})(x), (\mu^N_{A_1} \cap \mu^N_{A_2})(y)) = \min\left(\mu^N_{A_1}(x), \mu^N_{A_2}(y)\right).$$

This shows that G_{A_1} = (A_1, B_1) is a bipolar anti fuzzy graph. Similarly, we can show that G_{A_2} = (A_2, B_2) is a bipolar anti fuzzy graph.
The converse the above proposition is given by proposition 3.11.
As a consequence if propositions 3.16 and 3.17, we obtain.

Proposition 3.18: Let $G^*_{A_1}$= (V_1, E_1) and $G^*_{A_2}$ = (V_2, E_2) be anti crisp graphs and let $V_1 \cap V_2 = \emptyset$. Let A_1, A_2, B_1 and B_2 be bipolar fuzzy subsets of V_1, V_2, E_1 and E_2 respectively. Then $G_{A_1} + G_{A_2}$ = ($A_1 + A_2$, $B_1 + B_2$) is a bipolar anti fuzzy graph of G^*_A if and only if G_{A_1} = (A_1, B_1) and G_{A_2} = (A_2, B_2) are bipolar anti fuzzy graphs $G^*_{A_1}$ and $G^*_{A_2}$, respectively.

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