# Comparison of SLM based PAPR reduction methods without need of SI 

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#### Abstract

Orthogonal Frequency Division Multiplexing (OFDM) is considered to be a promising technique robust to multi-path fading and time varying channel behaviors of the wireless channels. However, the Peak-to-Average Power Ratio (PAPR) problem is a major drawback of this OFDM multicarrier transmission system which leads to power inefficiency in RF section of the transmitter. This paper present different PAPR reduction techniques developed based on selective mapping (SLM) algorithm without the side information (SI) being transmitted and conclude an overall comparison of these techniques. The SLM methods considered are Blind-SLM (b-SLM), Phase offset SLM (pSLM), extended SLM (eSLM), and low complexity eSLM (LC-eSLM). We assumed 2 transmitting antennas and corresponding SFBC coding. The CCDF (Complementary Cumulative Distribution Function) performance of PAPR for different schemes with $\mathrm{N}=512$ subcarriers per OFDM symbol is plotted and a comparison of the number of complex multiplications and complex additions involved in these SLM methods is done.


## Index terms: PAPR, OFDM, SLM, CCDF, SFBC.

## 1. Introduction:

Multi-Input-Multi-Output Orthogonal Frequency Division Multiplexing (MIMO-OFDM) systems have drawn significant attention due to its high data rate capacity and reliable performance. However, MIMO-OFDM systems suffer from high peak-toaverage power ratio (PAPR). The high PAPR demands design of the complex wide range linear performance power amplifier which drives the antenna. So reduction of PAPR becomes an important factor. There are many PAPR reduction techniques such as clipping, selected mapping (SLM), partial transmit sequence (PTS), constellation shaping and tone reservation etc [10]. Among these methods, the SLM scheme is an attractive and efficient technique, since it can achieve good PAPR reduction without signal distortion. However, in the conventional SLM (C-SLM) schemes, extra bits should be reserved for the explicit transmission of the phase rotation sequences as side information (SI), resulting in the decrease of the data rate. Moreover, if the SI is interfered in a frequency selective fading channel, it leads to the huge degradation of the bit error rate (BER) performance. Some of SLM schemes which involves explicit SI transmission are ordinary SLM (oSLM), Simplified SLM (sSLM) [8], and directed SLM (dSLM) [9]. The SLM schemes which do not require explicit transmission of SI are bSLM[2], pSLM[4] eSLM[1] and LC eSLM[1]. In the section 2 the bSLM, pSLM eSLM and LC eSLM realistion algorithms are presented. Section 3 contains simulation results and comparision of these techniques. Finally the performance analysis is concluded in section 4.

## 2. The procedure involved in the corresponding SLM techniques

2.1 bSLM: In bSLM technique a blind estimation of the index is done on simplified SLM. Steps involved in this technique are similar to that of sSLM but we don't transmit SI instead we estimate SI.
At the transmitter side, assume N subcarriers and Consider the data symbols (QAM/PSK) group them in to sets each containing N elements. We call it data symbol set or symbol vector. Apply SFBC [5], multiply each produced data symbol in a symbol set with the phase sequence $\mathrm{b}^{\mathrm{d}}=e^{i \theta}$ function $(0 \leq \mathrm{d} \leq \mathrm{D})$ and produce D symbol sets corresponding to the single symbol set. Apply N point IFFT to each of the D symbols sets. Calculate PAPR to each of the D symbols set. The symbol that has lowest PAPR is considered for transmitting. At receiver side, remove CP, apply FFT, Obtain equations for received signals. Estimate the phase sequence applied. Undo the phase rotation and determine the sent OFDM data symbols.
Case Study: No. of antennas considered for transmitting: 2, No. of subcarriers in an OFDM symbol considered: $\mathrm{N}_{\mathrm{c}}$, Antenna diversity technique: $S F B C$. Transmitted vector at $p^{\text {th }}$ antenna $-\mathbf{X}_{p}=\left[X_{p}(0), X_{p}(1), \ldots, X_{p}\left(N_{c}-1\right)\right]$ Received vector- $\mathbf{Y}=[Y(0)$, $Y$ (1), . . , Y ( $\mathrm{N}_{\mathrm{c}}-1$ )]

Channel assumptions: It is assumed that channel coefficients are known to receiver and remain unchanged for two adjacent subcarriers i.e $\left(H_{p}(2 v)=H_{p}(2 v+1), p=1,2,0 \leq v \leq N_{c} / 2-1\right)$. Noise is assumed to be additive zero-mean white Gaussian noise. Estimating the phase rotations has three methods 1 . General method 2. Low complexity method and 3. Blind method. We discussed the blind method.


Fig. 1 Block diagram of the SFBC-OFDM transmitter with two transmitter antennas and the SLM method for PAPR reduction.
The data symbols after SFBC for 2 transmit antennas is as below. Each row corresponds to an antenna.

$$
\left[\begin{array}{ll}
X_{1}(2 v) & X_{1}(2 v+1) \\
X_{2}(2 v) & X_{2}(2 v+1)
\end{array}\right]=\left[\begin{array}{cc}
X_{1}(2 v) & X_{1}(2 v+1) \\
X_{2}^{*}(2 v+1) & -X_{2}{ }^{*}(2 v)
\end{array}\right]
$$

Where ' $v$ ' represents the index within the symbol set $v=0,1, \ldots, \mathrm{~N}_{\mathrm{c}} / 2-1$
'D' different representations of the signals $\mathbf{x}_{1}$ and $\mathbf{x}_{2}$ are generated as follows:

$$
\mathbf{x}_{1}^{d}=\mathbf{I F F T}_{\mathbf{N}}\left\{\mathbf{X}_{\mathbf{1}} \otimes \mathbf{b}^{\mathbf{d}}\right\}
$$

$$
\mathbf{x}_{1}^{d}=\operatorname{IFFT}_{\mathbf{N}}\left\{\mathbf{X}_{\mathbf{1}} \otimes \mathbf{b}^{\mathbf{d}}\right\} \quad \text { Where } 0 \leq d \leq D-1
$$

The multiplication in $\{\otimes\}$ denotes element wise multiplication.
Here we considered only 2 values for $\theta$ are 0 and $2 \pi$. So the pairs $\left[b^{d}(2 v), b^{d}(2 v+1)\right], 0 \leq v<N_{c} / 2,0 \leq d<D-1$, can take the values $[+1,-1][-1,+1],[-1,-1]$, and $[+1,+1]$ with equal probabilities. The SFBC coded block is given as below

$$
\mathbf{C}^{d}=\left[\begin{array}{cc}
b^{d}(2 v) X_{1}(2 v) & b^{d}(2 v+1) X_{1}(2 v+1) \\
b^{d}(2 v) X_{2}^{*}(2 v+1) & -b^{d}(2 v+1) X_{2}^{*}(2 v)
\end{array}\right]
$$

It is noteworthy that matrix $\mathbf{C}^{d}$ is also orthogonal, as SFBC symbols i.e

$$
\mathbf{C}^{d}\left(\mathbf{C}^{d}\right)^{H}=\left(|X(2 V)|^{2}+|X(2 V+1)|^{2}\right) \mathbf{I}_{2}
$$

At the receiver antenna we have

$$
\begin{gathered}
Y(2 v)=H_{1}(2 v) b^{\hat{a}}(2 v) X(2 v)+H_{2}(2 v) b^{\hat{d}}(2 v) X^{*}(2 v+1)+V(2 v) \\
Y(2 v+1)=H_{1}(2 v) b^{\hat{d}}(2 v+1) X(2 v+1)+H_{2}(2 v) b^{\hat{d}}(2 v+1) X^{*}(2 v)+V(2 v+1)
\end{gathered}
$$

## Blind detection of $\widehat{\boldsymbol{d}}$ :

The steps of the proposed algorithm for the blind detection of $\hat{\mathrm{d}}$ and the OFDM frame $\mathbf{X}$ can be summarized as

- Calculate $\dot{\mathrm{Z}}^{+}(2 v)$ and $\dot{\mathrm{Z}}^{+}(2 v+1)$ obtained by putting $\hat{f}(v)=+1$, in $($ a $)$ and, $\dot{\mathrm{Z}}^{-}(2 v)$ and $\dot{\mathrm{Z}}^{-}(2 v+1)$ obtained by putting $\hat{f}(v)=$ -1 in (a), for $v=0,1, \ldots, N_{c} / 2-1$.
- Evaluate $\dot{f}(v), v=0,1, \ldots, N_{c} / 2-1$, using (b), and construct the vector $\dot{\mathbf{f}}$.
- Determine using $\hat{d}$ using (c)
- Determine the transmitted symbols $\left\{\hat{X}(k)_{\mathrm{k}=0}^{\mathrm{k}=\mathrm{Nc}-1}\right\}$ using (f).

$$
\begin{gather*}
\varsigma(2 v)=\frac{H_{1}(2 v) Y^{*}(2 v)-H_{2}^{*}(2 v) \hat{f}(v) Y(2 v+1)}{\left|H_{1}(2 v)\right|^{2}+\left|H_{2}(2 v)\right|^{2}} \\
\varsigma(2 v+1)=\frac{H_{2}^{*}(2 v) Y(2 v)-H_{2}^{*}(2 v) \hat{f}(v) Y(2 v+1)}{\left|H_{1}(2 v)\right|^{2}+\left|H_{2}(2 v)\right|^{2}} \\
\dot{Z}(2 v)=Q(\varsigma(2 v)) \quad \text { and } \quad \dot{Z}(2 v+1)=Q(\varsigma(2 v+1)) \tag{a}
\end{gather*}
$$

Where $Q($.$) is the mapping to the nearest constellation point$

$$
\begin{equation*}
\dot{f}(v)=\operatorname{sign}\left(\left|\dot{Z}^{-}(2 v)-\varsigma^{-}(2 v)\right|^{2}+\left|\dot{Z}^{-}(2 v+1)-\varsigma^{-}(2 v+1)\right|^{2}-\left|\dot{Z}^{+}(2 v)-\varsigma^{+}(2 v)\right|^{2}-\left|\dot{z}^{+}(2 v+1)-\varsigma^{+}(2 v+1)\right|^{2}\right) \tag{b}
\end{equation*}
$$

where $\zeta(2 v)$ and $\zeta(2 v+1)$ are defined in (a). After the detection of $\dot{f}(v)$ for $v=0,1, \ldots, N_{c} / 2-1$, the vector $\dot{\mathbf{f}}=[\dot{\mathrm{f}}(0), \dot{\mathrm{f}}(1), \ldots$, $\left.\dot{\mathrm{f}}\left(\frac{N C}{2}-1\right)\right]^{\mathrm{T}}$ is constructed. The vectors $\mathbf{f}^{\mathrm{d}}, \mathrm{d}=0,1, \ldots, \mathrm{D}-1$ have been calculated from (e) and stored at the receiver as

$$
\begin{align*}
& \mathbf{f}^{\mathrm{d}}=\left[\mathrm{f}^{\mathrm{d}}(0), \mathrm{f}^{\mathrm{d}}(0), \ldots \mathrm{f}^{\mathrm{d}}\left(\frac{N c}{2}-1\right)\right]^{\mathrm{T}}  \tag{d}\\
& f^{\mathrm{d}}(v)=\mathrm{b}^{\mathrm{d}}(2 \mathrm{v}) \mathrm{b}^{\mathrm{d}}(2 \mathrm{v}+1) \tag{e}
\end{align*}
$$

Then this vector is mapped to closest sequence among $\mathbf{f}^{\mathrm{d}}, 0 \leq \mathrm{d} \leq \mathrm{D}-1$ i.e.

$$
\begin{equation*}
\hat{d}=\underset{0 \leq d \leq D-1}{\arg . \min } \operatorname{dist}\left(\dot{\mathbf{f}}, \mathbf{f}^{\mathbf{d}}\right) \tag{c}
\end{equation*}
$$

Where $\operatorname{dist}(\mathbf{A}, \mathbf{B})$ denotes the Hamming distance between the vectors $\mathbf{A}$ and $\mathbf{B}$.
the received symbols are estimated using ( f )

$$
(\hat{X}(2 v), \hat{X}(2 v+1))=\left\{\begin{align*}
\left(\dot{Z}^{+}(2 v), \dot{Z}^{+}(2 v+1)\right) & \text { if } b^{\hat{a}}(2 v)=+1, f^{\hat{a}}(v)=+1  \tag{f}\\
\left(-\dot{Z}^{+}(2 v),-\dot{Z}^{+}(2 v+1)\right) & \text { if } b^{\hat{a}}(2 v)=-1, f^{\hat{a}}(v)=+1 \\
\left(\dot{Z}^{-}(2 v),-\dot{Z}^{-}(2 v+1)\right) & \text { if } b^{\hat{a}}(2 v)=+1, f^{\hat{a}}(v)=-1 \\
\left(-\dot{Z}^{-}(2 v),-\dot{Z}^{-}(2 v+1)\right) & \text { if } b^{\hat{a}}(2 v)=-1, f^{\hat{a}}(v)=-1
\end{align*}\right.
$$

## 2.2 pSLM:

In pSLM technique the estimation of the index is done using Minimum Eucledian Distance decoder.


Fig. 2. The transmitter Block diagram of the P-SLM scheme.
At the transmitter side, Assume available subcarriers to be N . Consider the data symbols, group them in to sets each containing N elements. We call it as data symbol set. Multiply each data symbol with the phase rotation sequences function $\mathrm{X}^{\mathrm{u}}(\mathrm{k})=\mathrm{P}^{\mathrm{u}}(\mathrm{k}) \mathrm{X}(\mathrm{k})$ where $\mathrm{P}^{\mathrm{u}}=e^{i \theta}$ and produce U sets for each data set.
Leave the data symbol set or vector reserved for $1^{\text {st }}$ antenna as it is, but for $2^{\text {nd }}$ antenna data symbol sets multiply each set data subcarrier symbol by phase offset function $\left\{e^{\frac{i 2 \pi u}{U}}, u=0,1, \ldots, U-1\right\}$. Apply SFBC, then IFFT, select OFDM data symbol with minimum PAPR for transmission. At the receiver side the index ' $u$ ' applied will be detected using MED (Minimum Euclidean Distance) method. If we don't multiply with phase offset function then it becomes conventional SLM (cSLM) [4]
Case Study: No. of antennas considered for transmitting: 2. No. of subcarriers in an OFDM symbol considered: N. Antenna diversity technique: SFBC. Considered data symbols for $\mathrm{p}^{\text {th }}$ transmitting antenna $-\mathbf{X}_{\mathrm{p}}=\left[\mathrm{X}_{\mathrm{p}}(0), \mathrm{X}_{\mathrm{p}}(1), \ldots, \mathrm{X}_{\mathrm{p}}\left(\mathrm{N}_{\mathrm{c}}-1\right)\right]$. Data symbols antenna after multiplying with phase rotation sequences: $\boldsymbol{X}_{p}^{u}=\left[X_{p}^{u}(0), X_{p}^{u}(1), X_{p}^{u}(2), \ldots X_{p}^{u}\left(\mathrm{~N}_{\mathrm{c}-} 1\right)\right] . \boldsymbol{X}_{1}^{u}$ is left as it is but $\boldsymbol{X}_{2}^{u}$ is multiplied with phase offset function $\left\{e^{\frac{i 2 \pi u}{U}}, u=0,1, \ldots, U-1\right\}$.

$$
C=\left(\begin{array}{cc}
X^{u}(2 l) & -X^{u *}(2 l+1) \\
e^{\frac{i 2 \pi u}{U}} X^{u}(2 l+1) & e^{\frac{i 2 \pi u}{U}} X^{u *}(2 l)
\end{array}\right)
$$

Space frequency matrix of the data symbols

Apply IFFT to each of the U symbol sets. Calculate PAPR to each of the U OFDM symbol sets. The OFDM symbol set that has lowest PAPR is considered for transmitting. Let the time domain symbol vector selected for transmitting denoted by $\widehat{\boldsymbol{x}}_{p}^{u}$ it's corresponding frequency domain representation is $\widehat{\boldsymbol{X}}_{p}^{u}$. Let $().(2 l)=(.)_{e}(l)$ and $().(2 l+1)=(.)_{\mathrm{o}}(l)$.
After SFBC, let $\quad \widehat{\boldsymbol{X}}_{1, e}^{u}=\boldsymbol{X}_{e}^{u}, \widehat{\boldsymbol{X}}_{1, o}^{u}=-\widehat{\boldsymbol{X}}_{o}^{u *}, \boldsymbol{X}_{2, e}^{u}=e^{\frac{i 2 \pi u}{U}} \boldsymbol{X}_{o}^{u}, \widehat{\boldsymbol{X}}_{2, o}^{u}=e^{\frac{i 2 \pi u}{U}} \boldsymbol{X}_{e}^{u *}$
Channel assumptions: It is assumed that channel coefficients are known to receiver and remain unchanged for two adjacent subcarriers. Noise is assumed to be additive zero-mean white Gaussian noise.
At receiver side:
Remove CP, apply FFT, obtain equations for received signals. Received vector- $\mathbf{Y}=[\mathrm{Y}(0), \mathrm{Y}(1), \ldots, Y(\mathrm{~N}-1)]$

$$
Y(k)=H_{l}(k) \hat{X}_{1}^{\widehat{u}}(k)+H_{2}(k) \hat{X}_{1}^{\widehat{u}}(k)+W(k) \quad(\mathrm{p} 2)
$$

where $W(k)$ represents the AWGN noise
Equation of received even data symbols using (p1) and (p2) is given as

$$
\begin{gather*}
Y_{e}(l)=H_{l, e}(l) \hat{X}_{e}^{\widehat{u}}(l)+e^{\frac{i 2 \pi \hat{u}}{U}} H_{2, e}(k) \hat{X}_{o}^{\widehat{u}}(l)+W_{e}(l) \\
Y_{o}(l)=-H_{l, o}(l) \widehat{X}_{o}^{\hat{u} *}(l)+e^{\frac{i 2 \pi \hat{u}}{U}} H_{2, o}(k) \widehat{X}_{e}^{\widehat{u}^{*}}(l)+W_{o}(l) \tag{p4}
\end{gather*}
$$

The Minimum Eucledean Distance ditector (MED) will estimate the sent symbols as below

$$
\begin{gather*}
\bar{X}_{e}^{u}(l)=H_{1, e}^{*}(l) Y_{e}(l)_{+} e^{\frac{i 2 \pi u}{U}} H_{2, o}(l) Y_{o}^{*}(l) \\
\bar{X}_{o}^{u}(l)=e^{-\frac{i 2 \pi u}{U}} H_{2, e}^{*}(l) Y_{e}(l)-H_{l, o}(l) Y_{o}^{*}(l) \tag{p5}
\end{gather*}
$$

We can assume $H_{l, e}(l) \approx H_{l, o}(l)$ and $H_{2, e}(l) \approx H_{2, o}(l)$ as the channel coeffiecients are constant for two consecutive data symbols.. Estimate the phase offset $\hat{u}$ applied using the equation below. We have to try different phase offsets to obtain the appropriate $\widehat{u}$, since $\hat{u}$ is unknown at the receiver
$\hat{u}=\underset{0 \leq u \leq U-1}{\arg \min }\left\{\sum_{l=0}^{\frac{N}{2}-1}\left(\left|\frac{\hat{X}_{e}^{u}(l)}{\left|H_{1, e}(l)\right|^{2}+\left|H_{2, e}(l)\right|^{2}}-X_{e}^{\hat{u}}(l)\right|^{2}+\left|\frac{\hat{X}_{o}^{u}(l)}{\left|H_{1, e}(l)\right|^{2}+\left|H_{2, e}(l)\right|^{2}}-X_{o}^{\widehat{u}}(l)\right|^{2}\right)\right\}$
After obtaining the index of the phase offset, we could easily obtain the phase rotation sequence $P^{\widehat{u}}$. Therefore, the P-SLM scheme does not need to transmit the SI. Undo the phase rotation and determine the sent OFDM data symbols. For detail derivation refer the paper [4]
2.3 eSLM:


Fig. 3 Block diagram of the SFBC MIMO-OFDM system employing the eSLM scheme.
Steps involved in producing eSLM:
At the transmitter side: Assume available subcarriers to be N . Consider the data symbols group them in to sets each containing N elements. We call it data symbol set.Apply Alamouti SFBC [5].

$$
\left[\begin{array}{ll}
X_{1}(2 i) & X_{1}(2 i+1) \\
X_{2}(2 i) & X_{2}(2 i+1)
\end{array}\right]=\left[\begin{array}{ll}
X_{1}(2 i) & X_{1}{ }^{*}(2 i+1) \\
X_{2}(2 i) & X_{2}^{*}(2 i+1)
\end{array}\right]
$$

for $0 \leq \mathrm{i} \leq \mathrm{N} / 2-1$, where $\mathrm{X}_{\mathrm{p}}(\mathrm{n})$ is the Alamouti encoded symbol modulated by n -th subcarrier in the p -th transmitting antenna.

Define ' $v$ ' extension matrices $E^{(v)}$ which contains amplitude extension and phase rotation units. It consists of $\mathrm{N} / 2$ basic extension units, $U_{i}^{(v)}$ for $0 \leq \mathrm{i} \leq \mathrm{N} / 2-1$ shown below

$$
\begin{align*}
\mathrm{E}^{(v)} & =\left[U_{0}^{(v)}, U_{1}^{(v)}, U_{2}^{(v)}, \ldots, U_{\frac{N}{2}-1}^{(v)}\right], 0 \leq \mathrm{v} \leq \mathrm{V}-1  \tag{a}\\
U_{i}^{(v)} & =\left[\begin{array}{ll}
U_{i, 1}^{(v)} e^{j \theta_{i, 1}^{(v)}} & U_{i, 2}^{(v)} e^{j \theta_{i, 2}^{(v)}} \\
U_{i, 3}^{(v)} e^{j \theta_{i, 3}^{(v)}} & U_{i, 4}^{(v)} e^{j \theta_{i, 4}^{(v)}}
\end{array}\right], \quad 0 \leq \mathrm{i} \leq \mathrm{N} / 2-1
\end{align*}
$$

(b)

Multiply each produced SFBC data block with the rows of extension matrices $\mathrm{E}^{\mathrm{v}}$ function and produce ' v ' sets of symbols to the single symbol set. In case studies for $\mathrm{V}=3$ the blocks is represented in (h) (i) (j)

$$
D_{i}^{(v)}=\left[\begin{array}{cc}
U_{i, 1}^{(v)} e^{j \theta_{i, 1}^{(v)}} X(2 i) & U_{i, 2}^{(v)} e^{j \theta_{i, 2}^{(v)}} X^{*}(2 i+1)  \tag{e}\\
U_{i, 3}^{(v)} e^{j \theta_{i, 3}^{(v)} X(2 i+1)} & -U_{i, 4}^{(v)} e^{j \theta_{i, 4}^{(v)}} X^{*}(2 i)
\end{array}\right]
$$

Two rows in the above are othogonal i.e (c) and (d) proves orthogognality in (e)

$$
\begin{align*}
& U_{i, 1}^{(v)} U_{i, 3}^{(v)}=U_{i, 2}^{(v)} U_{i, 4}^{(v)}  \tag{c}\\
& \left\langle\theta_{i, 1}^{(v)}-\theta_{i, 3}^{(v)}\right\rangle_{2 \pi}=\left\langle\theta_{i, 2}^{(v)}-\theta_{i, 4}^{(v)}\right\rangle_{2 \pi} \tag{d}
\end{align*}
$$

Where $\langle.\rangle_{2 \pi}$ denotes modulo $2 \pi$ operation. In the extension matrix for orthogonality we maintain that

$$
\begin{align*}
& U_{i, 1}^{(v)}=U_{i, 4}^{(v)}, \theta_{i, 1}^{(v)}=-\theta_{i, 4}^{(v)}  \tag{f}\\
& U_{i, 3}^{(v)}=U_{i, 2}^{(v)}, \theta_{i, 3}^{(v)}=-\theta_{i, 2}^{(v)} \quad, 0 \leq \mathrm{i} \leq \mathrm{N} / 2-1 \tag{g}
\end{align*}
$$

Apply IFFT to each of the V symbol sets and obtain V OFDM symbols for single symbol set. Calculate PAPR to the V OFDM symbol sets. The symbol set that has lowest PAPR is considered for transmitting. At receiver side, remove CP, apply FFT, obtain equations for received signals, detect the index, undo the amplitude extension and phase rotations and determine the sent OFDM data symbols. About extension matrices, the amplitude extension units are for detection of index and phase rotation units are to reduce the PAPR. The amplitude units are repeated periodically to improve the detection process. More specifically, the $N / 2$ extended sub-blocks are partitioned into $G$ groups, where each group comprises $M$ contiguous extended sub-blocks. In this work, it is assumed that $N / 2=M G$. Given any candidate signal $\mathbf{X}(v)$, pairs of amplitude factors are repeated with period $M$, i.e., $\left(\mathrm{U}^{(v)}{ }_{\mathrm{i}, 1}\right.$ , $\left.\mathrm{U}^{(\mathrm{v})}{ }_{\mathrm{i}, 2}\right)=\left(\mathrm{U}^{(\mathrm{v})}{ }_{\mathrm{i}+\mathrm{M}, 1}, \mathrm{U}^{(\mathrm{v})}{ }_{\mathrm{i}+\mathrm{M}, 2}\right)$. Information about extension matrices is known to transceiver beforehand.

| $\mathrm{X}_{1} \mathrm{X}_{2}$ | $\mathrm{B}_{0}$ | $\mathrm{B}_{1}$ | $\mathrm{B}_{2}$ | $\mathrm{B}_{3}$ | $\mathrm{B}_{4}$ | $\mathrm{B}_{5}$ | $\mathrm{B}_{6}$ | $\mathrm{B}_{7}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Multiply with extension matrix |  |  |  |  |  |
| $\mathrm{v}=0$ | $U_{0,1}^{(0)}$ | $U_{1,0}^{(0)}$ | $U_{2,0}^{(0)}$ | $U_{3,0}^{(0)}$ | $U_{4,1}^{(0)}$ | $U_{5,0}^{(0)}$ | $U_{6,0}^{(0)}$ | $U_{7,0}^{(0)}$ |
| $\mathrm{v}=1$ | $U_{0,2}^{(1)}$ | $U_{1,0}^{(1)}$ | $U_{2,0}^{(1)}$ | $U_{3,0}^{(1)}$ | $U_{4,2}^{(1)}$ | $U_{5,0}^{(1)}$ | $U_{6,0}^{(1)}$ | $U_{7,0}^{(1)}$ |
| $\mathrm{v}=2$ | $U_{0,0}^{(2)}$ | $U_{1,1}^{(2)}$ | $U_{2,0}^{(2)}$ | $U_{3,0}^{(2)}$ | $U_{4,0}^{(2)}$ | $U_{5,1}^{(2)}$ | $U_{6,0}^{(2)}$ | $U_{7,0}^{(2)}$ |
| $\mathrm{v}=3$ | $U_{0,0}^{(3)}$ | $U_{1,2}^{(3)}$ | $U_{2,0}^{(3)}$ | $U_{3,0}^{(3)}$ | $U_{4,0}^{(3)}$ | $U_{5,2}^{(3)}$ | $U_{6,0}^{(3)}$ | $U_{7,0}^{(3)}$ |
| $\mathrm{v}=4$ | $U_{0,0}^{(4)}$ | $U_{1,0}^{(4)}$ | $U_{2,1}^{(4)}$ | $U_{3,0}^{(4)}$ | $U_{4,0}^{(4)}$ | $U_{5,0}^{(4)}$ | $U_{6,1}^{(4)}$ | $U_{7,0}^{(4)}$ |
| $\mathrm{v}=5$ | $U_{0,0}^{(5)}$ | $U_{1,0}^{(5)}$ | $U_{2,1}^{(5)}$ | $U_{3,0}^{(5)}$ | $U_{4,0}^{(5)}$ | $U_{5,0}^{(5)}$ | $U_{6,2}^{(5)}$ | $U_{7,0}^{(5)}$ |
| $\mathrm{v}=6$ | $U_{0,0}^{(6)}$ | $U_{1,0}^{(6)}$ | $U_{2,0}^{(6)}$ | $U_{3,1}^{(6)}$ | $U_{4,0}^{(6)}$ | $U_{5,0}^{(6)}$ | $U_{6,0}^{(6)}$ | $U_{7,1}^{(6)}$ |
| $\mathrm{v}=7$ | $U_{0,0}^{(7)}$ | $U_{1,0}^{(7)}$ | $U_{2,0}^{(7)}$ | $U_{3,2}^{(7)}$ | $U_{4,0}^{(7)}$ | $U_{5,0}^{(7)}$ | $U_{6,0}^{(7)}$ | $U_{7,2}^{(7)}$ |

Fig. 4. Illustrative example showing design of extended sub-blocks for $(N, K, M)=(16,1,4)$.
Case Study: No. of antennas considered for transmitting: 2. No. of subcarriers in an OFDM symbol considered: N. Antenna diversity technique: SFBC for 2 antennas [5], Transmitted vector at $\mathrm{P}^{\text {th }}$ antenna $-\mathbf{X}_{p}=\left[X_{p}(0), X_{p}(1), \ldots, X_{p}\left(N_{c}-1\right)\right]$. Received vector- the received vector $\mathbf{Y}=[Y(0), Y(1), \ldots, Y(N-1)]$. It is assumed that channel coefficients are known to receiver and remain unchanged for two adjacent subcarriers. Noise is assumed to be additive zero-mean white Gaussian noise.

$$
D_{i, 0}^{(v)}=\left[\begin{array}{cc}
e^{j \theta_{i, 1}^{(v)}} X(2 i) & e^{j \theta_{i, 2}^{(v)} X^{*}(2 i+1)}  \tag{h}\\
e^{-j \theta_{i, 2}^{(v)}} X(2 i+1) & -e^{-j \theta_{i, 1}^{(v)} X^{*}(2 i)}
\end{array}\right]
$$

$$
\begin{align*}
& D_{i, 1}^{(v)}=\left[\begin{array}{cc}
e^{j \theta_{i, 1}^{(v)} X(2 i)} & U e^{j \theta_{i, 2}^{(v)} X^{*}(2 i+1)} \\
U e^{-j \theta_{i, 2}^{(v)}} X(2 i+1) & -e^{-j \theta_{i, 1}^{(v)} X^{*}(2 i)}
\end{array}\right]  \tag{i}\\
& D_{i, 2}^{(v)}=\left[\begin{array}{cc}
U e^{j \theta_{i, 1}^{(v)}} X(2 i) & e^{j \theta_{i, 2}^{(v)} X^{*}(2 i+1)} \\
e^{-j \theta_{i, 2}^{(v)}} X(2 i+1) & -U e^{-j \theta_{i, 1}^{(v)} X^{*}(2 i)}
\end{array}\right] \tag{j}
\end{align*}
$$

The symbol energy $\mathrm{E}_{\mathrm{s}}$ of the transmitter should be normalized to $\widehat{E}_{s}$ to fulfill the average power constraint as given by

$$
\hat{E}_{S}=\frac{2 M}{\left(U^{2}+1\right) K+2(M-K)} \cdot E_{s}
$$

The received symbols is given as below [7]

$$
Y_{i} \triangleq\left[\begin{array}{c}
Y(2 i)  \tag{k}\\
Y(2 i+1)
\end{array}\right]=\left[\begin{array}{c}
\left(U_{i, 1}^{(v)}\right)^{2}\left\|H_{i}\right\|^{2} X(2 i) \\
\left(U_{i, 1}^{(v)}\right)^{2}\left\|H_{i}\right\|^{2} X(2 i+1)
\end{array}\right]+\left[\begin{array}{c}
W(2 i) \\
W(2 i+1)
\end{array}\right]
$$

Where W(i) denotes additive white Gaussian noise at the receiver.
By approximating the amplitude of the data symbols as the square root of the symbol energy, the energy disparity detection of the index is given as below

$$
\begin{equation*}
\hat{v}=\underset{v=0,1, \ldots, V-1}{\arg \min }\left\{\sum_{i=0}^{\frac{N}{2}-1} \sum_{k=0}^{1} \frac{|Y(2 i+k)|}{\left\|H_{i}\right\|^{2} \sqrt{E_{s}}}-\left.\left(U_{i, k+1}^{(v)}\right)^{2}\right|^{2}\right\} \tag{1}
\end{equation*}
$$

As phase rotations and $V$ extension matrices are one-to-one correspondence, after detecting the index, the phase rotations can be compensated.

### 2.4 LC eSLM [4]:


fig. 5 Architecture of candidate signal generating block (CSGB) in the LC-eSLM scheme.
The LC-eSLM is the reduced version of eSLM in this method, after applying data modulation and SFBC encoding, we divide the N element data symbol in to 2 M disjoint sub-vectors groups each containing N elements with $\mathrm{N} / 2 \mathrm{M}$ nonzero elements. The entries of these nonzero elements is done by following the below equation.
Let the data symbol set for pth antenna be $\mathbf{X}_{\mathrm{p}}=\left\{\mathrm{x}_{\mathrm{p} 0}, \mathrm{x}_{\mathrm{p} 1}, \mathrm{x}_{\mathrm{p} 2}, \mathrm{x}_{\mathrm{p} 3}, \ldots \mathrm{x}_{\mathrm{pn}-1}\right\}$ where n is index in $\mathrm{X}_{\mathrm{p}}$.
Let the $\mathrm{m}^{\text {th }}$ disjoint set be denoted by $\mathbf{X}_{\mathrm{p}, \mathrm{m}}$. Now the entries in to this $\mathrm{m}^{\text {th }}$ set will be as below.

$$
\left[X_{p, m, i}\right]=\left\{\begin{array}{rcc}
{\left[X_{p, n=i}\right],} & \text { for } g \cdot 2 M+m=i . & \text { Where ' } \mathrm{i} \text { ' is the index in } \mathrm{m}^{\mathrm{th}} \\
& \forall g=0,1, \ldots, \frac{N}{2 M}-1
\end{array} \quad \begin{array}{l}
\text { subgroup and the condition is } \\
0,
\end{array}\right.
$$

Now do SFBC coding on 2M data symbols. The SFBC encoded symbol vector can be represented as below

$$
\boldsymbol{X}_{p}=\sum_{m=0}^{2 M-1} X_{i, m}
$$

Apply N point IDFT on the 2 M sub-vector elements. Now pass the time domain signals corresponding to each transmit antenna through candidate signal generating block. (CSGB) The $v^{\text {th }}$ candidate signal of the $p^{\text {th }}$ antenna, $\mathbf{x}^{(v)}{ }_{p}$, is obtained as

$$
\boldsymbol{X}_{p}^{(v)}=\sum_{m=0}^{2 M-1} A_{p, m}^{(v)} e^{i \phi_{p, m}^{(v)}} X_{p, m}
$$

Where $A^{(v)}{ }_{p, m}$ and $\phi^{(v)}{ }_{p, m}$ are respectively the $m^{\text {th }}$ amplitude extension and phase rotation corresponding to the $v^{\text {th }}$ candidate signal at the $p^{\text {th }}$ antenna. The candidate signals produced in the LC-eSLM scheme are identical to those produced by the eSLM scheme given the settings.

$$
\begin{aligned}
& A_{p, 2 i+j}^{(v)}=U_{i, 2 p+j-1}^{(v)}=U_{i+M, 2 p+j-1}^{(v)}=\cdots U_{i+1(G-1) M, 2 p+j-1}^{(v)} \quad \text { where } p=1,2 ; j=0,1 . \\
& \phi_{p, 2 i+j}^{(v)}=\theta_{i, 2 p+j-1}^{(v)}=\theta_{i+M, 2 p+j-1}^{(v)}=\cdots=\theta_{i+(G-1) M, 2 p+j-1}^{(v)} \text { where } p=1,2 ; j=0,1 .
\end{aligned}
$$

The relation between the eSLM and lc-eSLM becomes clear with the following matrix comparing with (b)

$$
U_{i}^{(v)}=\left[\begin{array}{lll}
A_{1,2\langle i\rangle_{M}}^{(v)} e^{i \phi_{p, 2 i i)_{M}}^{(v)}} & A_{1,2\langle i\rangle_{M+1}}^{(v)} e^{i \phi_{p, 2(i)_{M+1}}^{(v)}} \\
A_{2,2\langle i\rangle_{M}}^{(v)} e^{i \phi_{p, 2\langle i)_{M}}^{(v)}} & A_{2,2\langle i\rangle_{M+1}}^{(v)} e^{i \phi_{2,2\langle i)_{M+1}}^{(v)}}
\end{array}\right]
$$

In the LC-eSLM scheme, we maintain the amplitude extensions and phase rotations same as that for the eSLM scheme, i.e.,

$$
\begin{aligned}
A_{p, 2 i}^{(v)} & =A_{p, 2 i+1}^{(v)}, \phi_{1,2 i}^{(v)}=-\phi_{2,2 i+j}^{(v)} \\
A_{2,2 i}^{(v)} & =A_{p, 2 i+j}^{(v)}, \phi_{2,2 i}^{(v)}=-\phi_{1,2 i+1}^{(v)}
\end{aligned}
$$

As the LCeSLM is special case of eSLM the same detection which is followed for eSLM can be considered forLCeSLM also

## 3. Simulation results and conclusions:

The PAPR value of the transmitted candidate signal of each antenna $\mathbf{x}^{(v)}{ }_{p}$ is given as

$$
\operatorname{PAPR}\left(\boldsymbol{X}_{p}^{(v)}\right)=\frac{\max _{0 \leq k \leq L N-1}\left|\boldsymbol{X}_{p}^{(v)}(k)\right|^{2}}{E\left[\left|\boldsymbol{X}_{p}^{(v)}(k)\right|^{2}\right]}, \mathrm{p}=1,2
$$

The oversampling factor is $\mathrm{L}=4$ to approximate the true PAPR. The PAPR reduction performance is assessed by evaluating the complementary cumulative distribution function (CCDF) of the PAPR values, which is defined mathematically as

$$
\operatorname{CCDFPAPR}(\mathbf{x})(\gamma)=\operatorname{Prob}[\operatorname{PAPR}(\mathbf{x})>\gamma]
$$

Table 1. Comparison of various PAPR reduction schemes for SFBC MIMO-OFDM systems [1]

| SLM scheme | Number of complex <br> multiplications | Number of complex <br> additions | Number of <br> complex <br> SI bits | Orthogonolity <br> of SFBC |
| :--- | :--- | :--- | :--- | :--- |
| oSLM | $\mathrm{VLNlog}_{2}(L N)$ | $2 \mathrm{VLNlog}_{2}(L N)$ | $\log _{2}(V)$ | No |
| sSLM | VLNlog $_{2}(L N)$ | $2 \mathrm{VLNlog}_{2}(L N)$ | $\log _{2}(V)$ | No |
| bSLM | $\mathrm{VLNlog}_{2}(L N)$ | $2 \mathrm{VLNlog}_{2}(L N)$ | 0 | Yes |
| pSLM | $\mathrm{VN+VLN} \mathrm{\log }_{2}(L N)$ | $2 \mathrm{VLNlog}_{2}(L N)$ | 0 | Yes |
| eSLM | $(\mathrm{VKN} / \mathrm{M})+\mathrm{VLNlog} 2(L N)$ | $2 \mathrm{VLNlog}_{2}(L N)$ | 0 | Yes |
| LC-eSLM | $\mathrm{LNlog}_{2}\left(\frac{L N}{2 M}\right)+2 \mathrm{LN}(2 \mathrm{M}+\mathrm{KY})$ | $2 \mathrm{LNlog}_{2}\left(\frac{L N}{2 M}\right)+(4 \mathrm{M}-2) \mathrm{VLN}$ | 0 | Yes |

The above Table compares the six schemes considered in the Fig. 5 in terms of the required number of complex multiplications and additions, the number of SI bits, and the ability of the schemes to preserve the orthogonality of the SFBC coding structure. The bSLM and pSLM schemes and eSLM and LC-eSLM schemes preserve the orthogonality of the SFBC code, and therefore facilitate decoding at the receiver side using simple low-complexity linear operations.


Fig. 5 CCDF performance of PAPR for different schemes with $N=512$.
The above Figure 5 compares the CCDFs of the eSLM and LC-eSLM schemes with pSLM and sSLM, given the use of $V=8$ candidate signals. For all schemes, the simulations are done for a MIMO-OFDM system with $N=512$ subcarriers and 16-QAM modulation. The CCDF for a system with no PAPR reduction is also presented. In simulating the performance of the eSLM and LC-eSLM schemes, the phase rotations are chosen from the finite set $\{0, \pi / 2, \pi, 3 \pi / 2\}$, and the $(M, K)$ parameters are set as $(4,1)$. It is seen that the sSLM and pSLM schemes coincide i.e provide an equal PAPR reduction performance since they all apply equal phase rotations to the candidate signals at both antennas. The eSLM scheme achieves a better PAPR reduction performance than the sSLM, pSLM, and LC-eSLM (even bSLM[1]) schemes as a result of its greater degree of freedom in generating the candidate signals. However, the LC-eSLM scheme has a slightly poorer performance than the eSLM, sSLM and pSLM (also bSLM[1]) schemes due to its use of repetitive phase rotation sequences for each group.

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