SOME GRACEFUL LABELING FOR THE EXTENDED DUPLICATE GRAPH OF SPLITTING GRAPH OF PATH

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Abstract : In this paper, we prove that the extended duplicate graph of splitting graph of path admits graceful, odd-graceful and even graceful labeling.

***IndexTerms* - Graph labeling, Duplicate graph, splitting graph, Graceful labeling , Odd graceful labeling , Even graceful labeling.**

I. INTRODUCTION:

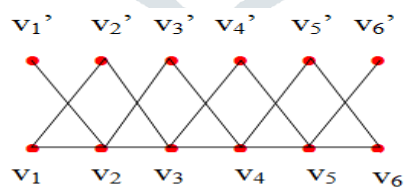
Labeling in graph theory is an active area of research which gained popularity after the mid sixties of 20th century.
Labeled graphs serve as useful models for a broad range of applications such as coding theory including design of good radar type codes,missle guidance codes,convolution codes and communication network.Most graph labeling trace their origin to one introduced by Rosa[4,6] in 1967 called β valuations renamed as graceful labeling by Golomb. For an extensive survey on graph labeling and bibliographic references we refer to Gallian[3]. For all terminology and notations we follow Harary [5]. E.Sampthkumar [1,2] introduced the idea of splitting graph and duplicate graph. K.Thirusangu, B. Selvam and P.P. Ulaganathan have proved that the extended duplicate graph of twig graphs is odd and even graceful labeling.[7].

II. PRELIMINARIES

In this section, we give the basic definitions relevant to this paper. Let G(V,E) be a finite, simple and undirected graph with p vertices and q edges.

Definition 2.1 Splitting graph: For each vertex v of a graph G, take a new vertex v'. Join v' to all the vertices of G adjacent to v. The graph Spl(G) thus obtained is called splitting graph of G.

Illustration 1 THE SPLITTING GRAPH OF PATH Spl(P_6)

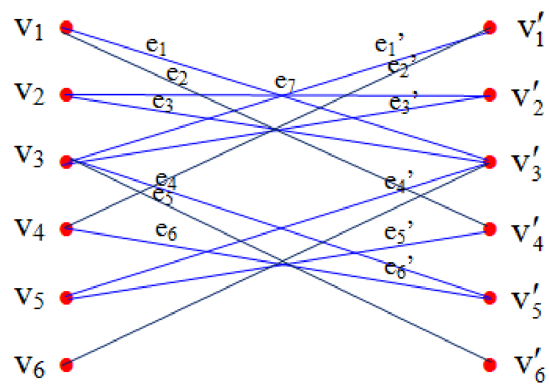


Spl(P_6)

Definition 2.2 Duplicate Graph: Let G (V,E) be a simple graph and the duplicate graph of G is DG = (V_1, E_1), where the vertex set $V_1 = V \cup V'$ and $V \cap V' = \phi$ and $f : V \to V'$ is bijective (for $v \in V$, we write $f(v) = v'$ for convenience) and the edge set E_1 of DG is defined as the edge ab is in E if and only if both ab' and $a'b$ are edges in E_1.

Definition2.3 Extended duplicate graph of Splitting graph of Path: Let DG = (V_1,E_1) be a duplicate graph of splitting graph of path G(V,E). Extended duplicate graph of splitting graph of path is obtained by adding the edge v_2v_2' to the duplicate graph. It is denoted by EDG Spl(P_m). Clearly it has 4m vertices and 6m-5 edges, where $m \geq 2$ is the number of length.

Illustration 2: EXTENDED DUPLICATE GRAPH OF SPLITTING GRAPH OF A PATH



EDG(Spl(P₃))

Definition 2.4 Graceful labeling: A function f is said to be graceful of a graph G with q-edges, if $f: V \rightarrow \{0,1,2,.....q\}$ such that each edge v_iv_j is assigned the label $\left|f(v_i) - f(v_j)\right|$ v_i, v_j and the resulting edge labels are distinct numbers $\{1,2,3,......q\}$.

Definition 2.5 Even Graceful labeling: A function f is said to be even graceful of a graph G with q-edges, if $f: V \rightarrow \{0,1,2,.....2q\}$ such that each edge v_iv_j is assigned the label $\left|f(v_i) - f(v_j)\right|$ and the resulting edge labels are distinct even numbers $\{2,4,6,......2q\}$.

Definition 2.6 Odd **Graceful labeling:** A function f is said to be odd graceful of a graph G with q-edges, if $f: V \rightarrow \{0,1,2,.....2q-1\}$ such that each edge v_iv_j is assigned the label $\left|f(v_i) - f(v_j)\right|$ and the resulting edge labels are distinct odd numbers $\{1,3,5,......2q-1\}$.

III. MAIN RESULTS

3.1 Graceful labeling

Algorithm: 3.1

Procedure [Graceful labeling for EDG(Spl(Pₘ)), m≥2]

$V \leftarrow \{v_1, v_2, \ldots, v_{2m}, v'_1, v'_2, \ldots, v'_{2m}\}$
$E \leftarrow \{e_1, e_2, \ldots, e_{3m-3}, e_{3m-2}, e_1', e_2', \ldots\ldots\ldots\ldots e_{3m-3}'\}$
$v_1 \leftarrow 6m-7, v_2 \leftarrow 0, v_1' \leftarrow 6m-6, v_2' \leftarrow 6m-5$
for i = 0 to [(m-2)/2] do
$v_{3+4i} \leftarrow 1+6i$
$v_{4+4i} \leftarrow 2+6i$
$v'_{3+4i} \leftarrow 3+6i$
$v'_{4+4i} \leftarrow 4+6i$
end for
for i = 0 to [(m-3/2] do
$v_{5+4i} \leftarrow 6m-13-6i$
$v_{6+4i} \leftarrow 6m-9-6i$
$v'_{5+4i} \leftarrow 6m-12-6i$
$v'_{6+4i} \leftarrow 6m-8-6i$
end for
end procedure

Theorem 3.1: The extended duplicate graph of splitting path graph Spl(P_m), $m \geq 2$ is graceful labeling.

Proof: Let $Spl(P_m)$, $m \geq 2$ be a splitting path graph . In order to label the vertices, define a function

$f : V \rightarrow \{0,1,2,.....q\}$ as given in algorithm 3.1.

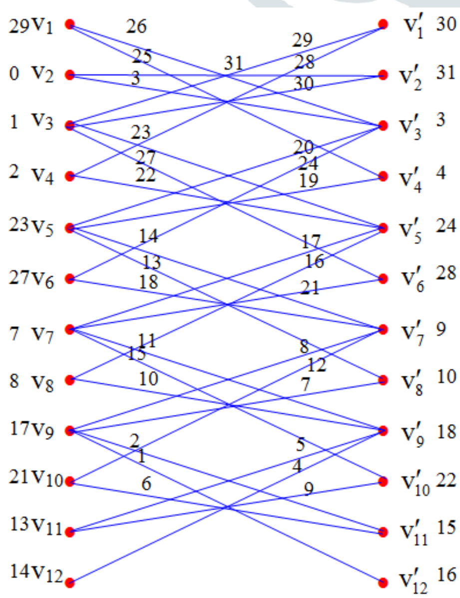
The vertices v_1, v_2, v'_1 and v'_2 receive label '6m-7' , '0' , '6m-6' and ' 6m-5' respectively **;**

For $0 \leq i \leq [(m-2)/2]$, the vertices v_{3+4i} receive label '1+6i' ; the vertices v_{4+4i} receive label '2+6i' ; the vertices v'_{3+4i} receive label '3+6i' ; the vertices v'_{4+4i} receive label '4+6i' ;
For $0 \leq i \leq [(m-3)/2]$, the vertices v_{5+4i} receive label '6m-13-6i' ; the vertices v_{6+4i} receive label '6m-9-6i' ; the vertices v'_{5+4i} receive label '6m-12-6i' ; the vertices v'_{6+4i} receive label '6m-8-6i' ;
Thus all the 4m vertices are labeled as {0,1,2......,6m-5} which are all distinct.
Hence the entire 4m vertices are labeled .
Now to compute the edge labeling , we define the induced function $f^* : E \rightarrow N$ such that
$f^*(v_i v_j) = | f(v_i) - f(v_j) |$ where $v_i, v_j \in V$.
The edge functions are as follows:
The induced function yields the label '6m-5' for the edges e_{3m-2}; the label '6m-10' for the edge e_1; the label '6m-11' for the edge e_2; the label '3' for the edge e_3;the label 6m-7 for the edge e'_1 ;the label 6m-8 for the edge e'_2; the label 6m-6 for the edge e'_3 ;

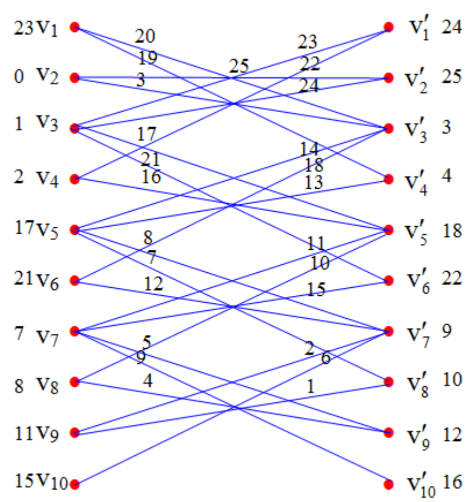
For i=0 to [(m-3/2]	For i=0 to [(m-4/2]
$f^*(e_{4+6i})$=6m-13-12i	$f^*(e_{7+6i})$=6m-22-12i
$f^*(e_{5+6i})$=6m-9-12i	$f^*(e_{8+6i})$=6m-23-12i
$f^*(e_{6+6i})$=6m-14-12i	$f^*(e_{9+6i})$=6m-18-12i
$f^*(e'_{4+6i})$=6m-16-12i	$f^*(e'_{7+6i})$=6m-19-12i
$f^*(e'_{5+6i})$=6m-12-12i	$f^*(e'_{8+6i})$=6m-20-12i
$f^*(e'_{6+6i})$=6m-17-12i	$f^*(e'_{9+6i})$=6m-15-12i

Thus all the 6m-5 edges are labeled as {1,2.3,.....,6m-5 } which are all distinct.
Hence the extended duplicate graph of splitting path graph $SplP_m$), $m \geq 2$ admits graceful labeling.

Illustration 3 : GRACEFUL LABELING FOR EXTENDED DUPLICATE SPILITTING GRAPH OF PATH



EDG(Spl(P₆))



EDG(Spl(P₅))

3.2 Even Graceful labeling

Algorithm: 3.2

Procedure [Even Graceful labeling for EDG Spl(P_m)), m≥ 2]

V ← {v_1, v_2, … ,v_{2m}, v'_1, v'_2, … ,v'_{2m}}
E ← {e_1, e_2, …., e_{3m-3}, e_{3m-2}, e_1',e_2',…………..e_{3m-3}'}
v_1←12m-14, v_2←0,v_1'←12m-12,v_2'←12m-10
for i = 0 to [(m-2)/2] do
v_{3+4i}← 2+12i
v_{4+4i}← 4+12i
v'_{3+4i} ←6+12i
v'_{4+4i} ← 8+12i
end for
for i = 0 to [(m-3/2] do
v_{5+4i}← 12m-26-12i
v_{6+4i}← 12m-18-12i
v'_{5+4i} ← 12m-24-12i
v'_{6+4i} ← 12m-16-12i
end for
end procedure

Theorem 3.2: The extended duplicate graph of splitting path graph Spl*(P_m)*, m ≥ 2 is even graceful labeling.

Proof: Let *Spl(P_m)*, m ≥ 2 be a splitting path graph . In order to label the vertices, define a function

f : V → {0,1,2,.....2q} as given in algorithm 3.2.

The vertices v_1, v_2, , v'_1 and v'_2 receive label '12m-14' , '0' , '12m-12' and ' 12m-10' respectively **;**

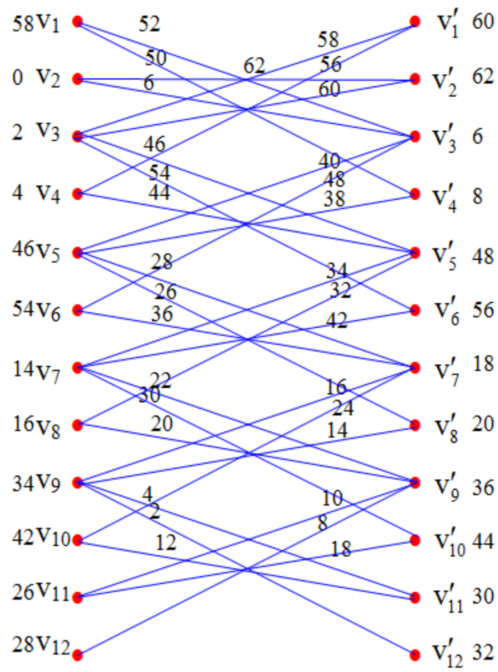
For 0≤ i ≤ [(m-2)/2] , the vertices v_{3+4i} receive label '2+12i' ; the vertices v_{4+4i} receive label '4+12i' ; the vertices v'_{3+4i} receive label '6+12i' ; the vertices v'_{4+4i} receive label '8+12i' ;
For 0≤ i ≤ [(m-3)/2] , the vertices v_{5+4i} receive label '12m-26-12i' ; the vertices v_{6+4i} receive label '12m-18-12i' ; the vertices v'_{5+4i} receive label '12m-24-12i'; the vertices v'_{6+4i} receive label '12m-16-12i' ;
Thus all the 4m vertices are labeled as {0,1,2......,12m-10} which are all distinct.
Hence the entire 4m vertices are labeled.
Now to compute the edge labeling , we define the induced function f * : E → N such that
f * (v_i v_j) = | f(v_i) - f(v_j) | where v_i , v_j ∈ V.
The edge functions are as follows:
The induced function yields the label '12m-10' for the edges e_{3m-2}; the label '12m-20' for the edges e_1; the label '12m-22' for the edges e_2; the label '6' for the edges e_3; ; the label '12m-14' for the edges e'_1; the label '12m-16' for the edges e'_2; the label '12m-12' for the edges e'_3;

For i=0 to [(m-3/2]

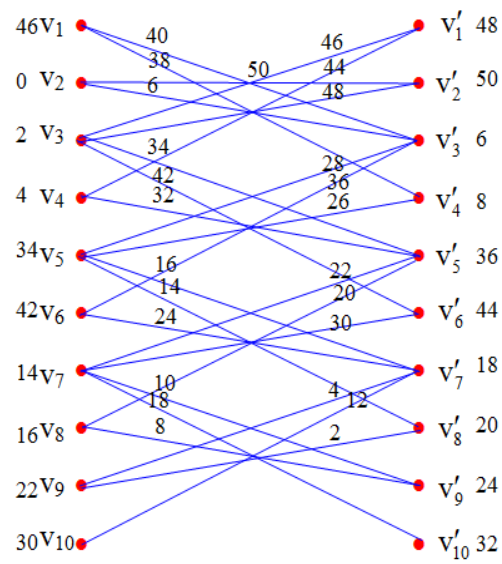
f *(e_{4+6i})=12m-26-24i
f *(e_{5+6i})= 12m-18-24i
f *(e_{6+6i})= 12m-28-24i
f *(e'_{4+6i})= 12m-32-24i
f *(e'_{5+6i})= 12m-24-24i
f *(e'_{6+6i})= 12m-34-24i
f *(e_{7+6i})= 12m-44-24i
f *(e_{8+6i})= 12m-46-24i
f *(e_{9+6i})= 12m-36-24i
f *(e'_{7+6i})= 12m-38-24i
f *(e'_{8+6i})= 12m-40-24i
f *(e'_{9+6i})= 12m-30-24

For i=0 to [(m-4/2]

Illustration 4 : EVEN GRACEFUL LABELING FOR EXTENDED DUPLICATE SPILITTING GRAPH OF PATH



EDG(Spl(P₆))



EDG(Spl(P₅))

Thus all the 6m-5 edges are labeled as {2,4,6,.....,12 m-10 } which are all distinct.
Hence the extended duplicate graph of splitting path graph S*plP*$_m$*)*, m ≥ 2 admits even graceful labeling.

3.3 Odd Graceful labeling

Algorithm: 3.3

Procedure [Odd Graceful labeling for EDG ($Spl(P_m)$), $m\geq 2$]

V ← {$v_1, v_2, \dots, v_{2m}, v'_1, v'_2, \dots, v'_{2m}$}

E ← {$e_1, e_2, \dots, e_{3m-3}, e_{3m-2}, e_1', e_2', \dots e_{3m-3}'$}

v_1←12m-16, v_2←0, v_1'←12m-13, v_2'←12m-11

for i = 0 to [(m-2)/2] do

v_{3+4i}← 2+12i

v_{4+4i}← 4+12i

v'_{3+4i} ←5+12i

v'_{4+4i} ← 7+12i

end for

for i = 0 to [(m-3/2] do

v_{5+4i}← 12m-28-12i

v_{6+4i}← 12m-20-12i

v'_{5+4i} ← 12m-25-12i

v'_{6+4i} ← 12m-17-12i

end for

end procedure

Theorem 3.3: The extended duplicate graph of splitting path graph Spl(P_m), $m \geq 2$ is odd graceful labeling.

Proof: Let $Spl(P_m)$, $m \geq 2$ be a splitting path graph . In order to label the vertices, define a function

$f : V \rightarrow \{0,1,2,.....2q-1\}$ as given in algorithm 3.3.

The vertices v_1, v_2, v'_1 and v'_2 receive label '12m-16' , '0' , '12m-13' and ' 12m-11' respectively **;**

For $0\leq i \leq$ [(m-2)/2] , the vertices v_{3+4i} receive label '2+12i' ; the vertices v_{4+4i} receive label '4+12i' ; the vertices v'_{3+4i} receive label '5+12i' ; the vertices v'_{4+4i} receive label '7+12i' ;

For $0\leq i \leq$ [(m-3)/2] , the vertices v_{5+4i} receive label '12m-28-12i' ; the vertices v_{6+4i} receive label '12m-20-12i' ; the vertices v'_{5+4i} receive label '12m-25-12i'; the vertices v'_{6+4i} receive label '12m-17-12i' ;

Thus all the 4m vertices are labeled as {0,1,2......,12m-11} which are all distinct.

Hence the entire 4m vertices are labeled .

Now to compute the edge labeling , we define the induced function $f^* : E \rightarrow N$ such that

$f^*(v_i v_j) = | f(v_i) - f(v_j) |$ where $v_i, v_j \in V$.

The edge functions are as follows:

The induced function yields the label '12m-11' for the edges e_{3m-2}; the label '12m-21' for the edges e_1; the label '12m-23' for the edges e_2; the label '5' for the edges e_3; the label '12m-15' for the edges e'_1; the label '12m-17' for the edges e'_2; the label '12m-13' for the edges e'_3;

For i=0 to [(m-3/2]

$f^*(e_{4+6i})$=12m-27-24i

$f^*(e_{5+6i})$= 12m-19-24i

$f^*(e_{6+6i})$= 12m-29-24i

$f^*(e'_{4+6i})$= 12m-33-24i

$f^*(e'_{5+6i})$= 12m-25-24i

$f^*(e'_{6+6i})$= 12m-35-24i

For i=0 to [(m-4/2]

$f^*(e_{7+6i})$= 12m-45-24i

$f^*(e_{8+6i})$= 12m-47-24i

$f^*(e_{9+6i})$= 12m-37-24i

$f^*(e'_{1+6i})$= 12m-39-24i

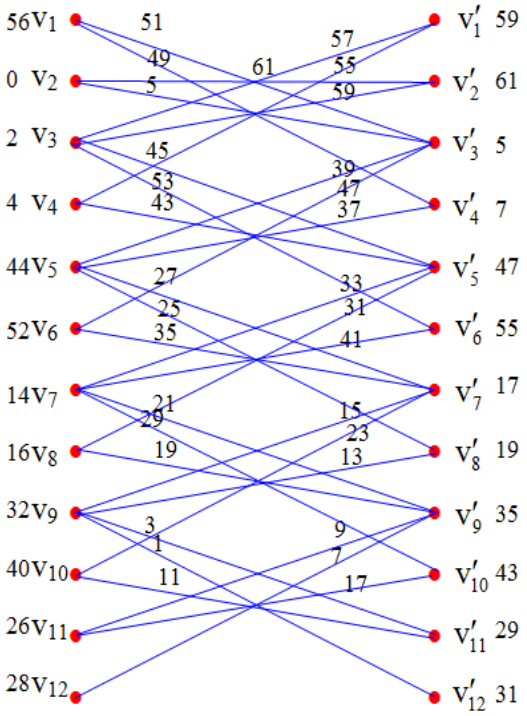
$f^*(e'_{2+6i})$= 12m-41-24i

$f^*(e'_{3+6i})$= 12m-31-24i

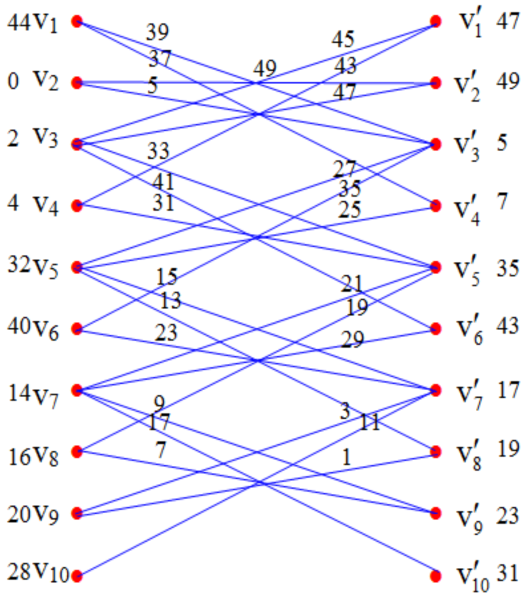
Thus all the 6m-5 edges are labeled as {1,3,5,,.....,12 m-11 } which are all distinct.

Hence the extended duplicate graph of splitting path graph SplP_m), $m \geq 2$ admits odd graceful labeling.

Illustration 5 : ODD GRACEFUL LABELING FOR EXTENDED DUPLICATE SPILITTING GRAPH OF PATH



EDG(Spl(P₆))



EDG(Spl(P₅))

IV . CONCLUSION

In this paper, we presented algorithms and prove that the extended duplicate graph of splitting graph path $Spl(P_m)$, $m \geq 2$ admits graceful labeling, even graceful labeling , odd graceful labeling.

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