A MODIFIED EPQ MODEL FOR DETERIORATING ITEMS HAVING VARYING DEMAND WITH IMPERFECT PRODUCTION SYSTEM

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ABSTRACT: Economic Production Quantity (EPQ) model is generally used by the researchers in production inventory management to assist in making decision on production lot size. In most of the cases, it is assumed that all items or units produced are perfectly good. But practically, defective or imperfect units are produced in each cycle of production. They must be screened out before delivery to the customers through a 100% screening process, which produces some reworkable, some non-reworkable but slightly defective and scrap items besides the major perfectly good items. Keeping this view in mind, present paper proposes an Economic Production Quantity (EPQ) model for deteriorating and imperfect quality items with varying demand. To achieve this objective a mathematical model is developed with optimal production lot size which minimizes the total cost of the system. Numerical illustration and sensitivity analysis are also carried out to analyze the performance of the model.

Keywords- EPQ, imperfect quality, inventory, reworkable, deteriorating items.

I. INTRODUCTION

Inventory plays a vital role in every manufacturing organization. An economic production quantity (EPQ) model is an inventory control model that determines the amount of product to be produced on a single facility so as to meet a deterministic demand over an infinite planning horizon.

EPQ model is widely used in industries to study the optimal production lot size that minimizes the overall cost of the system or maximizes the profit. In classical EPQ model all items produced are assumed to be of perfect quality on the basis of this criterion they continuously satisfy the demand for that product. But, this is not always true in practice. Generally in any production process, defective products are also produced along with the good or useful products. The production process cannot be considered as perfect or fully under control due to the random occurrences of many factors, like there can be overheating or vibration in the machinery system or there can be problem due to the failure of machinery parts, the power supply may fluctuate or even may discontinue.

Initially the production process is fully under control and produces 100% perfect quality items but with the passage of time the process may deteriorate and produces some percentage of imperfect items. With the result the final products may contain perfect and imperfect items both.

This requires an inspection of the products after the completion of production process. The idea behind inspection or screening is not to create good quality products but to control the quality of the product by finding and eliminating the imperfect items before they are supplied to the customers. Screening separates the perfect quality product by three types of poor quality products: reworkable items, non-reworkable but slightly defective and scrap items. Reworkable items are reworked to recreate perfect items and the non-reworkable but slightly defective items are sold under markdown policy. During the rework process, a fraction of defectives becomes scrap and disposed off from the inventory.

The imperfect quality products which are the result of out of control of machine can be reproduced and made available in the market. This process is known as rework (see Schardy (1967)).

EPQ model with imperfect production process was firstly studied by Rosenblatt and Lee (1986) which showed that the production run time was shorter than the one in classical model. The model was also extended by taking set up cost dependent deterioration rate.

Optimal production and rework lot size inventory models incorporating two lot sizing policies have been proposed by Teunter (2004) and Widyadana and Wee (2010). Yoo et. al. (2009) developed EPQ model under rework for imperfect quality items and imperfect inspection plan.

The rework phenomenon is useful for the items that decay with time and lose their effectiveness with time due to deterioration. Food items medicines chemicals, fertilizers etc come in this category. An economic production quantity model for deteriorating items with shortages for random machine unavailability time has been developed by Chung et. al. (2011) and Wee and Widyadana (2012).

Felix et. al. (2015) proposed a modified EPQ model incorporating both product and process deterioration at the same time with rework and backordering.

Krishnamoorthi and Panayappan (2012) suggested an EPQ model that incorporates both imperfect production quality and falsely not screening out a proportion of defects, thereby passing them on to customers, resulting in defect sales returns.

Mukhopadhyay and Goswami (2014) developed an EPQ model with imperfect quality with varying set up cost. Shah et al. (2015) proposed an EPQ model for imperfect production processes with rework and random preventing machine time for deteriorating items and trended demand.

In this proposed study, an economic production quantity model for deteriorating items with imperfect production system having varying demand is considered. The imperfect production system produces some reworkable, some non- reworkable but slightly defective and scrap items besides the major perfectly good items. There are three types of demand pattern throughout in one replenishment cycle. The demand is constant for production period, then in the rework up process, demand is exponentially decreasing function of time and after that in rework down and production down time the demand is a linear function of selling price under markdown policy. By applying markdown (or discount), the demand of the product is automatically increased.

This model is basically for those perishable products whose demand is low as compared to other perishable products and for those imperfect products which are not repairable but slightly defective.

The main objective of the suggested EPQ model is to determine the optimum lot size and optimum cycle time which minimizes the total cost of the system. In addition to this, an attempt has been made to develop the mathematical expression for total cost, total profit and order quantity. All the theoretical developments are illustrated with the help of a numerical example.

The formulation of the paper is as follows:

In section 2, notations and assumptions are given. The mathematical development is presented in section 3. Its application and sensitivity analysis have been done in section 4, which is carried out to study the effects of various parameters on the decision variables and objective functions. Section 5, concludes the study.

II. NOTATIONS AND ASSUMPTIONS:

The following notations and assumptions are made to develop the model:

Notations:

$I(t)$	=	On hand inventory level at time ‘t’
T	=	Length of replenishment cycle.
Q_1	=	Inventory level in production up time.
Q_2	=	Inventory level in rework-up time.
Q_3	=	Inventory level in production down time.
h	=	Inventory holding cost per unit per unit time.
C	=	Unit cost or initial price of the unit.
A	=	Set up cost per cycle
θ	=	A constant deterioration rate (unit/unit time); $0<\theta<1$
P_1	=	Production rate per unit time for production up time.
K_1	=	Production cost per unit for production up time.
P_2	=	Rework process rate per unit time
K_2	=	Rework cost per unit for rework up time.
x	=	product defect rate
x_1	=	product scrap rate
x_2	=	non-renewable but slightly defective product rate.
δ	=	Production percentage

γ = rework process percentage
λ = markdown percentage
R = markdown rate
T^* = Optimum Cycle time
TC^* = Optimum System Cost
TP^* = Optimum profit

Assumptions :

1. Production process is imperfect and it produces only single type of product.
2. Production rate is constant, finite and greater than demand rate.
3. Demand is considered to be constant in production up time. Then it is exponentially decreasing function of time in rework up time and after that it is a liner function of selling price.

$$D = \begin{cases} a, & 0 \le t \le t_1 \\ \alpha e^{-\beta t}, & 0 \le t \le t_2 \\ (q - rC), & 0 \le t \le t_3 \quad \text{and} \quad 0 \le t \le t_4 \end{cases}$$

Where, t_1 : production up time
t_2 : rework up time
t_3 : rework down time
t_4 : Production down time.

Also a, α, β, q, and r are positive constant; $a > 0$, $\alpha > 0$, $q > 0$, $0 < \beta < 1$, $0 < r < 1$ and $r = 1 - R$, where R is markdown rate

4. The screening process outputs four types of items- perfect items, reworkable defective items, non-reworkable but slightly defective items, scrap items. Perfect items are sold at full price, reworkable defective items are to be reworked and converted into perfect items, scrap items are to be disposed off and non-reworkable but slightly defective items are to be sold with reduced price or sold out when the markdown policy is allowed to reduce the inventory or stock.
5. The screening process and demand proceeds simultaneously.
6. Screening rate is higher than demand rate.
7. The time horizon of inventory system is infinite.
8. Shortages are not allowed.
9. Rate of deterioration of unit at any time is constant.
10. The deteriorated units cannot be repaired or replaced during the period.
11. Only one time markdown price at one planning period is considered.
12. Markdown price is known in advance.

III. MATHEMATICAL MODEL: FORMULATION AND SOLUTION:

According to the assumptions and notations mentioned earlier, the behavior of inventory system of time't' can be depicted in the following inventory-time diagram. (Fig.1)

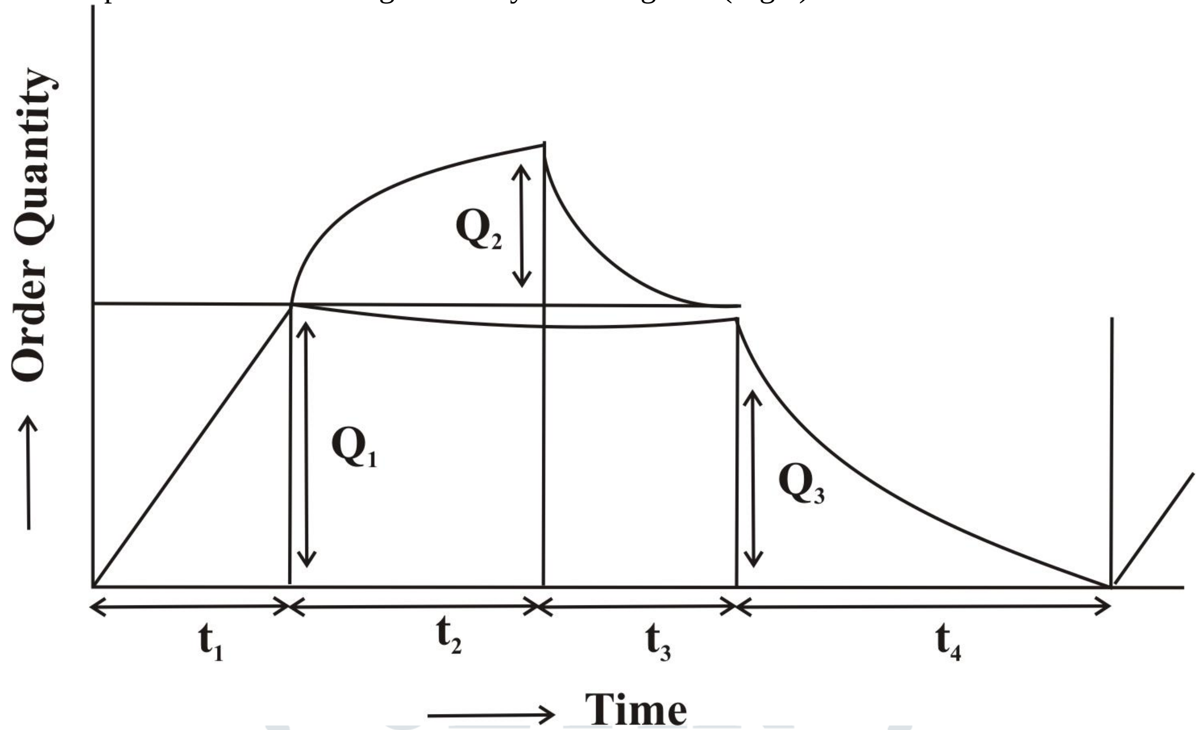

figure-1 : inventory time diagram

During the production period (0, t_1), defective item per unit time are to be reworked. In the production up time, demand is assumed to be constant and there is no deterioration. The rework process starts at the end of time t_1. The rework up process ends with time t_2 and demand is continuously decreasing with time and product starts deteriorating. During rework process some reworkable, some non-reworkable but slightly defective and scrap items are obtained. Scrap items are to be disposed off and according to the LIFO policy reworkable items during the rework up time are consumed before the fresh items from the production up time.

Since the demand is continuously decreasing with time, therefore to increase the demand or to reduce the inventory of reworked items and non-reworkable but slightly defective items, markdown policy is applied. Now, markdown is offered after time (t_1+t_2), as a result demand increases due to reduction in the selling price of the items. During rework down time demand is a function of selling price. When the inventory reduces to zero in rework down time, then the fresh items from the production up time are to be sold out in time t_4 with increased demand which is the function of selling price with markdown. After time t_4, the inventory reduces to zero and then the production run begins.

The inventory level of the product at time't' over period (0, T) can be represented by the following differential equations:

The inventory level during production up time can be described by the differential equation

$$\frac{dI(t)}{dt} = P_1 - a - x \quad ; \quad 0 < t \le t_1 \qquad \text{....(1)}$$

The inventory level during rework up time is

$$\frac{dI(t)}{dt} + \theta I(t) = P_2 - \alpha e^{-\beta t} - x_1 \qquad 0 < t \le t_2 \qquad \text{....(2)}$$

The inventory level during rework down time is

$$\frac{dI(t)}{dt} + \theta I(t) = -(q - rC) - x_2 \; ; \qquad 0 < t \le t_3 \qquad \text{....(3)}$$

The rate of change of inventory level during production down time is

$$\frac{dI(t)}{dt} + \theta I(t) = -(q - rC) \; ; \qquad 0 < t \le t_4 \qquad \text{....(4)}$$

Under the assumption of LIFO production system, the inventory level of fresh or good items during rework up time and rework down time is governed by

$$\frac{dI(t)}{dt}+\theta I(t)=0 \quad ; \quad 0<t\le (t_2+t_3) \qquad \text{….(5)}$$

With boundary conditions,

$t = 0$, $I(t) = 0$

The inventory level in a production up time is

$$I(t) = (P_1-a-x)t \qquad \text{….(6)}$$

at $t = t_1$, $I(t) = Q_1$

$$Q_1 = (P_1-a-x)t_1 \qquad \text{….(7)}$$

The inventory level in a rework up time with boundary conditions, $t = 0$, $I(t) = 0$ is

$$I(t)=\frac{(P_2-x_1)}{\theta}\left(1-e^{-\theta t}\right)+\frac{\alpha}{(\theta-\beta)}\left(e^{-\theta t}-e^{-\beta t}\right) \qquad \text{….(8)}$$

The inventory level in rework down time with boundary condition $t=t_3$, $I(t) = 0$ is

$$I(t)=\left[e^{\theta(t_3-t)}-1\right]\frac{(q-rC+x_2)}{\theta} \qquad \text{….(9)}$$

at $t=0$, $I(t)=Q_2$;

$$Q_2=\left[e^{\theta t_3}-1\right]\frac{(q-rC+x_2)}{\theta} \qquad \text{….(10)}$$

The inventory level in production down time with boundary condition $t=t_4$, $I(t)=0$ is

$$I(t)=\frac{(q-rC)}{\theta}\left(e^{\theta(t_4-t)}-1\right) \qquad \text{….(11)}$$

The inventory level of fresh items during rework up time and rework down time with boundary conditions

$t=0$, $I(t)=Q_1$ is

$$I(t)=Q_1 e^{-\theta t} \qquad \text{….(12)}$$

at $t=(t_2+t_3)$, $I(t)=Q_3$

$$Q_3=Q_1 e^{-\theta(t_2+t_3)} \qquad \text{….(13)}$$

Now, the holding cost of holding inventory per unit time is

$$HC=\frac{h}{T}\left[\int_0^{t_1}(P_1-a-x)t\,dt+\int_0^{t_2}\left\{\left(\frac{P_2-x_1}{\theta}\right)\left(1-e^{-\theta t}\right)+\frac{\alpha}{(\theta-\beta)}\left(e^{-\theta t}-e^{-\beta t}\right)\right\}dt\right.$$
$$\left.+\int_0^{t_3}\frac{(q-rC+x_2)}{\theta}\left(e^{\theta(t_3-t)}-1\right)dt+\int_0^{t_4}\frac{(q-rC)}{\theta}\left(e^{\theta(t_4-t)}-1\right)dt+Q_1\int_0^{t_2+t_3}e^{-\theta t}dt\right]$$

Since the deterioration rate is practically very small, the terms of the exponent involving θ^3, θ^4, … etc. become negligible and therefore can be neglected. Thus exponential terms are retained for θ and θ^2. Therefore, the required holding cost per unit time be

$$HC=\frac{h}{T}\left[(P_1-a-x)t_1+\left(\frac{P_2-x_1}{2}\right)t_2^2-\frac{\alpha t_2^2}{2}+(q-rC+x_2)\frac{t_3^2}{2}+\frac{(q-rC)t_4^2}{2}\right.$$
$$\left.+(P_1-a-x)t_1(t_2+t_3)-(P_1-a-x)t_1(t_2+t_3)^2\right]$$

Deterioration cost per unit time be

$$DC=\frac{C}{T}\left[P_2-\int_0^{t_2}\alpha e^{-\beta t}dt-\int_0^{t_3}(q-rc)dt-\int_0^{t_4}(q-rc)dt-(Q_1-Q_3)-x_1t_2\right]$$

As deterioration rate is very small, therefore the terms of exponent involving θ and θ^2 are retained and the other higher order terms involving θ^3, θ^4 etc. can be neglected.

Therefore,

$$DC=\frac{C}{T}\left[P_2-\alpha t_2+\frac{\alpha\beta t_2^2}{2}-(q-rC)(t_3+t_4)-x_1t_2-(P_1-a-x)\theta t_1(t_2+t_3)\right.$$
$$\left.+\frac{(P_1-a-x)\theta^2}{2}(t_2+t_3)^2\right] \qquad \text{….(15)}$$

Production cost per unit time for production up time PC = $P_1K_1t_1 / T$

Production cost per unit time for rework up time = PRC = $P_2K_2t_2/ T$

Set up cost per unit time SC = A/T

Total Cost of the system

TC = Holding Cost + Deterioration Cost + Production Cost + Set up cost + Scrap Cost

$$TC(T)=\frac{1}{T}\Bigg[h(P_1-a-x)t_1+h\frac{(P_2-x_1)t_2^2}{2}-\frac{h\alpha t_2^2}{2}+\frac{h(q-rC+x_3)t_3^2}{2}+\frac{h(q-rC)t_4^2}{2}$$
$$+h(P_1-a-x)t_1(t_2+t_3)-h(P_1-a-x)\theta t_1(t_2+t_3)^2+CP_2-C\alpha t_2+\frac{C\alpha\beta t_2^2}{2}$$
$$-C(q-rC)(t_3+t_4)-Cx_1t_2-C(P_1-a-x)\theta t_1(t_2+t_3)-\frac{C\theta^2(t_2+t_3)^2}{2}$$
$$+P_1K_1t_1+P_2K_2t_2+A+Cx_1t_2\Bigg] \qquad \text{….(16)}$$

Now, the objective is to minimize the total cost of the system. The necessary condition for TC(T) to be minimum is obtained by differentiating TC(T) with respect to T and equating it to zero. Optimum T, satisfies the condition:

$$\frac{d^2TC(T)}{dT}>0$$

For this purpose, eliminate t_1, t_2, t_3, t_4 in equation (16) by substituting the values of t_1, t_2, t_3, t_4 in terms of T.

Assume that $t_1 = \delta T$

$t_2 = \gamma(T–t_1) = \gamma(1–\delta)T$

$t_3=\lambda\{T-(t_1+t_2)\} =\lambda(1-\delta)(\lambda+\gamma)T$

$t_4=T-(t_1+t_2+t_3)=(1-\delta)[1-\gamma-\lambda(\lambda+\gamma)]T$

For simplification we assume that,

t_1 = ET

t_2 = FT where E = δ, F = g$\gamma(1–\delta)$

t_3 = GT G = $\lambda(1-\delta)(\lambda+\gamma)$

t_4 = HT H=$(1–\delta)[1–\gamma–\lambda(\lambda+\gamma)]$

Therefore, the resultant equation becomes:

$$TC(T)=\Bigg\{T^3\left[-h(P_1-a-x)\theta E(F+G)^2\right]+T^2\Bigg[h\frac{(P_2-x_1)F^2}{2}-\frac{h\alpha F^2}{2}$$
$$\frac{h(q-rC+x_2)G^2}{2}+\frac{h(q-rC)H^2}{2}+h(P_1-a-x)E(F+G)+\frac{C\alpha\beta F^2}{2}$$
$$-C(P_1-a-x)\theta E(F+G)-\frac{C\theta^2(F+G)^2}{2}\Bigg]+T\Big[h(P_1-a-x)E-C\alpha F$$
$$-C(q-rC)(G+H)+P_1K_1E+P_2K_2F\Big]+(CP_2+A)\Bigg\}$$

OR

$$TC(T)=\frac{1}{T}\left[UT^3+VT^2+WT+X\right]$$

$$TC(T)=UT^2+VT+W+\frac{X}{T} \qquad \text{….(17)}$$

Where

$$U=-h(P_1-a-x)\theta E(F+G)^2$$

$$V=\Bigg[h\frac{(P_2-x_1)F^2}{2}-\frac{h\alpha F^2}{2}\frac{h(q-rC+x_2)G^2}{2}+\frac{h(q-rC)H^2}{2}+h(P_1-a-x)E(F+G)$$

$$+\frac{C\alpha\beta F^2}{2}-C(P_1-a-x)\theta E(F+G)-\frac{C\theta^2(F+G)^2}{2}\Bigg]$$

$$W=\left[h(P_1-a-x)E-C\alpha F-C(q-rC)(G+H)+P_1K_1E+P_2K_2F\right]$$

$$X=(CP_2+A)$$

For finding optimum T, differentiate equation (17) with respect to T, and equate it to zero,

i.e. $$\frac{dTC(T)}{dt}=0$$

$$2UT+V-\frac{X}{T^2}=0$$

$$2UT^3+VT^2-X=0 \qquad \text{....(18)}$$

Equation (18) can be easily solved by Newton-Raphson's method for a positive $T=T^*$.
T^* will be an optimal solution, provided the following condition is satisfied for $T=T^*$.

$$\left(\frac{d^2TC(T)}{dT^2}\right)_{T=T^*}>0$$

$$\frac{d^2TC(T)}{dT^2}=\left(2U+\frac{2X}{T^3}\right)$$

Which is positive or greater than zero for the given values of the parameters and $T=T^*$.
The optimum order quantities Q_1^*, Q_2^* and Q_3^* are to be found by equation (7), (10) and (13).

Substituting the value of T^* in equation (17), the optimum system cost TC (T^*) can also be determined.

We can find the optimum profit of the system along with the optimum system cost. For that purpose total revenue has to be calculated.

Total revenue consists of the revenue before markdown and the revenue after markdown is realized. Therefore the revenue equation can be obtained as

$$TR=C\left[\int_0^{t_1}a\,dt+\int_0^{t_2}\alpha e^{-\beta t}dt+\int_0^{t_3}(q-rC)dt+\int_0^{t_4}(q-rC)dt\right]$$

$$TR=C\left[at_1+\alpha t_2-\frac{\alpha\beta t_2^2}{2}+(q-rC)(t_3+t_4)\right]$$

Total profit is equal to the total revenue minus total system cost

TP = TR – TC

By substituting the value of t_1, t_2, t_3 and t_4 the resultant equation of total profit is given by

TP = TR – TC

$$TP=CaET+C\alpha FT-\frac{C\alpha\beta F^2T^2}{2}+C(q-rC)(G+H)T-\left[UT^3+VT^2+WT+X\right]$$

$$TP=-UT^3+T^2\left[-V-\frac{C\alpha\beta F^2}{2}\right]+T\left[-W+CaE+C\alpha F+C(q-rC)(G+H)\right]-X$$

$$TP=U'T^3+V'T^2+W'T+X'$$

Where

$$U'=-U$$

$$V'=-\left(V+\frac{C\alpha\beta F^2}{2}\right)$$

$$W'=-W+CaE+C\alpha F+C(q-rC)(G+H)$$

$$X'=-X$$

Therefore,

Total profit rate

$$TPR(T) = \frac{1}{T}\left[U'T^3 + V'T^2 + W'T + X'\right]$$

$$TPR(T) = U'T^2 + V'T + W' + \frac{X'}{T} \quad ....(19)$$

For finding optimum T, differentiate equation (19) with respect T and equate it to zero i.e.

$$\frac{dTPR(T)}{dT} = 0$$

$$2U'T + V' - \frac{X'}{T^2} = 0$$

$$2U'T^3 + V'T^2 - X' = 0 \quad ....(20)$$

Equation (20) can be easily solved by Newton Rapshon method for a positive $T=T^*$.

For Optimality,

$$\left(\frac{d^2TPR(T)}{dT^2}\right)_{T=T^*} < 0$$

$$\left(\frac{d^2TPR(T)}{dT^2}\right)_{T=T^*} = \left(2U' + \frac{2X'}{T^3}\right)$$

IV. NUMERICAL ILLUSTRATION AND SENSITIVITY ANALYSIS:

To illustrate the results obtained for the suggested model, a numerical example with the following parameter values is considered;

h = Rs. 3/unit/unit time, C = Rs. 10/unit, a = 100 units, α = 50 units, β = 0.5, P_1 = 200 units / unit time, P_2 = 100 units per unit time, q = 50, θ = 0.08, x = 50 units/unit time, x_1 = 20 units / unit time, x_2 = 30 unit/unit time, K_1= Rs. 2/unit, K_2 = Rs. 3/unit, R = 0.5

We have assumed that the value of markdown rate 'R' is varying from 0.3 to 0.7, 'δ' known as the production percentage is changed by 0.2 to 0.4, 'γ' the rework process percentage is changed by 0.3 to 0.5 and 'λ' the markdown percentage is varying from 0.1 to 0.3

Tables 4.1, 4.2 and 4.3 show the experimental result on the sensitivity analysis for checking the effectiveness of the model with respect to the parameters δ, γ, λ and R on optimum cycle time, optimum order quantity and optimum system cost.

Similarly, Tables 4.4, 4.5, and 4.6 show the sensitivity analysis for the effectiveness of model an optimum cycle time optimum order quantity and optimum profit.

Effect of change of parameter on optimum cycle time, optimum order quantity and optimum cost

Table-4.1

Markdown Rate R	δ=0.2 γ-0.3 λ=0.1					δ=0.2 γ-0.3 λ=0.3				
	T^*	TC^*	Q_1^*	Q_2^*	Q_3^*	T^*	TC^*	Q_1^*	Q_2^*	Q_3^*
0.3	5.83	198.65	59	14	52	6.30	178.89	63	69	52
0.5	5.75	191.51	58	14	51	6.23	170.53	62	70	51
0.7	5.68	185.10	57	14	50	6.16	163.03	61	71	50

Table-4.2

Markdown Rate R	δ=0.2 γ-0.5 λ=0.1					δ=0.2 γ-0.5 λ=0.3				
	T^*	TC^*	Q_1^*	Q_2^*	Q_3^*	T^*	TC^*	Q_1^*	Q_2^*	Q_3^*
0.3	5.18	288.21	52	18	43	5.33	287.46	54	78	42
0.5	5.15	282.16	52	19	43	5.31	280.16	53	80	41
0.7	5.13	276.10	52	19	43	5.29	274.10	52	81	40

Table-4.3

Markdown Rate R	δ=0.4 γ-0.3 λ=0.1					δ=0.4 γ-0.3 λ=0.3				
	T^*	TC^*	Q_1^*	Q_2^*	Q_3^*	T^*	TC^*	Q_1^*	Q_2^*	Q_3^*
0.3	7.03	320.51	141	12	125	7.37	317.97	147	60	124
0.5	6.95	315.41	139	13	124	7.30	311.97	146	61	123
0.7	6.87	310.27	138	13	123	7.24	305.95	145	62	123

Effect of change of parameter on optimum cycle time, optimum order quantity and optimum profit

Table-4.4

Markdown Rate R	δ=0.2 γ-0.3 λ=0.1					δ=0.2 γ-0.3 λ=0.3				
	T^*	TP^*	Q_1^*	Q_2^*	Q_3^*	T^*	TP^*	Q_1^*	Q_2^*	Q_3^*
0.3	5.25	321.59	71	17	61	5.56	338.59	56	60	47
0.5	5.19	341.21	70	17	60	5.51	359.38	55	61	46
0.7	5.14	359.27	69	17	59	5.47	378.58	55	63	46

Table-4.5

Markdown Rate R	δ=0.2 γ-0.5 λ=0.1					δ=0.2 γ-0.5 λ=0.3				
	T^*	TP^*	Q_1^*	Q_2^*	Q_3^*	T^*	TP^*	Q_1^*	Q_2^*	Q_3^*
0.3	4.20	191.15	42	15	36	4.25	192.15	43	62	35
0.5	4.19	205.57	42	15	36	4.24	205.13	43	63	35
0.7	4.17	219.94	42	15	36	4.23	220.12	43	64	35

Table-4.6

Markdown Rate R	δ=0.4 γ-0.3 λ=0.1					δ=0.4 γ-0.3 λ=0.3				
	T^*	TP^*	Q_1^*	Q_2^*	Q_3^*	T^*	TP^*	Q_1^*	Q_2^*	Q_3^*
0.3	6.43	320.88	129	11	116	6.43	320.88	133	53	114
0.5	6.37	336.68	128	11	115	6.37	336.68	132	54	113
0.7	6.31	350.48	126	12	114	6.31	350.48	131	56	113

As per the above sensitivity analysis, it is clear that if the value of markdown price is different at a given markdown time, there is a very less or nominal variation in the replenishment time but system cost and profits are different with some amount.

Table-4.1 indicates that when markdown rate is increased by 40% of the initial rate then system cost is reduced by 3.5% approximately and given the values of other parameters when markdown rate is decreased by 40% than cost is increased by 3.7% approximately.

If markdown percentage is increased by 200% then there is very nominal reduction in the total system cost (say 11% approximately), whatever be the values of the other parameters.

Simultaneous perusal of table 4.1 and table 4.2 reveals that with the increase of rework process percentage by 66.67% approximately the cost is increased by 47% approximately,along with the other values of the parameters taken for illustration.

Viewing both the tables-4.1 and 4.3 simultaneously, it is seen that when production percentage is increased by 100% then system cost is increased by 65% approximately, irrespective of the values of other parameters.

Table-4.4 indicates that when markdown rate is increased to the 40% to the initial rate then profit is increased by 5.3% approximately, given other parameter values.

If markdown percentage is increased by 200% then profit is increased by 5.3% approximately and optimum order quantity Q_3* is reduced by 21% with the other values of the parameters as indicated in the table.

From both the Tables-4.4 and 4.5, it is clear that with the increase of rework process percentage by 66.67% approximately, the total profit is reduced by 40% approximately, with the other values of the parameters indicated in the tables.

When we observe Tables-4.4 and 4.6 together it is seen that when production percentage is increased by 100%, the profit is increased marginally by 1% to 2% only.

V. CONCLUSION:

In the proposed study, a modified economic production quantity model for deteriorating items having varying demand with imperfect production system has been developed. This model considers the imperfect production system which produces reworkable, non-reworkable but slightly defective and scraps items besides the major perfectly good items. Reworkable items are reworked into good quality products. Scrap items are disposed off and non-reworkable items are sold at least price under markdown.

Demand is constant in production cycle; exponentially decreasing function of time in rework process cycle and then after selling price dependent demand under markdown, to reduce the inventory.

The model developed is an imperfect production model applicable to those deteriorating products and non- reworkable defective products whose demand is low.

The important conclusion that can be drawn from the above sensitivity analysis is that the markdown policy should be applied at a later stage and total production cycle time should be minimized, in order to reduce the total system cost and to increase the total profit.

If the production is increased with a high percentage, there is only nominal increase in the profit. Thus, the management should produce only sufficient number of items as to strike the balance between the total system cost and total profit to the management. Excess product produced would only result in sheer loss to the system.

As the increment in the markdown rate does not have sufficient impact on the total system cost and total profit; it is better to discard the reworkable products rather than indulging in the rework process.

The defective items produced are basically a result of inefficiency of the technician who is handling the machine as well as improper functioning of the machine. Therefore, before starting the production process, the management should take consideration of these two important factors so that either the manufacturer does not completely adopt the rework process or the reworkable items are produced with a minimum percentage.

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