Mathematical Techniques Applied in Economics and Some Paradoxes Explained

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Abstract

Mathematical techniques have an important role to grasp the ideas of intellectual progress in economics but it is not an effective method for the new learners. Paradox in economics is a situation where the variables are failed to follow the general principles and assumptions and the common theory behave in opposite direction. The present paper tries to analyze some paradoxical results in economics in mathematical term with suitable examples.

Keywords: hypothesis, identity, paradox, parameter

History

The history of science analyses many paradigms or schools of thought. In physics we explain the motion of an object by Newton's classical laws of mechanics. Ptolemy explained the planetary motion in which earth was placed at the centre and was later replaced by Copernicus in which earth moved the origin to the sun. Even the Newtonian Classical mechanics was changed and replaced by the Einsteinian relative theory. Even at present the relative theory is also replaced by the very recent invention of Jandon particles whose velocity is higher than light. Now generally a basic simple question arises what is mathematics? The very lucid answer is that mathematics is the finest and very simple form of language for a precise presentation. Uses of mathematical relationships were necessary to build up theories. The application of analytical plane geometry is an important tool for mathematical analysis. But it has a limitation to explain only two or three variables. For more than three variables it is difficult to explain the theory in terms of Geometry. The use of calculus and some simple concepts of simultaneous-equation systems are necessary mathematical tools to explain social theories. Mathematical techniques have an important role to grasp the ideas of intellectual progress in economics but it is not an effective method for the new learners. Most of science background students have an idea about the basic knowledge of mathematics (like graph, equation, calculus etc.). But for ordinary students who generally come to study the subject economics is not so easy to grasp the idea of mathematics. In that case most of they spend so much time and effort to make sense for mathematics and the basic of fear escape from them.

The major difference between "mathematical economics" and "literary economics" lies principally in the fact that in mathematical economics the assumptions and conclusions are stated in mathematical symbols rather than words and equations rather than sentences and in place of literary logic we use mathematical logics and theorems. Mathematical deductive reasoning is more precise and meaningful to generalise theory. Mathematics is the ladder of modern Economics. Though it is tough but it is tricky and also funny. Hence to make Economics to be interesting it should be applied mathematical economics. Today's Economics is market based and information's are gathered through statistics and mathematical tools.

In short, the beauty of mathematical approach has the following advantages: the language used is more concise and precise, mathematical theorems are established, assumptions are more realistic, less time consuming process and can be generalise into n-variable case. Thus mathematical economics is more fundamental and relevant. Mathematical economics refers to the application of mathematics to the purely theoretical aspects of economic analysis. The theoretical analysis and empirical studies are mutually reinforcing and complementary in nature.

The postulates are *refutable* only through making logically valid prediction about real, observable events based on those postulates, under assumed test conditions, and then discovering that the predictions are false. If A→B and if B is true then we cannot conclude A is true. This is not a valid proposition. We can reach two theories about the shape of the earth. A round earth is postulated under the assumption that light waves travel in straight lines, ships coming in front a far become visible from the top down, as they approach the shore. This is confirmed by actual observation. However, this does not prove that the earth is round. Again a flat earth is postulated under the assumption that light waves travel in curves convex to the surface of the earth, the same events are predicted. Thus, on the basis of this experiment alone, no valid conclusion can be drawn about the shape of the earth (E. Silberberg 1990).

The basic difference between economics and pure science is that in science the laws are experimentally tested and verified but in case of economics most of the theories and laws are derived from human psychology which cannot be tested and are based upon certain assumptions. In this sense economics is more difficult but ornamental than pure science to conceptualise. The validity of these theories is largely depending upon the reliability of these assumptions from which conclusions are drawn. When is an inference from a premise, p, to a conclusion, c, reasonable? If an inference is made by the mainstream of the scientific community, it is not reasonable, for is not science the epitome of rationality? An inference from p to c is reasonable only if p supports c.

In empirical science, a theory is a set of explanations or predictions about various objects in the real world. Theory consists of 3 parts: A set of assertions or postulates or propositions concerning the behaviour of various theoretical constructs. The second part is the set of assumptions or test conditions. A theory will be observable if the assumptions are realistic and tested against a set of data. The third part comprises that are predicted by the theory. Like every social science evolve through the following stages: Descriptive stage, Analytical, Predictive and Control stage. Economics proceeds by looking at data, developing hypotheses, testing them and reaching a conclusion using mathematical and statistical tools. In mathematics uses the deductive methods of logic and geometry and the inductive methods of statistics. Since economics cannot use the controlled experiments like physicist, she faces the methodological problems. Economics uses the scientific methods consist of the process of observation, hypothesis formation, testing of hypothesis, interpretation and synthesis.

A model is asset of assumptions adopted to solve a prior problem. In any model variables are of two types: *exogenous* and *endogenous*. Exogenous variables are those variables whose values are not determined within the model. On the other hand Endogenous variables are those variables whose values are determined within the model. The exogenous variables affect the values of the endogenous variables. For example, the demand

for cold drinks, ice cream or air conditioners depend on weather condition. Hence weather is an exogenous variable which control these demands from outside. Now we should distinguish between constants and parameters. A constant is a magnitude that does not change and is therefore the antithesis of a variable. When a constant is used before a variable it is called the co-efficient of that variable. However, a co-efficient may be symbolic rather than numerical e.g. 2, 7 instead of a, b. In short, it is a constant that is variable! These parameters are changing in case of comparative static analysis. Suppose the demand curve is: D=f (P), D=a-bP, where D=dependent var. P=independent var. a and b are parameters.

Deductive and Inductive Approach

In economic analysis there are two types of methods: deductive and inductive used on the basis of logic. According to Wilson Gee, "By deductive method is meant the reasoning from general to particular or from universal to individual." Suppose man is rational. Now a rational man will try to purchase less quantity of a specific commodity whose price is increasing. Now we consider Rana is a man so he will be rational and behave to purchase less quantity of a specific commodity whose price is increasing. Here the logic is from general to particular and hence deductive approach in nature. In aggregate theories are based on deductive method and is also known as scientific method. According to Wilson Gee, "Inductive method is the process of reasoning from particular to general or from individual to universal." Suppose Sayan purchases more of a commodity whose price falls. We also find that Tanushree, Pampa, Indrani, Tutun and Kunal purchase more of a commodity whose price falls. Now we can generalise that we purchase more of a commodity whose price falls. Here the logic is from particular to general and hence inductive in nature. It uses a process of reasoning and generalisations from facts in order to establish a general principle and the derived laws are relative not universal. The basic logic of this method is that suppose a proposition is true for (r) and is also true for any (r+1) for any value of (r). All individual theories are based on inductive method and are also known as historical method, Concrete method, Empirical method, Analytical method and Realistic method. Now both the methods have some merits and demerits. Naturally a question arises which method is superior in economic analysis? Both the methods have some weakness when used alone. According to A. Marshall both deductive and inductive methods are necessary for scientific thought in the same way as we need both feet for effective walking. Thus in modern economic thought they are not exclusive rather complementary and so become equal partners rather than rivals. As a result both the methods are used in a very productive way as the situation to overcome the mathematical logic to establish general theory.

Equation and Identity

An equation is a mathematical statement that asserts the equality of two expressions. In notation: x+3=5. The equation holds true for a specific values of x=2 and only 2. The equality notation (=) was invented by Robert Recorde (1510-1558). According to types of operations equations can be divided into: Linear, Quadratic, Polynomial, Transcendental, Functional, Differential, Integral, Diophantine etc. On the other hand, an identity is a mathematical relation among several variables which is true for all possible values of the variables. For example: $(x+y)^2 \equiv x^2+2xy+y^2$ -is true for all possible values of x & y and hence it is an identity. An identity is always true by its definition. We cannot solve an identity for determining the

values of the variables since it is satisfied for all the values of the variables. In economics the identical situations are: Y ≡ C+S or Y ≡ C+I or I ≡ S. There are two approaches, static and dynamic, to explain the course of a system of variables through time. In a static analysis the variables are referred to the same period of time and in dynamic analysis the variables are refer to the different periods of time.

Paradoxes

In terms of mathematics, suppose a = b or a^2 = ab or a^2- b^2 = ab- b^2 or (a+b) (a-b) = b (a-b) or a+b = b or b+b= b or 2b = b or 2=1! And so on (this invalid conclusion arises because division of zero is undefined). *Paradox* in economics is a situation where the variables are failed to follow the general principles and assumptions and theory behave in opposite direction. There are so many paradoxical results in economics. Here some cases of economics paradoxes are analyzed. *Water-Diamond Paradox, Giffen's Paradox, Leontief Paradox, Paradox of Thrift, Scitovsky Paradox, Edgeworth Paradox Voting paradox and so on*.

Adam smith *water-diamond paradox* is rhetorical paradox of value. A man cannot live without water but it has lower price while diamond has very high price. The general shape of the demand curve is downward slopping but in Giffen paradoxical situation it is upward rising. In Leontief Paradoxical situation a country a higher capital per worker has a lower capital-labour ratio in export than in import. Another classic example of fallacy is that paradox of saving or paradox of thrift. It is belief that one individual can save more money by spending less, and then the society can save more money by spending less. But this is not commonly true. The real fact is that the society can save more only by spending more. *Scotovsky's* paradox is a result in welfare economics which states that there is no increase in social welfare by a return to the original part of the result. Suppose an allocation A is superior to another allocation B, we can prove that B is also superior to A when the gainer from the change of allocation A to B can compensate the looser for making the change but the looser can also compensate the gainer for going back to the original position. *Edgeworth* paradox describes a situation in which two players cannot reach a stable equilibrium with fixed price. Suppose two companies, A and B, sell an identical commodity product, and that customers choose the product solely on the basis of price. Each company faces capacity constraints, in that on its own it cannot satisfy demand at its zero-profit price, but together they can more than satisfy such demand.

Another classic example of paradox is the *voting paradox* in social choice developed by Kenneth J arrow. Suppose each individual voter has clear transitive preferences over alternative options. Then by majority voting-rule has no clear winner. Suppose there are three alternatives, say, A, B and C. Three individuals preferences are as follows: Individual 1- A> B>C, Individual 2- B> C>A and Individual 3- C> A>B. If transitive property holds true no winning candidate will be selected. In this peculiar situation of social choice no welfare decision can be achieved.

Some Invalid Conclusions

As for the mathematics, it comes in taking the results from these models, which we call theory, and seeing how these results match up with the data in the real world. Mathematics has an important role in demographic transition and population census. The Govt. has an important role to take economic decision using various statistical results and conclusion. Now we give an example which is very interesting. Suppose

the success rate of heart operation of a medical surgeon is 90%. He has successfully operated 90 patients. In this situation when he is going to operate the 91st person will must die. This conclusion is wrong because the survibility of the last person will also 90/100. This is nothing but the mathematical fallacy arising out by statistical counting.

There is another type of such mathematical fallacy. Suppose there are two doctors, say A and B. Doctor A having a success rate of 80% and doctor B having a success rate of 50%. Now if we conclude that A is a better doctor than B which is not necessarily a valid conclusion. Because there must be other factors which would be consider in this case. For example, the patient's conditions who admitted to these doctors. The patients for the A doctor may have some common diseases like fever, cold & cough, headache etc. and hence the success rate is high. On the contrary, the patients of doctor B may have some critical diseases like cancer, severe heart or lungs problems, brain tumour etc. and hence the success rate is very low. Practically, B may be the better physician than A with a high repute.

But again this is not a valid conclusion. A is better doctor compared to doctor B simply because in the long-run in 80% cases A is successful where only 50% cases B is success i.e. for large n, out of n cases in .8n number of cases A is successful where B will successful in .5n cases holding all other factors constants (i.e. ceteris paribus).

Another invalid conclusion can be drawn in case of *probability* theory. Suppose a coin is tossed. Then the appearance of either head or tail is equal to half if the coin is homogeneous and unbiased one. Now suppose a ceiling fan is fixed under the ceiling of a roof. Now the falling percentage of the fan is fifty percent is not a valid conclusion.

In *correlation* theory supposes the correlation coefficient, says r, and is zero for two variables X &Y. On the basis of this we cannot say that X and Y are independent to each other. This simply means that there is no linear relationship between these two variables but there may have other relation. For example, the relationship is like $y = x^2$ which have zero r but are not independent to each other. Another example like that there may be some correlation between intelligence and shoe-size. But it is non-sense to judge its utility in our practical life.

Similarly, in election there is high positive correlation between winning of a candidate and amount of money spend in his election purposes. This does not necessarily imply that spending a high amount of money means the candidate will win in the election. There are other factors like the quality and accessibility of the candidates and voters, political campaign, social factors, number of years for to be elected etc. money here is not only the measuring rod. For example candidate A will win in two consecutive elections spending different amounts of money.

Let us consider another example in IPL cricket match in India. Suppose there are two teams, say A and B is contesting each other in the semi-final match to reach the final in a must-win situation. In the final winner will get Rs.2 crore and runner will get Rs. 1 crore. Team A has a winning record of 7-7 and B has a winning record of 8-6. In this situation A has far more to gain from a victory than B has to lose. There may be a tie condition and the agreement made by those teams are quid pro quo agreement i.e. you let me to win today,

when I really need and I shall let you win in the next time. In this situation match fixing may occur and the players are threatened by the mafia for match fixing i.e. cricket match is now not corruption free at all. In this must winning situation the fixing rate is also very high compared to other ordinary games.

Conclusion

In short, the beauty of mathematical approach in economics has the following advantages: the language used is more concise and precise, mathematical theorems are established, assumptions are more realistic, less time consuming process and can be generalise into n-variable case. The theoretical analysis and empirical studies are mutually reinforcing and complementary in nature. It is an easy way to analyze economic analysis applying suitable mathematical techniques for economic result to be interesting. Economics is a soulful science and hence it is more interesting and ornamental than other science subjects.

References

[1] Prokhorov, A.B. 2001. Nonlinear Dynamics and Chaos Theory in Economics: a Historical Perspective. Available via DIALOG. https://www.msu.edu. Accessed 11July 2001

[2] Alpha C. Chiang 3rd Ed. Fundamental Methods of Mathematical Economics, McGraw-Hill, Singapore, 1984

[3] Bairagya R. and Sarkar S. 2012. Economics can be Interesting, Lambert Academic Publishing

[4] Black D.1958. The Theory of Committees and Elections. Cambridge University Press, London

[5] Coyle D. 2007. The Soulful Science. Available via DIALOG. www.amazon.com

[6] D. Salvatore. 1983. International Edition, Theory and Problems of Microeconomic Theory, Schaum's Outline Series, McGraw-Hill, Singapore

[7] E.T. Dowling. 1986. International Edition, Theory and Problems of Mathematics for Economists, Schaum's Outline Series, McGraw-Hill, Singapore

[8] Friedman D.1997. Hidden Order. Available via DIALOG. www.amazon.com

[9] Kenneth J. Arrow.1963. Social Choice and Individual Values

[10] Perelman, Y.1986. Physics Can be Fun. Mir Publishers, Moscow

[11] Silberberg E. 1990. The Structure of Economics-A Mathematical Analysis, 2nd edn. Tata McGraw-Hill, Singapore