RELIABILITY OF A POWER PLANT BY BOOLEAN FUNCTION TECHNIQUE

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Abstract: In this paper investigations have been carried out for the reliability analysis of a power plant by Boolean function technique. Failure times for the various components of the power plant have been assumed to follow either Weibull or exponential time distributions. Moreover, an important parameter of reliability, viz, M.T.T.F (mean time to failure) has been computed for exponential failure time distribution. Some numerical examples along with graphs on reliability and M.T.T.F have also been appended at the end to highlight the important results.

Index Terms: Reliability, M.T.T.F, Exponential Distribution, Weibull distribution, Boolean Function Technique.

1. INTRODUCTION:

In various reliability systems, of practical utility, we often come across the situation of evaluating the reliability by using supplementary variable and Laplace Transform technique, but so far very little work seems to have been done in the direction of evaluating the reliability by Boolean function technique. In the recent past various researchers have [1-5] considered numerous methods to determine symbolic reliability expressions for complex systems. Of course, in 1973 Fratta considered a Boolean algebra method for computing the terminal reliability in a communication network. In 1977 Nakagawa has also considered a decomposition method for computing system reliability by Boolean function technique.

Keeping these facts in view, we considered the reliability and MTTF analysis of a parallel redundant complex system by using Boolean function technique. The complex system under consideration consists of three generators G_1, G_2 and G_3 in parallel redundancy. These generators are connected with switching device $OPMS_4$ and main switch boards MSB_i by C_i. Cables C_i $(i=1-5)$; connect MSB_1, MSB_2 ; $MSB_2, MSB_3; MSB_3$, $MSB_4; MSB_1$, output switch $OPMS_4$; $MSB_2, OPMS_4$; MSB_3 , $OPMS_4$ respectively. Thus the complex system is nothing but a power plant, and consequently the reliability of power supply to critical consumers, fed from the main switch boards MSB_i, $i=1,2,3$ is estimated. Moreover, MTTF an important parameter of reliability, has also been computed for exponential failure rates of components. In particular, Weibull and exponential distributions have been considered for the evaluation of reliability. There is no service facility. Two graphs have also been given to highlight the important results. Thus, for a given set of parametric values of failure rates one can estimate the reliability and MTTF of such a parallel redundant system well in advance to forecast the operable behavior at any time.

2. ASSUMPTIONS:

(1) The reliabilities of all constituent components of the system are known.
(2) The states of all components are independent.
(3) The state of each component and of the whole system is either good or bad.
(4) The failure times of all components are arbitrary.
(5) All components are always operating (no stand by or switched redundancy)
(6) There is no repair facility.

3. SYSTEM TRANSITION DIAGRAM:

The system under consideration is shown in the following figure.

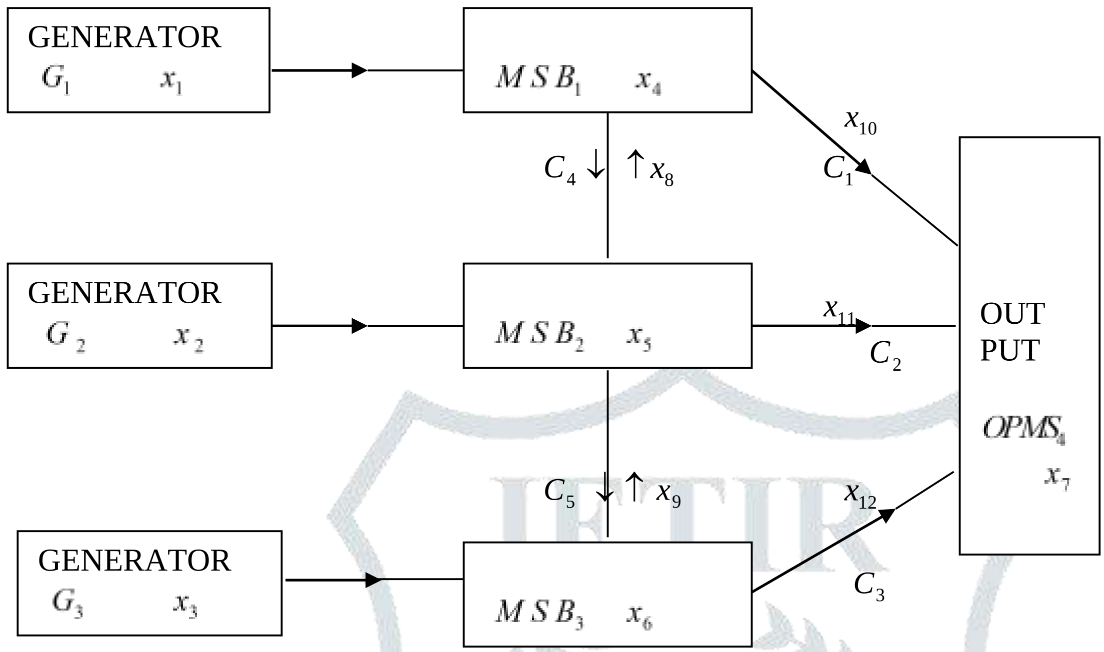

Fig.1 System Configuration

4. NOTATIONS:

x_1, x_2, x_3	States of sub-systems (generators) G_1, G_2 and G_3
x_4, x_5, x_6	States of MSB_1, MSB_2 and MSB_3
$x_8, x_9, x_{10}, x_{11}, x_{12}$	States of the cables C_1, C_2, C_3, C_4 and C_5
x_7	States of $OPMS_4$
x_k^{1}	Negation of x_k ($k = 1-12$)
$\wedge$	Conjunction
$\vee$	Disjunction

$$x_i = \begin{cases} 0, \text{ in bad state,} \\ 1, \text{ in good state,} \end{cases} \quad i = (1-12)$$

$\Pr(f = 1)$ The probability of the successful operation of the function f.

5. FORMATION OF THE MATHEMATICAL MODEL:

The object of the complex system is to supply power generated by generators G_1, G_2 and G_3 through switching devices $OPMS_4$. By using Boolean function technique, the conditions of capacity for the successful operation of the system in terms of logical matrix are expressed as

$$f(x_1, x_2, ...\,...x_{12}) = \begin{vmatrix} x_1 & x_4 & x_{10} & x_7 & & & & \\ x_1 & x_4 & x_8 & x_5 & x_{11} & x_7 & & \\ x_1 & x_4 & x_8 & x_5 & x_9 & x_6 & x_{12} & x_7 \\ x_2 & x_5 & x_8 & x_4 & x_{10} & x_7 & & \\ x_2 & x_5 & x_{11} & x_7 & & & & \\ x_2 & x_5 & x_9 & x_6 & x_{12} & x_7 & & \\ x_3 & x_6 & x_{12} & x_7 & & & & \\ x_3 & x_6 & x_9 & x_5 & x_{11} & x_7 & & \\ x_3 & x_6 & x_9 & x_5 & x_8 & x_4 & x_{10} & x_7 \end{vmatrix}$$

$$f(x_1, x_2, ...\,...x_{12}) = \begin{vmatrix} x_1 & x_4 & x_7 & x_{10} & & & & \\ x_1 & x_4 & x_5 & x_7 & x_8 & x_{11} & & \\ x_1 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{12} \\ x_2 & x_4 & x_5 & x_7 & x_8 & x_{10} & & \\ x_2 & x_5 & x_7 & x_{11} & & & & \\ x_2 & x_5 & x_6 & x_7 & x_9 & x_{12} & & \\ x_3 & x_6 & x_7 & x_{12} & & & & \\ x_3 & x_5 & x_6 & x_7 & x_9 & x_{11} & & \\ x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10} \end{vmatrix} \qquad (1)$$

6. SOLUTION OF THE MODEL:

By the application of algebra of logic, equation (1) may be written as

$$f(x_1, x_2, ...\,...x_{12}) = \left| x_7 \;\; n(x_1, x_2, x_3, x_4, x_5, x_6, x_8, x_9, x_{10}, x_{11}, x_{12}) \right| \qquad (2)$$

Where

$$n(x_1, x_2, x_3, ...\,...., x_8, x_9, x_{11}, x_{12}) = \begin{vmatrix} x_1 & x_4 & x_{10} & & & & \\ x_2 & x_5 & x_{11} & & & & \\ x_3 & x_6 & x_{12} & & & & \\ x_1 & x_4 & x_5 & x_8 & x_{11} & & \\ x_2 & x_4 & x_5 & x_8 & x_{10} & & \\ x_2 & x_5 & x_6 & x_9 & x_{12} & & \\ x_3 & x_5 & x_6 & x_9 & x_{11} & & \\ x_1 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{12} \\ x_3 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10} \end{vmatrix} \qquad (3)$$

Substituting, $K_1 = x_1\, x_4\, x_{10}$; $K_2 = x_2\, x_5\, x_{11}$; $K_3 = x_3\, x_6\, x_{12}$; $K_4 = x_1\, x_4\, x_5\, x_8\, x_{11}$

$K_5 = x_2\, x_4\, x_5\, x_8\, x_{10}$; $K_6 = x_2\, x_5\, x_6\, x_9\, x_{12}$; $K_7 = x_3\, x_5\, x_6\, x_9\, x_{11}$

$K_8 = x_1\, x_4\, x_5\, x_6\, x_8\, x_9\, x_{12}$; $K_9 = x_3\, x_4\, x_5\, x_6\, x_8\, x_9\, x_{10}$

From equation (3) we obtain

$$n(x_1, x_2, x_3, \ldots\ldots\ldots.., x_8, x_9, x_{11}, x_{12}) = \begin{vmatrix} K_1 \\ K_1^1 & K_2 \\ K_1^1 & K_2^1 & K_3 \\ K_1^1 & K_2^1 & K_3^1 & K_4 \\ K_1^1 & K_2^1 & K_3^1 & K_4^1 & K_5 \\ K_1^1 & K_2^1 & K_3^1 & K_4^1 & K_5^1 & K_6 \\ K_1^1 & K_2^1 & K_3^1 & K_4^1 & K_5^1 & K_6^1 & K_7 \\ K_1^1 & K_2^1 & K_3^1 & K_4^1 & K_5^1 & K_6^1 & K_7^1 & K_8 \\ K_1^1 & K_2^1 & K_3^1 & K_4^1 & K_5^1 & K_6^1 & K_7^1 & K_8^1 & K_9 \end{vmatrix} \qquad (4)$$

Using algebra of logic, we can determine the following

$$K_1^1 = \begin{vmatrix} x_1^1 \\ x_1 & x_4^1 \\ x_1 & x_4 & x_{10}^1 \end{vmatrix} \qquad (5)$$

$$K_2^1 = \begin{vmatrix} x_2^1 \\ x_2 & x_5^1 \\ x_2 & x_5 & x_{11}^1 \end{vmatrix} \qquad (6)$$

$$K_3^1 = \begin{vmatrix} x_3^1 \\ x_3 & x_6^1 \\ x_3 & x_6 & x_{12}^1 \end{vmatrix} \qquad (7)$$

$$K_4^1 = \begin{vmatrix} x_1^1 \\ x_1 & x_4^1 \\ x_1 & x_4 & x_5^1 \\ x_1 & x_4 & x_5 & x_8^1 \\ x_1 & x_4 & x_5 & x_8 & x_{11}^1 \end{vmatrix} \qquad (8)$$

$$K_5^1 = \begin{vmatrix} x_2^1 \\ x_2 & x_4^1 \\ x_2 & x_4 & x_5^1 \\ x_2 & x_4 & x_5 & x_8^1 \\ x_2 & x_4 & x_5 & x_8 & x_{10}^1 \end{vmatrix} \qquad (9)$$

$$K_6^1 = \left| \begin{array}{lllll} x_2^1 & & & & \\ x_2 & x_5^1 & & & \\ x_2 & x_5 & x_6^1 & & \\ x_2 & x_5 & x_6 & x_9^1 & \\ x_2 & x_5 & x_6 & x_9 & x_{12}^1 \end{array} \right| \quad (10)$$

$$K_7^1 = \left| \begin{array}{lllll} x_3^1 & & & & \\ x_3 & x_5^1 & & & \\ x_3 & x_5 & x_6^1 & & \\ x_3 & x_5 & x_6 & x_9^1 & \\ x_3 & x_5 & x_6 & x_9 & x_{11}^1 \end{array} \right| \quad (11)$$

$$K_8^1 = \left| \begin{array}{lllllll} x_1^1 & & & & & & \\ x_1 & x_4^1 & & & & & \\ x_1 & x_4 & x_5^1 & & & & \\ x_1 & x_4 & x_5 & x_6^1 & & & \\ x_1 & x_4 & x_5 & x_6 & x_8^1 & & \\ x_1 & x_4 & x_5 & x_6 & x_8 & x_9^1 & \\ x_1 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{12}^1 \end{array} \right| \quad (12)$$

$$K_9^1 = \left| \begin{array}{lllllll} x_3^1 & & & & & & \\ x_3 & x_4^1 & & & & & \\ x_3 & x_4 & x_5^1 & & & & \\ x_3 & x_4 & x_5 & x_6^1 & & & \\ x_3 & x_4 & x_5 & x_6 & x_8^1 & & \\ x_3 & x_4 & x_5 & x_6 & x_8 & x_9^1 & \\ x_3 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10}^1 \end{array} \right| \quad (13)$$

$$K_1^1 K_2 = \left| \begin{array}{lll} x_1^1 & & \\ x_1 & x_4^1 & \\ x_1 & x_4 & x_{10}^1 \end{array} \right| \wedge \left| x_2 \quad x_5 \quad x_{11} \right|$$

$$= \left| \begin{array}{llllll} x_1^1 & x_2 & x_5 & x_{11} & & \\ x_1 & x_2 & x_4^1 & x_5 & x_{11} & \\ x_1 & x_2 & x_4 & x_5 & x_{10}^1 & x_{11} \end{array} \right| \quad (14)$$

$$K_2^1 K_3 = \left| \begin{array}{lll} x_2^1 & & \\ x_2 & x_5^1 & \\ x_2 & x_5 & x_{11}^1 \end{array} \right| \wedge \left| x_3 \quad x_6 \quad x_{12} \right|$$

$$= \left| \begin{array}{llllll} x_2^1 & x_3 & x_6 & x_{12} & & \\ x_2 & x_3 & x_5^1 & x_6 & x_{12} & \\ x_2 & x_3 & x_5 & x_6 & x_{11}^1 & x_{12} \end{array} \right|$$

$$K_1^1\,K_2^1\,K_3 = \left|\begin{array}{l} x_2^1 \quad x_3 \quad x_6 \quad x_{12} \\ x_2 \quad x_3 \quad x_5^1 \quad x_6 \quad x_{12} \\ x_2 \quad x_3 \quad x_5 \quad x_6 \quad x_{11}^1 \quad x_{12} \end{array}\right| \wedge \left|\begin{array}{l} x_1^1 \\ x_1 \quad x_4^1 \\ x_1 \quad x_4 \quad x_{10}^1 \end{array}\right|$$

$$K_1^1\,K_2^1\,K_3 = \left|\begin{array}{l} x_1^1 \quad x_2^1 \quad x_3 \quad x_6 \quad x_{12} \\ x_1 \quad x_2^1 \quad x_3 \quad x_4^1 \quad x_6 \quad x_{12} \\ x_1 \quad x_2^1 \quad x_3 \quad x_4 \quad x_6 \quad x_{10}^1 \quad x_{12} \\ x_1^1 \quad x_2 \quad x_3 \quad x_5^1 \quad x_6 \quad x_{12} \\ x_1 \quad x_2 \quad x_3 \quad x_4^1 \quad x_5^1 \quad x_6 \quad x_{12} \\ x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5^1 \quad x_6 \quad x_{10}^1 \quad x_{12} \\ x_1^1 \quad x_2 \quad x_3 \quad x_5 \quad x_6 \quad x_8^1 \quad x_{12} \\ x_1 \quad x_2 \quad x_3 \quad x_4^1 \quad x_5 \quad x_6 \quad x_{11}^1 \quad x_{12} \\ x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad x_6 \quad x_{10}^1 \quad x_{11}^1 \quad x_{12} \end{array}\right| \tag{15}$$

$$K_3^1\,K_4 = \left|\begin{array}{l} x_3^1 \\ x_3 \quad x_6^1 \\ x_3 \quad x_6 \quad x_{12}^1 \end{array}\right| \wedge \left| x_1 \quad x_4 \quad x_5 \quad x_8 \quad x_{11} \right|$$

$$K_3^1\,K_4 = \left|\begin{array}{l} x_1 \quad x_3^1 \quad x_4 \quad x_5 \quad x_8 \quad x_{11} \\ x_1 \quad x_3 \quad x_4 \quad x_5 \quad x_6^1 \quad x_8 \quad x_{11} \\ x_1 \quad x_3 \quad x_4 \quad x_5 \quad x_6 \quad x_8 \quad x_{11} \quad x_{12}^1 \end{array}\right|$$

$$K_2^1\,K_3^1\,K_4 = \left|\begin{array}{l} x_2^1 \\ x_2 \quad x_5^1 \\ x_2 \quad x_5 \quad x_{11}^1 \end{array}\right| \wedge \left|\begin{array}{l} x_1 \quad x_3^1 \quad x_4 \quad x_5 \quad x_8 \quad x_{11} \\ x_1 \quad x_3 \quad x_4 \quad x_5 \quad x_6^1 \quad x_8 \quad x_{11} \\ x_1 \quad x_3 \quad x_4 \quad x_5 \quad x_6 \quad x_8 \quad x_{11} \quad x_{12}^1 \end{array}\right|$$

$$= \left|\begin{array}{l} x_1 \quad x_2^1 \quad x_3^1 \quad x_4 \quad x_5 \quad x_8 \quad x_{11} \\ x_1 \quad x_2^1 \quad x_3 \quad x_4 \quad x_5 \quad x_6^1 \quad x_8 \quad x_{11} \\ x_1 \quad x_2^1 \quad x_3 \quad x_4 \quad x_5 \quad x_6 \quad x_8 \quad x_{11} \quad x_{12}^1 \end{array}\right|$$

$$K_1^1\,K_2^1\,K_3^1\,K_4 = \left|\begin{array}{l} x_1^1 \\ x_1 \quad x_4^1 \\ x_1 \quad x_4 \quad x_{10}^1 \end{array}\right| \wedge \left|\begin{array}{l} x_1 \quad x_2^1 \quad x_3^1 \quad x_4 \quad x_5 \quad x_8 \quad x_{11} \\ x_1 \quad x_2^1 \quad x_3 \quad x_4 \quad x_5 \quad x_6^1 \quad x_8 \quad x_{11} \\ x_1 \quad x_2^1 \quad x_3 \quad x_4 \quad x_5 \quad x_6 \quad x_8 \quad x_{11} \quad x_{12}^1 \end{array}\right|$$

$$K_1^1\,K_2^1\,K_3^1\,K_4 = \left|\begin{array}{l} x_1 \quad x_2^1 \quad x_3^1 \quad x_4 \quad x_5 \quad x_8 \quad x_{10}^1 \quad x_{11} \\ x_1 \quad x_2^1 \quad x_3 \quad x_4 \quad x_5 \quad x_6^1 \quad x_8 \quad x_{10}^1 \quad x_{11} \\ x_1 \quad x_2^1 \quad x_3 \quad x_4 \quad x_5 \quad x_6 \quad x_8 \quad x_{10}^1 \quad x_{11} \quad x_{12}^1 \end{array}\right| \tag{16}$$

$$K_4^1\,K_5 = \left|\begin{array}{l} x_1^1 \\ x_1 \quad x_4^1 \\ x_1 \quad x_4 \quad x_5^1 \\ x_1 \quad x_4 \quad x_5 \quad x_8^1 \\ x_1 \quad x_4 \quad x_5 \quad x_8 \quad x_{11}^1 \end{array}\right| \wedge \left| x_2 \quad x_4 \quad x_5 \quad x_8 \quad x_{10} \right|$$

$$= \left| \begin{array}{lllllll} x_1^1 & x_2 & x_4 & x_5 & x_8 & x_{10} & \\ x_1 & x_2 & x_4 & x_5 & x_8 & x_{10} & x_{11}^1 \end{array} \right|$$

$$K_3^1 K_4^1 K_5 = \left| \begin{array}{lll} x_3^1 & & \\ x_3 & x_6^1 & \\ x_3 & x_6 & x_{12}^1 \end{array} \right| \wedge \left| \begin{array}{lllllll} x_1^1 & x_2 & x_4 & x_5 & x_8 & x_{10} & \\ x_1 & x_2 & x_4 & x_5 & x_8 & x_{10} & x_{11}^1 \end{array} \right|$$

$$K_3^1 K_4^1 K_5 = \left| \begin{array}{llllllllll} x_1^1 & x_2 & x_3^1 & x_4 & x_5 & x_8 & x_{10} & & & \\ x_1^1 & x_2 & x_3 & x_4 & x_5 & x_6^1 & x_8 & x_{10} & & \\ x_1^1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_8 & x_{10} & x_{12}^1 & \\ x_1 & x_2 & x_3^1 & x_4 & x_5 & x_8 & x_{10} & x_{11}^1 & & \\ x_1 & x_2 & x_3 & x_4 & x_5 & x_6^1 & x_8 & x_{10} & x_{11}^1 & \\ x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_8 & x_{10} & x_{11}^1 & x_{12}^1 \end{array} \right|$$

$$K_2^1 K_3^1 K_4^1 K_5 = \left| \begin{array}{lll} x_2^1 & & \\ x_2 & x_5^1 & \\ x_2 & x_5 & x_8^1 \end{array} \right| \wedge \left| \begin{array}{llllllllll} x_1^1 & x_2 & x_3^1 & x_4 & x_5 & x_8 & x_{10} & & & \\ x_1^1 & x_2 & x_3 & x_4 & x_5 & x_6^1 & x_8 & x_{10} & & \\ x_1^1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_8 & x_{10} & x_{12}^1 & \\ x_1 & x_2 & x_3^1 & x_4 & x_5 & x_8 & x_{10} & x_{11}^1 & & \\ x_1 & x_2 & x_3 & x_4 & x_5 & x_6^1 & x_8 & x_{10} & x_{11}^1 & \\ x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_8 & x_{10} & x_{11}^1 & x_{12}^1 \end{array} \right|$$

$$K_2^1 K_3^1 K_4^1 K_5 = \left| \begin{array}{llllllllll} x_1^1 & x_2 & x_3^1 & x_4 & x_5 & x_8 & x_{10} & x_{11}^1 & & \\ x_1^1 & x_2 & x_3 & x_4 & x_5 & x_6^1 & x_8 & x_{10} & x_{11}^1 & \\ x_1^1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_8 & x_{10} & x_{11}^1 & x_{12}^1 \\ x_1 & x_2 & x_3^1 & x_4 & x_5 & x_8 & x_{10} & x_{11}^1 & & \\ x_1 & x_2 & x_3 & x_4 & x_5 & x_6^1 & x_8 & x_{10} & x_{11}^1 & \\ x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_8 & x_{10} & x_{11}^1 & x_{12}^1 \end{array} \right|$$

$$K_1^1 K_2^1 K_3^1 K_4^1 K_5 = \left| \begin{array}{lll} x_1^1 & & \\ x_1 & x_4^1 & \\ x_1 & x_4 & x_7^1 \end{array} \right| \wedge \left| \begin{array}{llllllllll} x_1^1 & x_2 & x_3^1 & x_4 & x_5 & x_8 & x_{10} & x_{11}^1 & & \\ x_1^1 & x_2 & x_3 & x_4 & x_5 & x_6^1 & x_8 & x_{10} & x_{11}^1 & \\ x_1^1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_8 & x_{10} & x_{11}^1 & x_{12}^1 \\ x_1 & x_2 & x_3^1 & x_4 & x_5 & x_8 & x_{10} & x_{11}^1 & & \\ x_1 & x_2 & x_3 & x_4 & x_5 & x_6^1 & x_8 & x_{10} & x_{11}^1 & \\ x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_8 & x_{10} & x_{11}^1 & x_{12}^1 \end{array} \right|$$

$$K_1^1 K_2^1 K_3^1 K_4^1 K_5 = \left| \begin{array}{llllllllll} x_1^1 & x_2 & x_3^1 & x_4 & x_5 & x_8 & x_{10} & x_{11}^1 & & \\ x_1^1 & x_2 & x_3 & x_4 & x_5 & x_6^1 & x_8 & x_{10} & x_{11}^1 & \\ x_1^1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_8 & x_{10} & x_{11}^1 & x_{12}^1 \end{array} \right| \qquad (17)$$

$$K_5^1 K_6 = \begin{vmatrix} x_2^1 & & & & \\ x_2 & x_4^1 & & & \\ x_2 & x_4 & x_5^1 & & \\ x_2 & x_4 & x_5 & x_8^1 & \\ x_2 & x_4 & x_5 & x_8 & x_{10}^1 \end{vmatrix} \wedge \begin{vmatrix} x_2 & x_5 & x_6 & x_9 & x_{12} \end{vmatrix}$$

$$= \begin{vmatrix} x_2 & x_4^1 & x_5 & x_6 & x_9 & x_{12} & & \\ x_2 & x_4 & x_5 & x_6 & x_8^1 & x_9 & x_{12} & \\ x_2 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10}^1 & x_{12} \end{vmatrix}$$

$$K_4^1 K_5^1 K_6 = \begin{vmatrix} x_1^1 & & & & \\ x_1 & x_4^1 & & & \\ x_1 & x_4 & x_5^1 & & \\ x_1 & x_4 & x_5 & x_{11}^1 & \\ x_1 & x_4 & x_5 & x_8^1 & x_{11} \end{vmatrix} \wedge \begin{vmatrix} x_2 & x_4^1 & x_5 & x_6 & x_9 & x_{12} & & \\ x_2 & x_4 & x_5 & x_6 & x_8^1 & x_9 & x_{12} & \\ x_2 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10}^1 & x_{12} \end{vmatrix}$$

$$K_4^1 K_5^1 K_6 = \begin{vmatrix} x_1^1 & x_2 & x_4^1 & x_5 & x_6 & x_9 & x_{12} & & & \\ x_1 & x_2 & x_4^1 & x_5 & x_6 & x_9 & x_{12} & & & \\ x_1^1 & x_2 & x_4 & x_5 & x_6 & x_8^1 & x_9 & x_{12} & & \\ x_1 & x_2 & x_4 & x_5 & x_6 & x_8^1 & x_9 & x_{12} & & \\ x_1^1 & x_2 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10}^1 & x_{12} & \\ x_1 & x_2 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10}^1 & x_{11}^1 & x_{12} \end{vmatrix}$$

$$K_3^1 K_4^1 K_5^1 K_6 = \begin{vmatrix} x_3^1 & & \\ x_3 & x_6^1 & \\ x_3 & x_6 & x_9^1 \end{vmatrix} \wedge \begin{vmatrix} x_1^1 & x_2 & x_4^1 & x_5 & x_6 & x_9 & x_{12} & & & \\ x_1 & x_2 & x_4^1 & x_5 & x_6 & x_9 & x_{12} & & & \\ x_1^1 & x_2 & x_4 & x_5 & x_6 & x_8^1 & x_9 & x_{12} & & \\ x_1 & x_2 & x_4 & x_5 & x_6 & x_8^1 & x_9 & x_{12} & & \\ x_1^1 & x_2 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10}^1 & x_{12} & \\ x_1 & x_2 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10}^1 & x_{11}^1 & x_{12} \end{vmatrix}$$

$$K_3^1 K_4^1 K_5^1 K_6 = \begin{vmatrix} x_1^1 & x_2 & x_3^1 & x_4^1 & x_5 & x_6 & x_9 & x_{12} & & & \\ x_1 & x_2 & x_3^1 & x_4^1 & x_5 & x_6 & x_9 & x_{12} & & & \\ x_1^1 & x_2 & x_3^1 & x_4 & x_5 & x_6 & x_8^1 & x_9 & x_{12} & & \\ x_1 & x_2 & x_3^1 & x_4 & x_5 & x_6 & x_8^1 & x_9 & x_{12} & & \\ x_1^1 & x_2 & x_3^1 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10}^1 & x_{12} & \\ x_1 & x_2 & x_3^1 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10}^1 & x_{11}^1 & x_{12} \end{vmatrix}$$

$$K_2^1 K_3^1 K_4^1 K_5^1 K_6 =$$

$$\begin{vmatrix} x_2^1 & & \\ x_2 & x_5^1 & \\ x_2 & x_5 & x_8^1 \end{vmatrix} \wedge \begin{vmatrix} x_1^1 & x_2 & x_3^1 & x_4^1 & x_5 & x_6 & x_9 & x_{12} & & & \\ x_1 & x_2 & x_3^1 & x_4^1 & x_5 & x_6 & x_9 & x_{12} & & & \\ x_1^1 & x_2 & x_3^1 & x_4 & x_5 & x_6 & x_8^1 & x_9 & x_{12} & & \\ x_1 & x_2 & x_3^1 & x_4 & x_5 & x_6 & x_8^1 & x_9 & x_{12} & & \\ x_1^1 & x_2 & x_3^1 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10}^1 & x_{12} & \\ x_1 & x_2 & x_3^1 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10}^1 & x_{11}^1 & x_{12} \end{vmatrix}$$

$$K_2^1 K_3^1 K_4^1 K_5^1 K_6 = \begin{vmatrix} x_1^1 & x_2 & x_3^1 & x_4^1 & x_5 & x_6 & x_9 & x_{11}^1 & x_{12} & & \\ x_1 & x_2 & x_3^1 & x_4^1 & x_5 & x_6 & x_9 & x_{11}^1 & x_{12} & & \\ x_1^1 & x_2 & x_3^1 & x_4 & x_5 & x_6 & x_8^1 & x_9 & x_{11}^1 & x_{12} & \\ x_1 & x_2 & x_3^1 & x_4 & x_5 & x_6 & x_8^1 & x_9 & x_{11}^1 & x_{12} & \\ x_1^1 & x_2 & x_3^1 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10}^1 & x_{11}^1 & x_{12} \\ x_1 & x_2 & x_3^1 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10}^1 & x_{11}^1 & x_{12} \end{vmatrix}$$

$$K_1^1 K_2^1 K_3^1 K_4^1 K_5^1 K_6 =$$

$$\begin{vmatrix} x_1^1 & & \\ x_1 & x_4^1 & \\ x_1 & x_4 & x_7^1 \end{vmatrix} \wedge \begin{vmatrix} x_1^1 & x_2 & x_3^1 & x_4^1 & x_5 & x_6 & x_9 & x_{11}^1 & x_{12} & & \\ x_1 & x_2 & x_3^1 & x_4^1 & x_5 & x_6 & x_9 & x_{11}^1 & x_{12} & & \\ x_1^1 & x_2 & x_3^1 & x_4 & x_5 & x_6 & x_8^1 & x_9 & x_{11}^1 & x_{12} & \\ x_1 & x_2 & x_3^1 & x_4 & x_5 & x_6 & x_8^1 & x_9 & x_{11}^1 & x_{12} & \\ x_1^1 & x_2 & x_3^1 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10}^1 & x_{11}^1 & x_{12} \\ x_1 & x_2 & x_3^1 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10}^1 & x_{11}^1 & x_{12} \end{vmatrix}$$

$$K_1^1 K_2^1 K_3^1 K_4^1 K_5^1 K_6 = \begin{vmatrix} x_1^1 & x_2 & x_3^1 & x_4^1 & x_5 & x_6 & x_9 & x_{11}^1 & x_{12} & & \\ x_1 & x_2 & x_3^1 & x_4^1 & x_5 & x_6 & x_9 & x_{11}^1 & x_{12} & & \\ x_1^1 & x_2 & x_3^1 & x_4 & x_5 & x_6 & x_8^1 & x_9 & x_{11}^1 & x_{12} & \\ x_1 & x_2 & x_3^1 & x_4 & x_5 & x_6 & x_8^1 & x_9 & x_{10}^1 & x_{11}^1 & x_{12} \\ x_1^1 & x_2 & x_3^1 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10}^1 & x_{11}^1 & x_{12} \\ x_1 & x_2 & x_3^1 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10}^1 & x_{11}^1 & x_{12} \end{vmatrix} \quad (18)$$

$$K_6^1 K_7 = \begin{vmatrix} x_2^1 & & & & \\ x_2 & x_5^1 & & & \\ x_2 & x_5 & x_6^1 & & \\ x_2 & x_5 & x_6 & x_9^1 & \\ x_2 & x_5 & x_6 & x_9 & x_{12}^1 \end{vmatrix} \wedge \begin{vmatrix} x_3 & x_5 & x_6 & x_9 & x_{11} \end{vmatrix}$$

$$= \begin{vmatrix} x_2^1 & x_3 & x_5 & x_6 & x_9 & x_{11} & \\ x_2 & x_3 & x_5 & x_6 & x_9 & x_{11} & x_{12}^1 \end{vmatrix}$$

$$K_5^1 K_6^1 K_7 = \begin{vmatrix} x_2^1 & & & & \\ x_2 & x_4^1 & & & \\ x_2 & x_4 & x_5^1 & & \\ x_2 & x_4 & x_5 & x_8^1 & \\ x_2 & x_4 & x_5 & x_8 & x_{10}^1 \end{vmatrix} \wedge \begin{vmatrix} x_2^1 & x_3 & x_5 & x_6 & x_9 & x_{11} & \\ x_2 & x_3 & x_5 & x_6 & x_9 & x_{11} & x_{12}^1 \end{vmatrix}$$

$$K_5^1 K_6^1 K_7 = \begin{vmatrix} x_2^1 & x_3 & x_5 & x_6 & x_9 & x_{11} \\ x_2 & x_3 & x_4^1 & x_5 & x_6 & x_9 & x_{11} & x_{12}^1 \\ x_2 & x_3 & x_4 & x_5 & x_6 & x_8^1 & x_9 & x_{11} & x_{12}^1 \\ x_2 & x_3 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10}^1 & x_{11} & x_{12}^1 \end{vmatrix}$$

$$K_4^1 K_5^1 K_6^1 K_7 =$$

$$\begin{vmatrix} x_1^1 \\ x_1 & x_4^1 \\ x_1 & x_4 & x_5^1 \\ x_1 & x_4 & x_5 & x_8^1 \\ x_1 & x_4 & x_5 & x_8 & x_{11}^1 \end{vmatrix} \wedge \begin{vmatrix} x_2^1 & x_3 & x_5 & x_6 & x_9 & x_{11} \\ x_2 & x_3 & x_4^1 & x_5 & x_6 & x_9 & x_{11} & x_{12}^1 \\ x_2 & x_3 & x_4 & x_5 & x_6 & x_8^1 & x_9 & x_{11} & x_{12}^1 \\ x_2 & x_3 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10}^1 & x_{11} & x_{12}^1 \end{vmatrix}$$

$$K_4^1 K_5^1 K_6^1 K_7 = \begin{vmatrix} x_1^1 & x_2^1 & x_3 & x_5 & x_6 & x_9 & x_{11} \\ x_1 & x_2^1 & x_3 & x_4^1 & x_5 & x_6 & x_9 & x_{11} \\ x_1 & x_2^1 & x_3 & x_4 & x_5 & x_6 & x_8^1 & x_9 & x_{11} \\ x_1^1 & x_2 & x_3 & x_4^1 & x_5 & x_6 & x_9 & x_{11} & x_{12}^1 \\ x_1 & x_2 & x_3 & x_4^1 & x_5 & x_6 & x_9 & x_{11} & x_{12}^1 \\ x_1^1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_8^1 & x_9 & x_{11} & x_{12}^1 \\ x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_8^1 & x_9 & x_{11} & x_{12}^1 \\ x_1^1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10}^1 & x_{11} & x_{12}^1 \end{vmatrix}$$

$$K_3^1 K_4^1 K_5^1 K_6^1 K_7 = \begin{vmatrix} x_3^1 \\ x_3 & x_6^1 \\ x_3 & x_6 & x_9^1 \end{vmatrix} \wedge \begin{vmatrix} x_1^1 & x_2^1 & x_3 & x_5 & x_6 & x_9 & x_{11} \\ x_1 & x_2^1 & x_3 & x_4^1 & x_5 & x_6 & x_9 & x_{11} \\ x_1 & x_2^1 & x_3 & x_4 & x_5 & x_6 & x_8^1 & x_9 & x_{11} \\ x_1^1 & x_2 & x_3 & x_4^1 & x_5 & x_6 & x_9 & x_{11} & x_{12}^1 \\ x_1 & x_2 & x_3 & x_4^1 & x_5 & x_6 & x_9 & x_{11} & x_{12}^1 \\ x_1^1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_8^1 & x_9 & x_{11} & x_{12}^1 \\ x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_8^1 & x_9 & x_{11} & x_{12}^1 \\ x_1^1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10}^1 & x_{11} & x_{12}^1 \end{vmatrix}$$

$$K_3^1 K_4^1 K_5^1 K_6^1 K_7 = \begin{vmatrix} x_1^1 & x_2^1 & x_3 & x_5 & x_6 & x_9 & x_{11} & x_{12}^1 \\ x_1 & x_2^1 & x_3 & x_4^1 & x_5 & x_6 & x_9 & x_{11} & x_{12}^1 \\ x_1 & x_2^1 & x_3 & x_4 & x_5 & x_6 & x_8^1 & x_9 & x_{11} & x_{12}^1 \end{vmatrix}$$

$$K_2^1 K_3^1 K_4^1 K_5^1 K_6^1 K_7 =$$

$$\begin{vmatrix} x_2^1 \\ x_2 & x_5^1 \\ x_2 & x_5 & x_{11}^1 \end{vmatrix} \wedge \begin{vmatrix} x_1^1 & x_2^1 & x_3 & x_5 & x_6 & x_9 & x_{11} & x_{12}^1 \\ x_1 & x_2^1 & x_3 & x_4^1 & x_5 & x_6 & x_9 & x_{11} & x_{12}^1 \\ x_1 & x_2^1 & x_3 & x_4 & x_5 & x_6 & x_8^1 & x_9 & x_{11} & x_{12}^1 \end{vmatrix}$$

$$= \begin{vmatrix} x_1^1 & x_2^1 & x_3 & x_5 & x_6 & x_9 & x_{11} & x_{12}^1 \\ x_1 & x_2^1 & x_3 & x_4^1 & x_5 & x_6 & x_9 & x_{11} & x_{12}^1 \\ x_1 & x_2^1 & x_3 & x_4 & x_5 & x_6 & x_8^1 & x_9 & x_{11} & x_{12}^1 \end{vmatrix}$$

$$K_1^1\,K_2^1\,K_3^1\,K_4^1\,K_5^1\,K_6^1\,K_7 =$$

$$\left|\begin{array}{lll} x_1^1 & & \\ x_1 & x_4^1 & \\ x_1 & x_4 & x_7^1 \end{array}\right| \wedge \left|\begin{array}{llllllllll} x_1^1 & x_2^1 & x_3 & x_5 & x_6 & x_9 & x_{11} & x_{12}^1 & & \\ x_1 & x_2^1 & x_3 & x_4^1 & x_5 & x_6 & x_9 & x_{11} & x_{12}^1 & \\ x_1 & x_2^1 & x_3 & x_4 & x_5 & x_6 & x_8^1 & x_9 & x_{11} & x_{12}^1 \end{array}\right|$$

$$K_1^1\,K_2^1\,K_3^1\,K_4^1\,K_5^1\,K_6^1\,K_7 = \left|\begin{array}{lllllllllll} x_1^1 & x_2^1 & x_3 & x_5 & x_6 & x_9 & x_{11} & x_{12}^1 & & & \\ x_1 & x_2^1 & x_3 & x_4^1 & x_5 & x_6 & x_9 & x_{11} & x_{12}^1 & & \\ x_1 & x_2^1 & x_3 & x_4 & x_5 & x_6 & x_8^1 & x_9 & x_{10}^1 & x_{11} & x_{12}^1 \end{array}\right| \qquad (19)$$

$$K_7^1\,K_8 = \left|\begin{array}{lllll} x_3^1 & & & & \\ x_3 & x_5^1 & & & \\ x_3 & x_5 & x_6^1 & & \\ x_3 & x_5 & x_6 & x_9^1 & \\ x_3 & x_5 & x_6 & x_9 & x_{11}^1 \end{array}\right| \wedge \left|\begin{array}{lllllll} x_1 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{12} \end{array}\right|$$

$$= \left|\begin{array}{lllllllll} x_1 & x_3^1 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{12} & \\ x_1 & x_3 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{11}^1 & x_{12} \end{array}\right|$$

$$K_6^1\,K_7^1\,K_8 = \left|\begin{array}{lllll} x_2^1 & & & & \\ x_2 & x_5^1 & & & \\ x_2 & x_5 & x_6^1 & & \\ x_2 & x_5 & x_6 & x_9^1 & \\ x_2 & x_5 & x_6 & x_9 & x_{12}^1 \end{array}\right| \wedge \left|\begin{array}{lllllllll} x_1 & x_3^1 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{12} & \\ x_1 & x_3 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{11}^1 & x_{12} \end{array}\right|$$

$$= \left|\begin{array}{llllllllll} x_1 & x_2^1 & x_3^1 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{12} & \\ x_1 & x_2^1 & x_3 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{11}^1 & x_{12} \end{array}\right|$$

$$K_5^1\,K_6^1\,K_7^1\,K_8 =$$

$$\left|\begin{array}{lllll} x_2^1 & & & & \\ x_2 & x_4^1 & & & \\ x_2 & x_4 & x_5^1 & & \\ x_2 & x_4 & x_5 & x_8^1 & \\ x_2 & x_4 & x_5 & x_8 & x_{10}^1 \end{array}\right| \wedge \left|\begin{array}{llllllllll} x_1 & x_2^1 & x_3^1 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{12} & \\ x_1 & x_2^1 & x_3 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{11}^1 & x_{12} \end{array}\right|$$

$$K_5^1\,K_6^1\,K_7^1\,K_8 = \left|\begin{array}{llllllllll} x_1 & x_2^1 & x_3^1 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{12} & \\ x_1 & x_2^1 & x_3 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{11}^1 & x_{12} \end{array}\right|$$

$K_4^1 K_5^1 K_6^1 K_7^1 K_8 =$

$$\left| \begin{array}{lllll} x_1^1 & & & & \\ x_1 & x_4^1 & & & \\ x_1 & x_4 & x_5^1 & & \\ x_1 & x_4 & x_5 & x_8^1 & \\ x_1 & x_4 & x_5 & x_8 & x_{11}^1 \end{array} \right| \wedge \left| \begin{array}{llllllllll} x_1 & x_2^1 & x_3^1 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{12} & \\ x_1 & x_2^1 & x_3 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{11}^1 & x_{12} \end{array} \right|$$

$$= \left| \begin{array}{llllllllll} x_1 & x_2^1 & x_3^1 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{11}^1 & x_{12} \\ x_1 & x_2^1 & x_3 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{11}^1 & x_{12} \end{array} \right|$$

$K_3^1 K_4^1 K_5^1 K_6^1 K_7^1 K_8 =$

$$\left| \begin{array}{lll} x_3^1 & & \\ x_3 & x_6^1 & \\ x_3 & x_6 & x_{12}^1 \end{array} \right| \wedge \left| \begin{array}{llllllllll} x_1 & x_2^1 & x_3^1 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{11}^1 & x_{12} \\ x_1 & x_2^1 & x_3 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{11}^1 & x_{12} \end{array} \right|$$

$$= \left| x_1 \quad x_2^1 \quad x_3^1 \quad x_4 \quad x_5 \quad x_6 \quad x_8 \quad x_9 \quad x_{11}^1 \quad x_{12} \right|$$

$K_2^1 K_3^1 K_4^1 K_5^1 K_6^1 K_7^1 K_8 =$

$$\left| \begin{array}{lll} x_2^1 & & \\ x_2 & x_5^1 & \\ x_2 & x_5 & x_{11}^1 \end{array} \right| \wedge \left| x_1 \quad x_2^1 \quad x_3^1 \quad x_4 \quad x_5 \quad x_6 \quad x_8 \quad x_9 \quad x_{11}^1 \quad x_{12} \right|$$

$$K_2^1 K_3^1 K_4^1 K_5^1 K_6^1 K_7^1 K_8 = \left| x_1 \quad x_2^1 \quad x_3^1 \quad x_4 \quad x_5 \quad x_6 \quad x_8 \quad x_9 \quad x_{11}^1 \quad x_{12} \right|$$

$K_1^1 K_2^1 K_3^1 K_4^1 K_5^1 K_6^1 K_7^1 K_8 =$

$$\left| \begin{array}{lll} x_1^1 & & \\ x_1 & x_4^1 & \\ x_1 & x_4 & x_{10}^1 \end{array} \right| \wedge \left| x_1 \quad x_2^1 \quad x_3^1 \quad x_4 \quad x_5 \quad x_6 \quad x_8 \quad x_9 \quad x_{11}^1 \quad x_{12} \right|$$

$$K_1^1 K_2^1 K_3^1 K_4^1 K_5^1 K_6^1 K_7^1 K_8 = \left| x_1 \quad x_2^1 \quad x_3^1 \quad x_4 \quad x_5 \quad x_6 \quad x_8 \quad x_9 \quad x_{10}^1 \quad x_{11}^1 \quad x_{12} \right| \qquad (20)$$

$$K_8^1 K_9 = \left| \begin{array}{lllllll} x_1^1 & & & & & & \\ x_1 & x_4^1 & & & & & \\ x_1 & x_4 & x_5^1 & & & & \\ x_1 & x_4 & x_5 & x_6^1 & & & \\ x_1 & x_4 & x_5 & x_6 & x_8^1 & & \\ x_1 & x_4 & x_5 & x_6 & x_8 & x_9^1 & \\ x_1 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{12}^1 \end{array} \right| \wedge \left| x_3 \quad x_4 \quad x_5 \quad x_6 \quad x_8 \quad x_9 \quad x_{10} \right|$$

$$= \left| \begin{array}{lllllllll} x_1^1 & x_3 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10} & \\ x_1 & x_3 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10} & x_{12}^1 \end{array} \right|$$

$$K_7^1 K_8^1 K_9 = \begin{vmatrix} x_3^1 & & & & \\ x_3 & x_5^1 & & & \\ x_3 & x_5 & x_6^1 & & \\ x_3 & x_5 & x_6 & x_9^1 & \\ x_3 & x_5 & x_6 & x_9 & x_{11}^1 \end{vmatrix} \wedge \begin{vmatrix} x_1^1 & x_3 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10} & \\ x_1 & x_3 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10} & x_{12}^1 \end{vmatrix}$$

$$= \begin{vmatrix} x_1^1 & x_3 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10} & x_{11}^1 & \\ x_1 & x_3 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10} & x_{11}^1 & x_{12}^1 \end{vmatrix}$$

$$K_6^1 K_7^1 K_8^1 K_9 =$$

$$\begin{vmatrix} x_2^1 & & & & \\ x_2 & x_5^1 & & & \\ x_2 & x_5 & x_6^1 & & \\ x_2 & x_5 & x_6 & x_9^1 & \\ x_2 & x_5 & x_6 & x_9 & x_{12}^1 \end{vmatrix} \wedge \begin{vmatrix} x_1^1 & x_3 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10} & x_{11}^1 & \\ x_1 & x_3 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10} & x_{11}^1 & x_{12}^1 \end{vmatrix}$$

$$K_6^1 K_7^1 K_8^1 K_9 = \begin{vmatrix} x_1^1 & x_2^1 & x_3 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10} & x_{11}^1 & \\ x_1^1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10} & x_{11}^1 & x_{12}^1 \\ x_1 & x_2^1 & x_3 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10} & x_{11}^1 & x_{12}^1 \\ x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10} & x_{11}^1 & x_{12}^1 \end{vmatrix}$$

$$K_5^1 K_6^1 K_7^1 K_8^1 K_9 =$$

$$\begin{vmatrix} x_2^1 & & & & \\ x_2 & x_4^1 & & & \\ x_2 & x_4 & x_5^1 & & \\ x_2 & x_4 & x_5 & x_8^1 & \\ x_2 & x_4 & x_5 & x_8 & x_{10}^1 \end{vmatrix} \wedge \begin{vmatrix} x_1^1 & x_2^1 & x_3 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10} & x_{11}^1 & \\ x_1^1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10} & x_{11}^1 & x_{12}^1 \\ x_1 & x_2^1 & x_3 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10} & x_{11}^1 & x_{12}^1 \\ x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10} & x_{11}^1 & x_{12}^1 \end{vmatrix}$$

$$= \begin{vmatrix} x_1^1 & x_2^1 & x_3 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10} & x_{11}^1 & \\ x_1 & x_2^1 & x_3 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10} & x_{11}^1 & x_{12}^1 \end{vmatrix}$$

$$K_4^1 K_5^1 K_6^1 K_7^1 K_8^1 K_9 =$$

$$\begin{vmatrix} x_1^1 & & & & \\ x_1 & x_4^1 & & & \\ x_1 & x_4 & x_5^1 & & \\ x_1 & x_4 & x_5 & x_8^1 & \\ x_1 & x_4 & x_5 & x_8 & x_{11}^1 \end{vmatrix} \wedge \begin{vmatrix} x_1^1 & x_2^1 & x_3 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10} & x_{11}^1 & \\ x_1 & x_2^1 & x_3 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10} & x_{11}^1 & x_{12}^1 \end{vmatrix}$$

$$= \begin{vmatrix} x_1^1 & x_2^1 & x_3 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10} & x_{11}^1 & \\ x_1 & x_2^1 & x_3 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10} & x_{11}^1 & x_{12}^1 \end{vmatrix}$$

$$K_3^1 K_4^1 K_5^1 K_6^1 K_7^1 K_8^1 K_9 =$$

$$\begin{vmatrix} x_3^1 & & \\ x_3 & x_6^1 & \\ x_3 & x_6 & x_{12}^1 \end{vmatrix} \wedge \begin{vmatrix} x_1^1 & x_2^1 & x_3 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10} & x_{11}^1 & \\ x_1 & x_2^1 & x_3 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10} & x_{11}^1 & x_{12}^1 \end{vmatrix}$$

$$K_3^1 K_4^1 K_5^1 K_6^1 K_7^1 K_8^1 K_9 = \begin{vmatrix} x_1^1 & x_2^1 & x_3 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10} & x_{11}^1 & x_{12}^1 \\ x_1 & x_2^1 & x_3 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10} & x_{11}^1 & x_{12}^1 \end{vmatrix}$$

$$K_2^1 K_3^1 K_4^1 K_5^1 K_6^1 K_7^1 K_8^1 K_9 =$$

$$\begin{vmatrix} x_2^1 & & \\ x_2 & x_5^1 & \\ x_2 & x_5 & x_{11}^1 \end{vmatrix} \wedge \begin{vmatrix} x_1^1 & x_2^1 & x_3 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10} & x_{11}^1 & x_{12}^1 \\ x_1 & x_2^1 & x_3 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10} & x_{11}^1 & x_{12}^1 \end{vmatrix}$$

$$= \begin{vmatrix} x_1^1 & x_2^1 & x_3 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10} & x_{11}^1 & x_{12}^1 \\ x_1 & x_2^1 & x_3 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10} & x_{11}^1 & x_{12}^1 \end{vmatrix}$$

$$K_1^1 K_2^1 K_3^1 K_4^1 K_5^1 K_6^1 K_7^1 K_8^1 K_9 =$$

$$\begin{vmatrix} x_1^1 & & \\ x_1 & x_4^1 & \\ x_1 & x_4 & x_{10}^1 \end{vmatrix} \wedge \begin{vmatrix} x_1^1 & x_2^1 & x_3 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10} & x_{11}^1 & x_{12}^1 \\ x_1 & x_2^1 & x_3 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10} & x_{11}^1 & x_{12}^1 \end{vmatrix}$$

$$K_1^1 K_2^1 K_3^1 K_4^1 K_5^1 K_6^1 K_7^1 K_8^1 K_9 = \begin{vmatrix} x_1^1 & x_2^1 & x_3 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10} & x_{11}^1 & x_{12}^1 \end{vmatrix} \qquad (21)$$

Making use of (14) to (21) in equation (4), one can obtain

$$
n(x_1, x_2, x_3, x_4, x_5, x_6, x_8, x_9, x_{10}, x_{11}, x_{12}) = \left|
\begin{array}{lllllllllll}
x_1 & x_4 & x_{10} \\
x_1^1 & x_2 & x_5 & x_{11} \\
x_1 & x_2 & x_4^1 & x_5 & x_{11} \\
x_1 & x_2 & x_4 & x_5 & x_{10}^1 & x_{11} \\
x_1^1 & x_2^1 & x_3 & x_6 & x_{12} \\
x_1 & x_2^1 & x_3 & x_4^1 & x_6 & x_{12} \\
x_1 & x_2^1 & x_3 & x_4 & x_6 & x_{10}^1 & x_{12} \\
x_1^1 & x_2 & x_3 & x_5^1 & x_6 & x_{12} \\
x_1 & x_2 & x_3 & x_4^1 & x_5^1 & x_6 & x_{12} \\
x_1 & x_2 & x_3 & x_4 & x_5^1 & x_6 & x_{10}^1 & x_{12} \\
x_1^1 & x_2 & x_3 & x_5 & x_6 & x_{11}^1 & x_{12} \\
x_1 & x_2 & x_3 & x_4^1 & x_5 & x_6 & x_{11}^1 & x_{12} \\
x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_{10}^1 & x_{11}^1 & x_{12} \\
x_1 & x_2^1 & x_3^1 & x_4 & x_5 & x_8 & x_{10}^1 & x_{11} \\
x_1 & x_2^1 & x_3 & x_4 & x_5 & x_6^1 & x_8 & x_{10}^1 & x_{11} \\
x_1 & x_2^1 & x_3 & x_4 & x_5 & x_6 & x_8 & x_{10}^1 & x_{11} & x_{12}^1 \\
x_1^1 & x_2 & x_3 & x_4 & x_5 & x_8 & x_{10} & x_{11}^1 \\
x_1^1 & x_2 & x_3 & x_4 & x_5 & x_6^1 & x_8 & x_{10} & x_{11}^1 \\
x_1^1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_8 & x_{10} & x_{11}^1 & x_{12}^1 \\
x_1^1 & x_2 & x_3^1 & x_4^1 & x_5 & x_6 & x_9 & x_{11}^1 & x_{12} \\
x_1 & x_2 & x_3^1 & x_4^1 & x_5 & x_6 & x_9 & x_{11}^1 & x_{12} \\
x_1^1 & x_2 & x_3^1 & x_4 & x_5 & x_6 & x_8^1 & x_9 & x_{11}^1 & x_{12} \\
x_1 & x_2 & x_3^1 & x_4 & x_5 & x_6 & x_8^1 & x_9 & x_{10}^1 & x_{11}^1 & x_{12} \\
x_1^1 & x_2 & x_3^1 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10}^1 & x_{11}^1 & x_{12} \\
x_1 & x_2 & x_3^1 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10}^1 & x_{11}^1 & x_{12} \\
x_1^1 & x_2^1 & x_3 & x_5 & x_6 & x_9 & x_{11} & x_{12}^1 \\
x_1 & x_2^1 & x_3 & x_4^1 & x_5 & x_6 & x_9 & x_{11} & x_{12}^1 \\
x_1 & x_2^1 & x_3 & x_4 & x_5 & x_6 & x_8^1 & x_9 & x_{10}^1 & x_{11} & x_{12}^1 \\
x_1 & x_2^1 & x_3^1 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10}^1 & x_{11}^1 & x_{12} \\
x_1^1 & x_2^1 & x_3 & x_4 & x_5 & x_6 & x_8 & x_9 & x_{10} & x_{11}^1 & x_{12}^1
\end{array}
\right|
$$

$$f(x_1, x_2, \ldots\ldots\ldots x_{12}) = \left| \begin{array}{l}
x_1\ x_4\ x_7\ x_{10} \\
x_1^1\ x_2\ x_5\ x_7\ x_{11} \\
x_1\ x_2\ x_4^1\ x_5\ x_7\ x_{11} \\
x_1\ x_2\ x_4\ x_5\ x_7\ x_{10}^1\ x_{11} \\
x_1^1\ x_2^1\ x_3\ x_6\ x_7\ x_{12} \\
x_1\ x_2^1\ x_3\ x_4^1\ x_6\ x_7\ x_{12} \\
x_1\ x_2^1\ x_3\ x_4\ x_6\ x_7\ x_{10}^1\ x_{12} \\
x_1^1\ x_2\ x_3\ x_5^1\ x_6\ x_7\ x_{12} \\
x_1\ x_2\ x_3\ x_4^1\ x_5^1\ x_6\ x_7\ x_{12} \\
x_1\ x_2\ x_3\ x_4\ x_5^1\ x_6\ x_7\ x_{10}^1\ x_{12} \\
x_1^1\ x_2\ x_3\ x_5\ x_6\ x_7\ x_{11}^1\ x_{12} \\
x_1\ x_2\ x_3\ x_4^1\ x_5\ x_6\ x_7\ x_{11}^1\ x_{12} \\
x_1\ x_2\ x_3\ x_4\ x_5\ x_6\ x_7\ x_{10}^1\ x_{11}^1\ x_{12} \\
x_1\ x_2^1\ x_3^1\ x_4\ x_5\ x_7\ x_8\ x_{10}^1\ x_{11} \\
x_1\ x_2^1\ x_3\ x_4\ x_5\ x_6^1\ x_7\ x_8\ x_{10}^1\ x_{11} \\
x_1\ x_2^1\ x_3\ x_4\ x_5\ x_6\ x_7\ x_8\ x_{10}^1\ x_{11}\ x_{12}^1 \\
x_1^1\ x_2\ x_3^1\ x_4\ x_5\ x_7\ x_8\ x_{10}\ x_{11}^1 \\
x_1^1\ x_2\ x_3\ x_4\ x_5\ x_6^1\ x_7\ x_8\ x_{10}\ x_{11}^1 \\
x_1^1\ x_2\ x_3\ x_4\ x_5\ x_6\ x_7\ x_8\ x_{10}\ x_{11}^1\ x_{12}^1 \\
x_1^1\ x_2\ x_3^1\ x_4^1\ x_5\ x_6\ x_7\ x_9\ x_{11}^1\ x_{12} \\
x_1\ x_2\ x_3^1\ x_4^1\ x_5\ x_6\ x_7\ x_9\ x_{11}^1\ x_{12} \\
x_1^1\ x_2\ x_3^1\ x_4\ x_5\ x_6\ x_7\ x_8^1\ x_9\ x_{11}^1\ x_{12} \\
x_1\ x_2\ x_3^1\ x_4\ x_5\ x_6\ x_7\ x_8^1\ x_9\ x_{10}^1\ x_{11}^1\ x_{12} \\
x_1^1\ x_2\ x_3^1\ x_4\ x_5\ x_6\ x_7\ x_8\ x_9\ x_{10}^1\ x_{11}^1\ x_{12} \\
x_1\ x_2\ x_3^1\ x_4\ x_5\ x_6\ x_7\ x_8\ x_9\ x_{10}^1\ x_{11}^1\ x_{12} \\
x_1^1\ x_2\ x_3\ x_5\ x_6\ x_7\ x_9\ x_{11}\ x_{12}^1 \\
x_1\ x_2^1\ x_3\ x_4^1\ x_5\ x_6\ x_7\ x_9\ x_{11}\ x_{12}^1 \\
x_1\ x_2^1\ x_3\ x_4\ x_5\ x_6\ x_7\ x_8^1\ x_9\ x_{10}^1\ x_{11}\ x_{12}^1 \\
x_1\ x_2^1\ x_3^1\ x_4\ x_5\ x_6\ x_7\ x_8\ x_9\ x_{10}^1\ x_{11}^1\ x_{12} \\
x_1^1\ x_2^1\ x_3\ x_4\ x_5\ x_6\ x_7\ x_8\ x_9\ x_{10}\ x_{11}^1\ x_{12}^1
\end{array} \right|$$

Finally, the probability of the successful operation (i.e. reliability) of the complex system is given by

$$R_S = P_r\{f(X_1, X_2, X_3, \ldots\ldots\ldots, X_{10}, X_{11}, X_{12}) = 1\}$$

$$= R_1 R_4 R_7 R_{10} + Q_1 R_2 R_5 R_7 R_{11} + R_1 R_2 Q_4 R_5 R_7 R_{11} + R_1 R_2 R_4 R_5 R_7 Q_{10} R_{11} +$$

$$Q_1 Q_2 R_3 R_6 R_7 R_{12} + R_1 Q_2 R_3 Q_4 R_6 R_7 R_{12} + R_1 Q_2 R_3 R_4 R_6 R_7 Q_{10} R_{12}$$

$$Q_1 R_2 R_3 Q_5 R_6 R_7 R_{12} + R_1 R_2 R_3 Q_4 Q_5 R_6 R_7 R_{12} + R_1 R_2 R_3 R_4 Q_5 R_6 R_7 Q_{10} R_{12} +$$

$$
\begin{aligned}
& Q_1 R_2 R_3 R_5 R_6 R_7 Q_{11} R_{12} + R_1 R_2 R_3 Q_4 R_5 R_6 R_7 Q_{11} R_{12} + R_1 Q_2 Q_3 R_4 R_5 R_7 R_8 Q_{10} R_{11} \\
& R_1 R_2 R_3 R_4 R_5 R_6 R_7 Q_{10} Q_{11} R_{12} + R_1 Q_2 R_3 R_4 R_5 Q_6 R_7 R_8 Q_{10} R_{11} + \\
& R_1 Q_2 R_3 R_4 R_5 R_6 R_7 R_8 Q_{10} R_{11} Q_{12} + Q_1 R_2 Q_3 R_4 R_5 R_7 R_8 R_{10} Q_{11} + \\
& Q_1 R_2 R_3 R_4 R_5 Q_6 R_7 R_8 R_{10} Q_{11} + Q_1 R_2 R_3 R_4 R_5 R_6 R_7 R_8 R_{10} Q_{11} Q_{12} + \\
& Q_1 R_2 Q_3 Q_4 R_5 R_6 R_7 R_9 Q_{11} R_{12} + R_1 R_2 Q_3 Q_4 R_5 R_6 R_7 R_9 Q_{11} R_{12} + \\
& Q_1 R_2 Q_3 R_4 R_5 R_6 R_7 Q_8 R_9 Q_{11} R_{12} + R_1 R_2 Q_3 R_4 R_5 R_6 R_7 Q_8 R_9 Q_{10} Q_{11} R_{12} + \\
& Q_1 R_2 Q_3 R_4 R_5 R_6 R_7 R_8 R_9 Q_{10} Q_{11} R_{12} + R_1 R_2 Q_3 R_4 R_5 R_6 R_7 R_8 R_9 Q_{10} Q_{11} R_{12} + \\
& Q_1 R_2 R_3 R_5 R_6 R_7 R_9 R_{11} Q_{12} + R_1 Q_2 R_3 Q_4 R_5 R_6 R_7 R_9 R_{11} Q_{12} + \\
& R_1 Q_2 R_3 R_4 R_5 R_6 R_7 Q_8 R_9 Q_{10} R_{11} Q_{12} + R_1 Q_2 Q_3 R_4 R_5 R_6 R_7 R_8 R_9 Q_{10} Q_{11} R_{12} + \\
& Q_1 Q_2 R_3 R_4 R_5 R_6 R_7 R_8 R_9 R_{10} Q_{11} Q_{12} . \qquad (22)
\end{aligned}
$$

$$
\begin{aligned}
R_S(t) = {} & R_1 R_4 R_7 R_{10} + R_2 R_5 R_7 R_{11} - R_1 R_2 R_4 R_5 R_7 R_{10} R_{11} + R_3 R_6 R_7 R_{12} \\
& - R_1 R_3 R_4 R_6 R_7 R_{10} R_{12} - R_2 R_3 R_5 R_6 R_7 R_{11} R_{12} + R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_{10} R_{11} R_{12} \\
& + R_1 R_4 R_5 R_7 R_8 R_{11} - R_1 R_2 R_4 R_5 R_7 R_8 R_{11} - R_1 R_4 R_5 R_7 R_8 R_{10} R_{11} \\
& + 2R_1 R_2 R_4 R_5 R_7 R_8 R_{10} R_{11} + R_1 R_3 R_4 R_5 R_6 R_7 R_8 R_{10} - R_1 R_3 R_4 R_5 R_6 R_7 R_8 R_{10} R_{11} \\
& - 2 R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_8 R_{10} R_{11} R_{12} + R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_8 R_{11} R_{12} \\
& + R_1 R_3 R_4 R_5 R_6 R_7 R_8 R_{10} R_{11} R_{12} - R_1 R_3 R_4 R_5 R_6 R_7 R_8 R_{11} R_{12} + R_2 R_4 R_5 R_7 R_8 R_{10} \\
& - R_1 R_2 R_4 R_5 R_7 R_8 R_{10} - R_2 R_4 R_5 R_7 R_8 R_{10} R_{11} - R_2 R_3 R_4 R_5 R_6 R_7 R_8 R_{10} R_{12} \\
& + R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_8 R_{10} R_{12} + R_2 R_3 R_4 R_5 R_6 R_7 R_8 R_{10} R_{11} R_{12} + R_2 R_5 R_6 R_7 R_9 R_{12} \\
& - R_2 R_5 R_6 R_7 R_9 R_{11} R_{12} - R_2 R_3 R_5 R_6 R_7 R_9 R_{12} - R_2 R_3 R_4 R_5 R_6 R_7 R_8 R_9 R_{11} R_{12} \\
& + R_1 R_2 R_4 R_5 R_6 R_7 R_9 R_{10} R_{11} R_{12} - R_1 R_2 R_4 R_5 R_6 R_7 R_9 R_{10} R_{12} \\
& - 2 R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_9 R_{10} R_{11} R_{12} + R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_9 R_{10} R_{12} \\
& + R_2 R_4 R_5 R_6 R_7 R_8 R_9 R_{10} R_{11} R_{12} - R_2 R_4 R_5 R_6 R_7 R_8 R_9 R_{10} R_{12} \\
& + 4 R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_8 R_9 R_{10} R_{11} R_{12} - 3 R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_8 R_9 R_{10} R_{12} \\
& - R_1 R_2 R_4 R_5 R_6 R_7 R_8 R_9 R_{12} - 2 R_1 R_2 R_4 R_5 R_6 R_7 R_8 R_9 R_{10} R_{11} R_{12} \\
& + 2 R_1 R_2 R_4 R_5 R_6 R_7 R_8 R_9 R_{10} R_{12} + R_1 R_2 R_4 R_5 R_6 R_7 R_8 R_9 R_{11} R_{12} \\
& - 2 R_2 R_3 R_4 R_5 R_6 R_7 R_8 R_9 R_{10} R_{11} R_{12} + 2 R_2 R_3 R_4 R_5 R_6 R_7 R_8 R_9 R_{10} R_{12} \\
& + R_2 R_3 R_4 R_5 R_6 R_7 R_8 R_9 R_{11} R_{12} + R_2 R_3 R_5 R_6 R_7 R_9 R_{11} \\
& + 2 R_1 R_2 R_3 R_5 R_6 R_7 R_9 R_{11} R_{12} - R_1 R_2 R_3 R_5 R_6 R_7 R_9 R_{11} \\
& + R_1 R_3 R_5 R_6 R_7 R_9 R_{11} + R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_9 R_{11} - R_1 R_2 R_3 R_5 R_6 R_7 R_8 R_{11} \\
& + R_1 R_3 R_5 R_6 R_7 R_9 R_{11} + R_1 R_3 R_4 R_5 R_6 R_7 R_9 R_{10} R_{11} R_{12} \\
& - R_1 R_3 R_4 R_5 R_6 R_7 R_9 R_{10} R_{11} + R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_8 R_9 R_{11} \\
& - R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_8 R_9 R_{10} R_{11} - R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_9 R_{11} \\
& + R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_9 R_{10} R_{11} - R_1 R_3 R_4 R_5 R_6 R_7 R_8 R_9 R_{11} \\
& - 2 R_1 R_3 R_4 R_5 R_6 R_7 R_8 R_9 R_{10} R_{11} R_{12} + 2 R_1 R_3 R_4 R_5 R_6 R_7 R_8 R_9 R_{10} R_{11} \\
& + 2 R_1 R_3 R_4 R_5 R_6 R_7 R_8 R_9 R_{11} R_{12} - 2 R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_8 R_9 R_{11} R_{12} \\
& + R_1 R_4 R_5 R_6 R_7 R_8 R_9 R_{12} + R_1 R_4 R_5 R_6 R_7 R_8 R_9 R_{10} R_{11} R_{12} \\
& - R_1 R_4 R_5 R_6 R_7 R_8 R_9 R_{10} R_{12} - R_1 R_4 R_5 R_6 R_7 R_8 R_9 R_{11} R_{12} \\
& - R_1 R_3 R_4 R_5 R_6 R_7 R_8 R_9 R_{12} + 2 R_1 R_3 R_4 R_5 R_6 R_7 R_8 R_9 R_{10} R_{12} \\
& + R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_8 R_9 R_{12} + R_3 R_4 R_5 R_6 R_7 R_8 R_9 R_{10} \\
& + R_3 R_4 R_5 R_6 R_7 R_8 R_9 R_{10} R_{11} R_{12} - R_3 R_4 R_5 R_6 R_7 R_8 R_9 R_{10} R_{11}
\end{aligned}
$$

$$-R_3 R_4 R_5 R_6 R_7 R_8 R_9 R_{10} R_{12} + R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_8 R_9 R_{10}$$
$$-R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_8 R_9 R_{10} R_{11} - R_1 R_3 R_4 R_5 R_6 R_7 R_8 R_9 R_{10}$$
$$-R_1 R_3 R_4 R_5 R_6 R_7 R_8 R_9 R_{10} R_{11} R_{12} - R_2 R_3 R_4 R_5 R_6 R_7 R_8 R_9 R_{10}$$
$$+R_2 R_3 R_4 R_5 R_6 R_7 R_8 R_9 R_{10} R_{11} \quad (23)$$

Where, $R_1, R_2, R_3, R_4, R_5, R_6, R_7, R_8, R_9, R_{10}, R_{11}, R_{12}$ are reliabilities of the generators G_1, G_2 and G_3 switching devices MSB_1, MSB_2, MSB_3 and $OPMS_4$ and cables C_1, C_2, C_3, C_4 and C_5 respectively and $Q_i\, s$ $(i = 1-12)$, are corresponding unreliability's.

7. PARTICULAR CASES:

CASE 1:

If reliabilities of each component of the complex system is R, equation (23) yields

$$R_S = 3R^4 + 3R^6 - 7R^7 + 2R^8 - 13R^9 + 27R^{10} - 18R^{11} + 4R^{12} \quad (24)$$

CASE 2: When failure rates follows Weibull Distribution:

Let failure rates of the generators G_1, G_2 and G_3 switching devices MSB_1, MSB_2, MSB_3 and $OPMS_4$ and cables C_1, C_2, C_3, C_4 and C_5 be $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7, \lambda_8, \lambda_9$ $\lambda_{10}, \lambda_{11}, \lambda_{12}$ respectively, then from equation (23) reliability of the complex system at an instant t is given by

$$\begin{aligned}
R_{SW}(t) = {} & \exp(-a_1 t^p) + \exp(-a_2 t^p) - \exp(-a_3 t^p) + \exp(-a_4 t^p) \\
& - \exp(-a_5 t^p) - \exp(-a_6 t^p) + \exp(-a_7 t^p) + \exp(-a_8 t^p) \\
& - \exp(-a_9 t^p) - \exp(-a_{10} t^p) + 2\exp(-a_{11} t^p) + \exp(-a_{12} t^p) \\
& - \exp(-a_{13} t^p) - 2\exp(-a_{14} t^p) + \exp(-a_{15} t^p) + \exp(-a_{16} t^p) \\
& - \exp(-a_{17} t^p) + \exp(-a_{18} t^p) - \exp(-a_{19} t^p) - \exp(-a_{20} t^p) \\
& - \exp(-a_{21} t^p) + \exp(-a_{22} t^p) + \exp(-a_{23} t^p) + \exp(-a_{24} t^p) \\
& - \exp(-a_{25} t^p) - \exp(-a_{26} t^p) - \exp(-a_{27} t^p) + \exp(-a_{28} t^p) \\
& - \exp(-a_{29} t^p) - 2\exp(-a_{30} t^p) + \exp(-a_{31} t^p) + \exp(-a_{32} t^p) \\
& - \exp(-a_{33} t^p) + 4\exp(-a_{34} t^p) - 3\exp(-a_{35} t^p) - \exp(-a_{36} t^p) \\
& - 2\exp(-a_{37} t^p) + 2\exp(-a_{38} t^p) + \exp(-a_{39} t^p) - 2\exp(-a_{40} t^p)
\end{aligned}$$

$$+2\exp\left(-a_{41}t^{p}\right)+\exp\left(-a_{42}t^{p}\right)+\exp\left(-a_{43}t^{p}\right)+2\exp\left(-a_{44}t^{p}\right)$$

$$-\exp\left(-a_{45}t^{p}\right)+\exp\left(-a_{46}t^{p}\right)+\exp\left(-a_{47}t^{p}\right)-\exp\left(-a_{48}t^{p}\right)$$

$$+\exp\left(-a_{49}t^{p}\right)+\exp\left(-a_{50}t^{p}\right)-\exp\left(-a_{51}t^{p}\right)+\exp\left(-a_{52}t^{p}\right)$$

$$-\exp\left(-a_{53}t^{p}\right)-\exp\left(-a_{54}t^{p}\right)+\exp\left(-a_{55}t^{p}\right)-\exp\left(-a_{56}t^{p}\right)$$

$$-2\exp\left(-a_{57}t^{p}\right)+2\exp\left(-a_{58}t^{p}\right)+2\exp\left(-a_{59}t^{p}\right)-2\exp\left(-a_{60}t^{p}\right)$$

$$+\exp\left(-a_{61}t^{p}\right)+\exp\left(-a_{62}t^{p}\right)-\exp\left(-a_{63}t^{p}\right)-\exp\left(-a_{64}t^{p}\right)$$

$$-\exp\left(-a_{65}t^{p}\right)+2\exp\left(-a_{66}t^{p}\right)+\exp\left(-a_{67}t^{p}\right)+\exp\left(-a_{68}t^{p}\right)$$

$$+\exp\left(-a_{69}t^{p}\right)-\exp\left(-a_{70}t^{p}\right)-\exp\left(-a_{71}t^{p}\right)+\exp\left(-a_{72}t^{p}\right)$$

$$-\exp\left(-a_{73}t^{p}\right)-\exp\left(-a_{74}t^{p}\right)-\exp\left(-a_{75}t^{p}\right)-\exp\left(-a_{76}t^{p}\right)$$

$$+\exp\left(-a_{77}t^{p}\right) \qquad (25)$$

Where p is a positive parameter and a_i are given by

$a_1=\lambda_1+\lambda_4+\lambda_7+\lambda_{10}$; $a_2=\lambda_2+\lambda_5+\lambda_7+\lambda_{11}$; $a_3=\lambda_1+\lambda_2+\lambda_4+\lambda_5+\lambda_7+\lambda_{10}+\lambda_{11}$

$a_4=\lambda_3+\lambda_6+\lambda_7+\lambda_{12}$; $a_5=\lambda_1+\lambda_3+\lambda_4+\lambda_6+\lambda_7+\lambda_{10}+\lambda_{12}$; $a_6=\lambda_2+\lambda_3+\lambda_5+\lambda_6+\lambda_7+\lambda_{11}+\lambda_{12}$

$a_7=\lambda_1+\lambda_2+\lambda_3+\lambda_4+\lambda_5+\lambda_6+\lambda_7+\lambda_{10}+\lambda_{11}+\lambda_{12}$; $a_8=\lambda_1+\lambda_4+\lambda_5+\lambda_7+\lambda_8+\lambda_{11}$

$a_9=\lambda_1+\lambda_2+\lambda_4+\lambda_5+\lambda_7+\lambda_8+\lambda_{11}$; $a_{10}=\lambda_1+\lambda_4+\lambda_5+\lambda_7+\lambda_8+\lambda_{10}+\lambda_{11}$

$a_{11}=\lambda_1+\lambda_2+\lambda_4+\lambda_5+\lambda_7+\lambda_8+\lambda_{10}+\lambda_{11}$; $a_{12}=\lambda_1+\lambda_3+\lambda_4+\lambda_5+\lambda_6+\lambda_7+\lambda_8+\lambda_{10}$

$a_{13}=\lambda_1+\lambda_3+\lambda_4+\lambda_5+\lambda_6+\lambda_7+\lambda_8+\lambda_{10}+\lambda_{11}$;

$a_{15}=\lambda_1+\lambda_2+\lambda_3+\lambda_4+\lambda_5+\lambda_6+\lambda_7+\lambda_8+\lambda_{11}+\lambda_{12}$;

$a_{16}=\lambda_1+\lambda_3+\lambda_4+\lambda_5+\lambda_6+\lambda_7+\lambda_8+\lambda_{10}+\lambda_{11}+\lambda_{12}$;

$a_{17}=\lambda_1+\lambda_3+\lambda_4+\lambda_5+\lambda_6+\lambda_7+\lambda_8+\lambda_{11}+\lambda_{12}$; $a_{18}=\lambda_2+\lambda_4+\lambda_5+\lambda_7+\lambda_8+\lambda_{10}$

$a_{19}=\lambda_1+\lambda_2+\lambda_4+\lambda_5+\lambda_7+\lambda_8+\lambda_{10}$; $a_{20}=\lambda_2+\lambda_4+\lambda_5+\lambda_7+\lambda_8+\lambda_{10}+\lambda_{11}$

$a_{21}=\lambda_2+\lambda_3+\lambda_4+\lambda_5+\lambda_6+\lambda_7+\lambda_8+\lambda_{10}+\lambda_{12}$; $a_{22}=\lambda_1+\lambda_2+\lambda_3+\lambda_4+\lambda_5+\lambda_6+\lambda_7+\lambda_8+\lambda_{10}+\lambda_{12}$

$a_{23}=\lambda_2+\lambda_3+\lambda_4+\lambda_5+\lambda_6+\lambda_7+\lambda_8+\lambda_{10}+\lambda_{11}+\lambda_{12}$; $a_{24}=\lambda_2+\lambda_5+\lambda_6+\lambda_7+\lambda_9+\lambda_{12}$

$a_{25} = \lambda_2 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_9 + \lambda_{11} + \lambda_{12}$; $a_{26} = \lambda_2 + \lambda_3 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_9 + \lambda_{12}$

$a_{27} = \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{11} + \lambda_{12}$;

$a_{28} = \lambda_1 + \lambda_2 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{12}$;

$a_{29} = \lambda_1 + \lambda_2 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_9 + \lambda_{10} + \lambda_{12}$;

$a_{30} = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{12}$;

$a_{31} = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_9 + \lambda_{10} + \lambda_{12}$;

$a_{32} = \lambda_2 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{12}$;

$a_{33} = \lambda_2 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{12}$;

$a_{34} = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{12}$;

$a_{35} = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{12}$;

$a_{36} = \lambda_1 + \lambda_2 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{12}$;

$a_{37} = \lambda_1 + \lambda_2 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{12}$;

$a_{38} = \lambda_1 + \lambda_2 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{12}$;

$a_{39} = \lambda_1 + \lambda_2 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{11} + \lambda_{12}$;

$a_{40} = \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{12}$;

$a_{41} = \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{12}$;

$a_{42} = \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{11} + \lambda_{12}$;

$a_{43} = \lambda_2 + \lambda_3 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_9 + \lambda_{11}$; $a_{44} = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_9 + \lambda_{11} + \lambda_{12}$

$a_{45} = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_9 + \lambda_{11}$; $a_{46} = \lambda_1 + \lambda_3 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_9 + \lambda_{11}$

$a_{47} = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_9 + \lambda_{11}$; $a_{48} = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_{11}$

$a_{49} = \lambda_1 + \lambda_3 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_9 + \lambda_{11}$; $a_{50} = \lambda_1 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{12}$

$a_{51} = \lambda_1 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_9 + \lambda_{10} + \lambda_{11}$; $a_{52} = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{11}$

$a_{53} = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{11}$;

$$a_{54} = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_9 + \lambda_{11}$$

$$a_{55} = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_9 + \lambda_{10} + \lambda_{11}$$

$$a_{56} = \lambda_1 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{11}$$

$$a_{57} = \lambda_1 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{12}$$

$$a_{58} = \lambda_1 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{11}$$

$$a_{59} = \lambda_1 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{11} + \lambda_{12}$$

$$a_{60} = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{11} + \lambda_{12}$$

$$a_{61} = \lambda_1 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{12}$$

$$a_{62} = \lambda_1 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{12}$$

$$a_{63} = \lambda_1 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{12}$$

$$a_{64} = \lambda_1 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{11} + \lambda_{12}$$

$$a_{65} = \lambda_1 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{12}$$

$$a_{66} = \lambda_1 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{12}$$

$$a_{67} = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{12}$$

$$a_{68} = \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10}$$

$$a_{69} = \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{12}$$

$$a_{70} = \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{11}$$

$$a_{71} = \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{12}$$

$$a_{72} = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10}$$

$$a_{73} = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{11}$$

$$a_{74} = \lambda_1 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10}$$

$$a_{75} = \lambda_1 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{12}$$

$$a_{76} = \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10}$$

$a_{77} = \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{11}$

CASE 3: When failure rates follow exponential distribution

Exponential distribution is nothing but a particular case of Weibull distribution for $p = 1$ and is very useful in numerous practical problems, thus the reliability of the complex system in this case at an instant t is given by

$$R_{SE}(t) = \lfloor R_{SW}(t) \rfloor \text{ at } p = 1$$

$$\begin{aligned}
&= \exp(-a_1 t) + \exp(-a_2 t) - \exp(-a_3 t) + \exp(-a_4 t) - \exp(-a_5 t) - \exp(-a_6 t) \\
&+ \exp(-a_7 t) + \exp(-a_8 t) - \exp(-a_9 t) - \exp(-a_{10} t) + 2\exp(-a_{11} t) \\
&+ \exp(-a_{12} t) - \exp(-a_{13} t) - 2\exp(-a_{14} t) + \exp(-a_{15} t) + \exp(-a_{16} t) \\
&- \exp(-a_{17} t) + \exp(-a_{18} t) - \exp(-a_{19} t) - \exp(-a_{20} t) - \exp(-a_{21} t) \\
&+ \exp(-a_{22} t) + \exp(-a_{23} t) + \exp(-a_{24} t) - \exp(-a_{25} t) - \exp(-a_{26} t) \\
&- \exp(-a_{27} t) + \exp(-a_{28} t) - \exp(-a_{29} t) - 2\exp(-a_{30} t) + \exp(-a_{31} t) \\
&+ \exp(-a_{32} t) - \exp(-a_{33} t) + 4\exp(-a_{34} t) - 3\exp(-a_{35} t) - \exp(-a_{36} t) \\
&- 2\exp(-a_{37} t) + 2\exp(-a_{38} t) + \exp(-a_{39} t) - 2\exp(-a_{40} t) + 2\exp(-a_{41} t) \\
&+ \exp(-a_{42} t) + \exp(-a_{43} t) + 2\exp(-a_{44} t) - \exp(-a_{45} t) + \exp(-a_{46} t) \\
&+ \exp(-a_{47} t) - \exp(-a_{48} t) + \exp(-a_{49} t) + \exp(-a_{50} t) - \exp(-a_{51} t) \\
&+ \exp(-a_{52} t) - \exp(-a_{53} t) - \exp(-a_{54} t) + \exp(-a_{55} t) - \exp(-a_{56} t) \\
&- 2\exp(-a_{57} t) + 2\exp(-a_{58} t) + 2\exp(-a_{59} t) - 2\exp(-a_{60} t) + \exp(-a_{61} t) \\
&+ \exp(-a_{62} t) - \exp(-a_{63} t) - \exp(-a_{64} t) - \exp(-a_{65} t) + 2\exp(-a_{66} t) \\
&+ \exp(-a_{67} t) + \exp(-a_{68} t) + \exp(-a_{69} t) - \exp(-a_{70} t) - \exp(-a_{71} t) + \exp(-a_{72} t) \\
&- \exp(-a_{73} t) - \exp(-a_{74} t) - \exp(-a_{75} t) - \exp(-a_{76} t) + \exp(-a_{77} t) \qquad (26)
\end{aligned}$$

8. EVALUATION OF M.T.T.F. OF THE SYSTEM

The expression for $M.T.T.F$ in this case is given by

$$M.T.T.F = \int_0^\infty R_{SE}(t)\,dt$$

$$\begin{aligned}
&= \frac{1}{a_1} + \frac{1}{a_2} - \frac{1}{a_3} + \frac{1}{a_4} - \frac{1}{a_5} - \frac{1}{a_6} + \frac{1}{a_7} + \frac{1}{a_8} - \frac{1}{a_9} - \frac{1}{a_{10}} + \frac{2}{a_{11}} + \frac{1}{a_{12}} \\
&- \frac{1}{a_{13}} - \frac{2}{a_{14}} + \frac{1}{a_{15}} + \frac{1}{a_{16}} - \frac{1}{a_{17}} + \frac{1}{a_{18}} - \frac{1}{a_{19}} - \frac{1}{a_{20}} - \frac{1}{a_{21}} + \frac{1}{a_{22}} + \frac{1}{a_{23}} \\
&+ \frac{1}{a_{24}} - \frac{1}{a_{25}} - \frac{1}{a_{26}} - \frac{1}{a_{27}} + \frac{1}{a_{28}} - \frac{1}{a_{29}} - \frac{2}{a_{30}} + \frac{1}{a_{31}} + \frac{1}{a_{32}} - \frac{1}{a_{33}} + \frac{4}{a_{34}}
\end{aligned}$$

$$-\frac{3}{a_{35}}-\frac{1}{a_{36}}-\frac{2}{a_{37}}+\frac{2}{a_{38}}+\frac{1}{a_{39}}-\frac{2}{a_{40}}+\frac{2}{a_{41}}+\frac{1}{a_{42}}+\frac{1}{a_{43}}+\frac{2}{a_{44}}-\frac{1}{a_{45}}$$

$$+\frac{1}{a_{46}}+\frac{1}{a_{47}}-\frac{1}{a_{48}}+\frac{1}{a_{49}}+\frac{1}{a_{50}}-\frac{1}{a_{51}}+\frac{1}{a_{52}}-\frac{1}{a_{53}}-\frac{1}{a_{54}}+\frac{1}{a_{55}}-\frac{1}{a_{56}}$$

$$-\frac{2}{a_{57}}+\frac{2}{a_{58}}+\frac{2}{a_{59}}-\frac{2}{a_{60}}+\frac{1}{a_{61}}+\frac{1}{a_{62}}-\frac{1}{a_{63}}-\frac{1}{a_{64}}-\frac{1}{a_{65}}+\frac{2}{a_{66}}+\frac{1}{a_{67}}$$

$$+\frac{1}{a_{68}}+\frac{1}{a_{69}}-\frac{1}{a_{70}}-\frac{1}{a_{71}}+\frac{1}{a_{72}}-\frac{1}{a_{73}}-\frac{1}{a_{74}}-\frac{1}{a_{75}}-\frac{1}{a_{76}}+\frac{1}{a_{77}} \qquad (27)$$

9. NUMERICAL COMPUTATION FOR RELIABILITY and MTTF:

Setting $\lambda_i = 0.1$ for $i = 1-12$ and $p = 2$ in equations (25) and (26), one can compute Table 1 and Setting $\lambda_i = \lambda$ one can compute Table 2 from equation (27).

Table 1

S. No.	Time (t)	System Reliability	
		Exponential Distribution	Weibull Distribution
1	0	1.00000	1.00000
2	1	0.94039	0.94039
3	2	0.80276	0.48493
4	3	0.63969	0.08272
5	4	0.48493	5.09306E-3
6	5	0.35447	1.36944E-4
7	6	0.25232	1.673E-6
8	7	0.17618	9E-9
9	8	0.12132	2.28656E-11
10	9	0.0827	2.546713E-14
11	10	0.05601	1.274506E-17

Table 2

SI.NO	λ	MTTF
1	0.0	∞
2	0.1	4.52
3	0.2	2.26
4	0.3	1.50
5	0.4	1.13
6	0.5	0.90
7	0.6	0.75
8	0.7	0.64
9	0.8	0.56
10	0.9	0.50
11	1.0	0.45

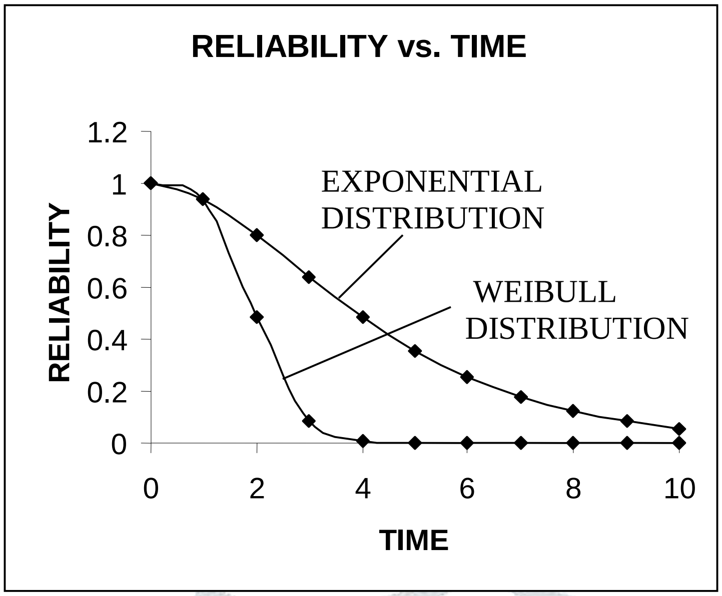

Fig .2

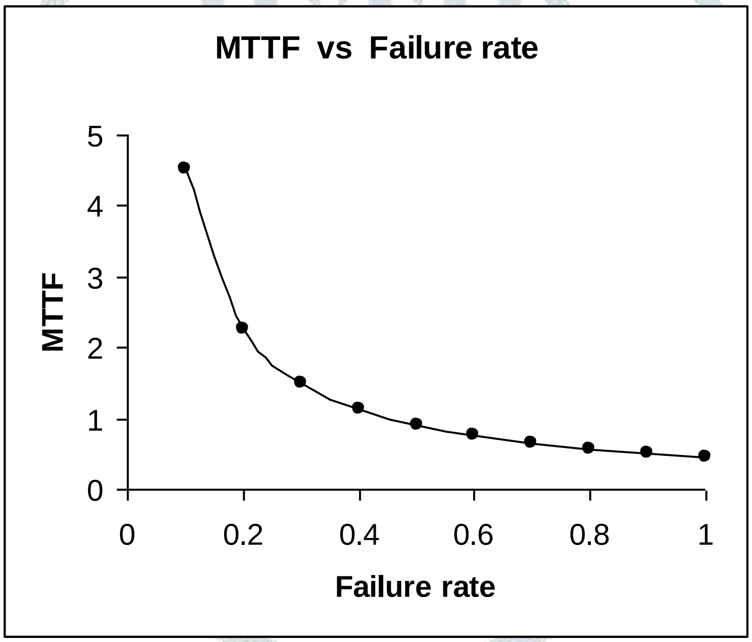

Fig .3

10. INTERPRETATION OF THE RESULTS:

Table 1 computes the variation of reliability with respect to time, when failure follows exponential and Weibull distributions. A critical examination of Reliability vs. time graph shows that the reliability of the complex system decreases approximately at a uniform rate in case of exponential distribution, where as it decreases very rapidly when failure follows Weibull distribution.

Further, Table 2 computes the mean time to failure i.e. MTTF at an instant *t*. An inspection of MTTF vs. failure rate graph reveals that in the beginning it decreases catastrophically, but later on, it decreases approximately at a uniform rate.

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