EVALUATION OF RELIABILITY OF A PARALLEL SYSTEM WITH COMMON CAUSE FAILURES AND CRITICAL HUMAN ERRORS

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Abstract:

This paper presents reliability and availability analysis of a n-unit parallel system with common-cause failures and human errors. Laplace transforms of state probabilities, system availability, and reliability are presented. A formula to estimate system mean time to failure is developed. Special case expressions for steady-state probabilities and system availability are derived.

Keywords:

Supplementary variable method, Common cause failures, Human error, Reliability, Availability, Mean time to failure.

Introduction:

Redundancy can generally be used to increase the reliability of a system with out any change in the reliability of the individual units that form the system. The parallel configuration is one form of redundancy. This form is often used to improve system reliability. However, in conventional reliability and availability analyses of parallel systems the occurrence of common-cause failures and critical human errors is not taken into consideration. Their impact on a system's reliability is much greater than many of us realize.

A common-cause failure is defined as any instance where multiple units fail due to a single cause. Some of the common-cause failures may occur due to : equipment design deficiency, operations and maintenance errors, external normal environment, external catastrophe, functional deficiency, common manufacturer, and common power source. This paper separates common-cause failures into two categories, i .e, common-cause failures resulting from a human error, and not from a human error. The common-cause failures resulting from a human action are called critical human errors and the remaining ones are simply referred to as common-cause failures. Many studies have indicated that a significant percentage of system failures are due to human error. Selective human error data and their sources are given in references [1-15].

In this paper we discuss about reliability and availability analysis of a parallel system with common-cause failures and critical human errors. The system may fail either due to a common-cause failure, or a critical human error or when all units fail. It is assumed that when only one unit operates, a common-cause failure or a critical human error may still occur. Here, it is important to note that only those failures/human errors occurring (i .e., when one unit is operating) are categorized as common-cause failures/critical human error, the ones which may have led to a total system failure, if there have been more than one unit working normally in the system.

Assumptions:

The following assumptions are associated with the system under study:

(a) Failures are statistically independent.
(b) All system units are active, identical and form a parallel network.
(c) A unit failure rate is constant.
(d) A common-cause failure or a critical human error leads to system failure.
(e) A common-cause failure or a critical human error can occur when one (or more) unit is operating.
(f) Critical human error and common-cause failure rates are constant.
(g) Failed system repair rates are constant/non-constant.
(h) At least one unit must operate normally for the system's success

***Symbols*:**

The following symbols are associated with this model:

n - Number of units in the parallel system.

λ - Constant failure rate of a unit.

i - System upstate as shown in boxes of fig 1:

$i = 0$ (all n units operating normally),

$i = 1$ (one unit failed, $(n-1)$ operating),

$i = 2$ (two units failed, $(n-2)$ operating).

$i = 3$ (three units failed, $(n-3)$ operating)

$i = k$ (k units failed, $(n-k)$/ one operating)

k - Number of failed units in the system and corresponding upstate of the system, for $k = 0,1,2,........., (n-1)$.

λ_{c1i}, λ_{c2i} - Constant common cause failure rate from system upstate i,

for $i = 0,1,2,\ldots\ldots\ldots,k$

λ_{h1i}, λ_{h2i} - Constant critical human error rate from system upstate i,

for $i = 0,1,2,\ldots\ldots\ldots,k$

j - System down-state as shown in boxes of fig 1:

$j = n$ (all units failed other than due to a common-cause failure or a critical human error)

$j = c1, c2$ (system failed due to a common cause failure)

$j = h1, h2$ (system failed due to a critical human error)

$P_i(t)$ - Probability that the system is in upstate i at time t, for $i = 0,1,2,\ldots\ldots.,k$.

$P_j(t)$ - Probability that the system is in down state j at the time t, for $j = n, c1, c2, h1, h2$.

. P_i - Steady state probability that the system is in upstate i, for $i = 0,1,2,\ldots\ldots.,k$

P_j - Steady state probability that the system is in down state j, for $j = n, c1, c2, h1, h2$.

$r_1(x), r_2(x), q_{c1}(x), q_{c2}(x)$ Repair rates and probability density functions of repair times, respectively, when the failed system is in states $c1$, $c2$ and has an elapsed repair time of x .

$z_1(x), z_2(x), q_{h1}(x), q_{h2}(x)$ Repair rates and probability density functions of repair times, respectively, when the failed system is in states $h1$, $h2$ and has an elapsed repair time of x .

$\mu(x), q_n(x)$ Repair rates and probability density functions of repair times, respectively, when the failed system is in states n and has an elapsed repair time of x .

s Laplace transform variable.

μ, r_1, r_2, z_1, z_2 Constant repair rates from failed system states

n, $c1$, $c2$, $h1$, $h2$ respectively.

SYSTEM TRANSITION DIAGRAM FOR GENERAL MODEL:

The system transition diagram is shown in fig 1:

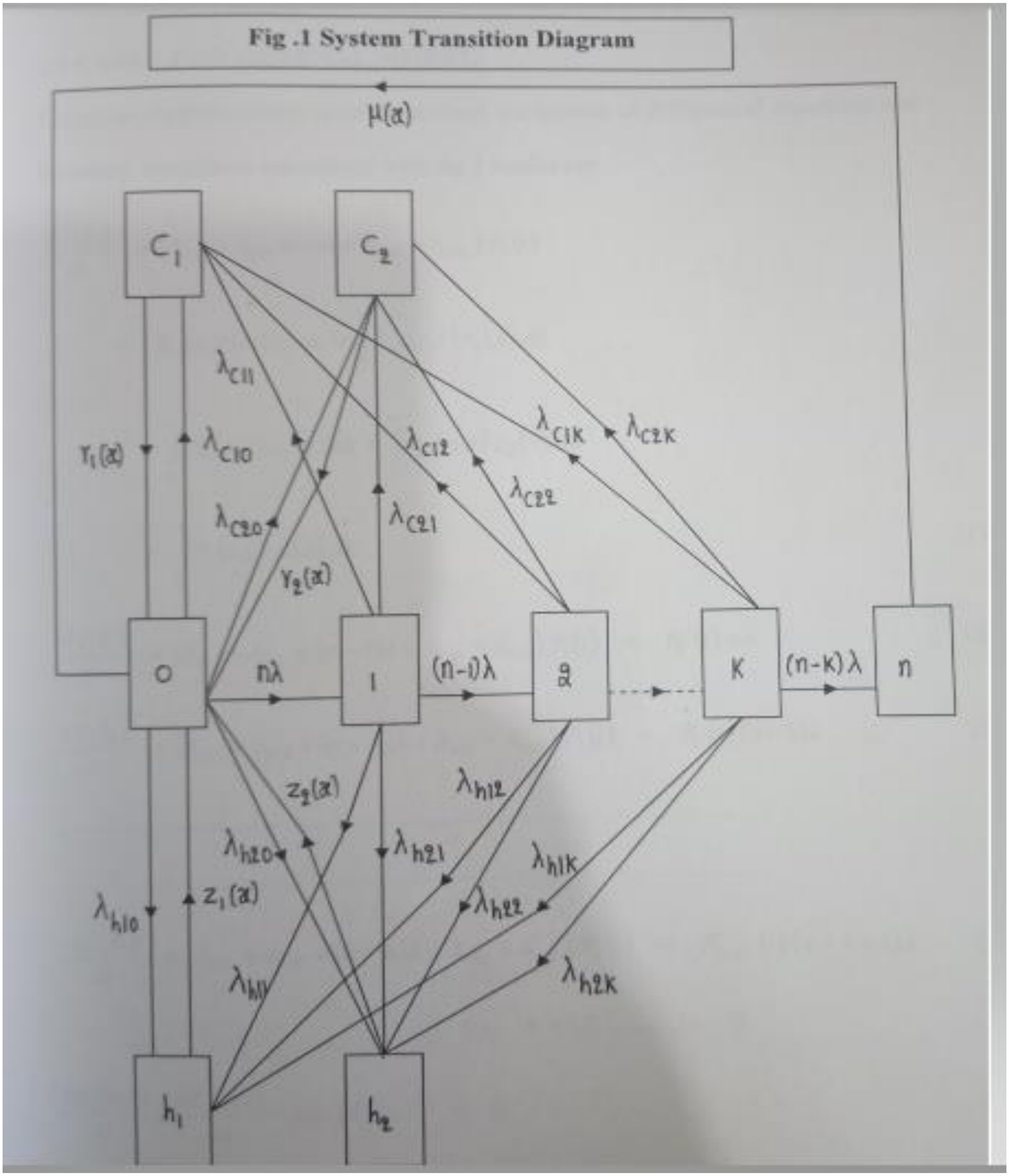

Fig .1 System Transition Diagram

Case 1:

For General Model:

Using the supplementary variable method, the system of differential equations and boundary conditions associated with model 1 are

$$\frac{dP_0(t)}{dt} + \left(\lambda_{c10} + \lambda_{h10} + n\lambda + \lambda_{c20} + \lambda_{h20}\right) P_0(t) = \int_0^\infty P_{c1}(x,t)\, r_1(x)\, dx + \int_0^\infty P_{c2}(x,t)\, r_2(x)\, dx$$

$$+ \int_0^\infty P_{h1}(x,t)\, z_1(x)\, dx + \int_0^\infty P_{h2}(x,t)\, z_2(x)\, dx$$

$$+ \int_0^\infty P_n(x,t)\, \mu(x)\, dx \qquad (1)$$

$$\frac{dP_1(t)}{dt} + \left(\lambda_{c11} + \lambda_{h11} + (n-1)\lambda + \lambda_{c21} + \lambda_{h21}\right) P_1(t) = P_0(t)\, n\lambda \qquad (2)$$

$$\frac{dP_2(t)}{dt} + \left(\lambda_{c12} + \lambda_{h12} + (n-2)\lambda + \lambda_{c22} + \lambda_{h22}\right) P_2(t) = P_1(t)(n-1)\lambda \qquad (3)$$

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$$\frac{dP_k(t)}{dt} + \left(\lambda_{c1k} + \lambda_{h1k} + (n-k)\lambda + \lambda_{c2k} + \lambda_{h2k}\right) P_k(t) = P_{(k-1)}(t)(n-k+1)\lambda \qquad (4)$$

for $k = 1,2,3,......, (n-1)$

$$\frac{\partial P_n(x,t)}{\partial t} + \frac{\partial P_n(x,t)}{\partial x} + \mu(x)\, P_n(x,t) = 0 \qquad (5)$$

$$\frac{\partial P_{c1}(x,t)}{\partial t} + \frac{\partial P_{c1}(x,t)}{\partial x} + r_1(x)\, P_{c1}(x,t) = 0 \qquad (6)$$

$$\frac{\partial P_{c2}(x,t)}{\partial t} + \frac{\partial P_{c2}(x,t)}{\partial x} + r_2(x)\, P_{c2}(x,t) = 0 \qquad (7)$$

$$\frac{\partial P_{h1}(x,t)}{\partial t} + \frac{\partial P_{h1}(x,t)}{\partial x} + z_1(x)\, P_{h1}(x,t) = 0 \qquad (8)$$

$$\frac{\partial P_{h2}(x,t)}{\partial t} + \frac{\partial P_{h2}(x,t)}{\partial x} + z_2(x)\, P_{h2}(x,t) = 0 \qquad (9)$$

$$P_n(0,t) = (n-k)\,\lambda\, P_k(t) \qquad (10)$$

$$P_{c1}(0,t) = \sum_{i=0}^{k} P_i(t)\, \lambda_{c1i} \qquad (11)$$

$$P_{c2}(0,t) = \sum_{i=0}^{k} P_i(t)\, \lambda_{c2i} \qquad (12)$$

$$P_{h1}(0,t) = \sum_{i=0}^{k} P_i(t)\,\lambda_{h1i} \tag{13}$$

$$P_{h2}(0,t) = \sum_{i=0}^{k} P_i(t)\,\lambda_{h2i} \tag{14}$$

$$P_i(0) = 1 \quad \text{for} \quad i = 0$$

$$P_i(0) = 0 \quad \text{for} \quad i = 1,2,3,\ldots\ldots,k$$

$$P_j(x,0) = 0 \quad \text{for} \quad j = n, c1, c2, h1, h2$$

Solving above equations using Laplace transforms leads to the following Laplace transforms of the probabilities:

$$sP_0(s) - 1 + \left(\lambda_{c10} + \lambda_{h10} + n\lambda + \lambda_{c20} + \lambda_{h20}\right) P_0(s) = \int_0^\infty P_{c1}(x,s)\, r_1(x)\, dx + \int_0^\infty P_{c2}(x,s)\, r_2(x)\, dx$$

$$+ \int_0^\infty P_{h1}(x,s)\, z_1(x)\, dx + \int_0^\infty P_{h2}(x,s)\, z_2(x)\, dx$$

$$+ \int_0^\infty P_n(x,s)\, \mu(x)\, dx \tag{15}$$

$$sP_1(s) + \left(\lambda_{c11} + \lambda_{h11} + (n-1)\lambda + \lambda_{c21} + \lambda_{h21}\right) P_1(s) = P_0(s)\, n\lambda \tag{16}$$

$$sP_2(s) + \left(\lambda_{c12} + \lambda_{h12} + (n-2)\lambda + \lambda_{c22} + \lambda_{h22}\right) P_2(s) = P_1(s)(n-1)\lambda \tag{17}$$

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$$sP_k(s) + \left(\lambda_{c1k} + \lambda_{h1k} + (n-k)\lambda + \lambda_{c2k} + \lambda_{h2k}\right) P_k(s) = P_{k-1}(s)(n-k+1)\lambda \tag{18}$$

$$\left[s + \frac{\partial}{\partial x} + \mu(x)\right] P_n(x,s) = 0 \tag{19}$$

$$\left[s + \frac{\partial}{\partial x} + r_1(x)\right] P_{c1}(x,s) = 0 \tag{20}$$

$$\left[s + \frac{\partial}{\partial x} + r_2(x)\right] P_{c2}(x,s) = 0 \tag{21}$$

$$\left[s + \frac{\partial}{\partial x} + h_1(x)\right] P_{h1}(x,s) = 0 \tag{22}$$

$$\left[s+\frac{\partial}{\partial x}+h_2(x)\right]P_{h2}(x,s) \;=\; 0 \tag{23}$$

$$P_n(0,s) \;=\; (n-k)\,\lambda\; P_k(s) \tag{24}$$

$$P_{c1}(0,s) \;=\; \sum_{i=0}^{k} P_i(s)\,\lambda_{c1i} \tag{25}$$

$$P_{c2}(0,s) \;=\; \sum_{i=0}^{k} P_i(s)\,\lambda_{c2i} \tag{26}$$

$$P_{h1}(0,s) \;=\; \sum_{i=0}^{k} P_i(s)\,\lambda_{h1i} \tag{27}$$

$$P_{h2}(0,s) \;=\; \sum_{i=0}^{k} P_i(s)\,\lambda_{h2i} \tag{28}$$

From (16),

$$P_1(s)\left(s+\lambda_{c11}+\lambda_{h11}+(n-1)\lambda+\lambda_{c21}+\lambda_{h21}\right) \;=\; P_0\,(s)\,n\lambda$$

$$\Rightarrow P_1(s) \;=\; \frac{n\lambda P_0(s)}{s+\lambda_{c11}+\lambda_{h11}+(n-1)\lambda+\lambda_{c21}+\lambda_{h21}} \tag{29}$$

From (17),

$$P_2(s)\left(s+\lambda_{c12}+\lambda_{h12}+(n-2)\lambda+\lambda_{c22}+\lambda_{h22}\right) \;=\; P_1\,(s)\,(n-1)\lambda$$

$$\Rightarrow P_2(s) \;=\; \frac{n(n-1)\lambda^2 P_0(s)}{\left(s+\lambda_{c11}+\lambda_{h11}+(n-1)\lambda+\lambda_{c21}+\lambda_{h21}\right)\left(s+\lambda_{c12}+\lambda_{h12}+(n-2)\lambda+\lambda_{c22}+\lambda_{h22}\right)} \tag{30}$$

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$$P_k(s) \;=\; \frac{\prod_{i=0}^{k-1}(n-i)\lambda^k P_0(s)}{\prod_{i=1}^{k}\left\{s+(n-i)\lambda+\lambda_{c1i}+\lambda_{c2i}+\lambda_{h1i}+\lambda_{h2i}\right\}} \tag{31}$$

for $k=1,2,3,\ldots\ldots,(n-1)$

From (19),

$$\frac{\partial}{\partial x}P_n(x,s) \;=\; -(s+\mu(x))\,P_n(x,s) \Rightarrow \frac{\frac{\partial}{\partial x}P_n(x,s)}{P_n(x,s)} \;=\; -(s+\mu(x))$$

Integrate on both sides

$$\Rightarrow \log[P_n(x,s)]_0^x = -sx - \int_0^x \mu(x)\,dx$$

$$\Rightarrow \log\left[\frac{P_n(x,s)}{P_n(0,s)}\right] = -sx - \int_0^x \mu(x)dx$$

$$\Rightarrow \left[\frac{P_n(x,s)}{P_n(0,s)}\right] = e^{-sx-\int_0^x \mu(x)dx}$$

$$\Rightarrow P_n(x,s) = P_n(0,s)\, e^{-sx-\int_0^x \mu(x)dx}$$

$$\Rightarrow P_n(x,s) = (n-k)\lambda\, P_k(s)\, e^{-sx-\int_0^x \mu(x)dx}$$

From (20),

$$\frac{\partial}{\partial x}P_{c1}(x,s) = -(s+r_1(x))\,P_{c1}(x,s) \Rightarrow \frac{\frac{\partial}{\partial x}P_{c1}(x,s)}{P_{c1}(x,s)} = -(s+r_1(x))$$

Integrate on both sides

$$\Rightarrow \log[P_{c1}(x,s)]_0^x = -sx - \int_0^x r_1(x)\,dx \Rightarrow \log\left[\frac{P_{c1}(x,s)}{P_{c1}(0,s)}\right] = -sx - \int_0^x r_1(x)dx$$

$$\Rightarrow \left[\frac{P_{c1}(x,s)}{P_{c1}(0,s)}\right] = e^{-sx-\int_0^x r_1(x)dx} \quad \Rightarrow P_{c1}(x,s) = P_{c1}(0,s)\, e^{-sx-\int_0^x r_1(x)dx}$$

$$\Rightarrow P_{c1}(x,s) = \sum_{i=0}^{k} P_i(s)\,\lambda_{c1i}\, e^{-sx-\int_0^x r_1(x)dx}$$

Similarly, $P_{c2}(x,s) = \sum_{i=0}^{k} P_i(s)\,\lambda_{c2i}\, e^{-sx-\int_0^x r_2(x)dx}$

$$P_{h1}(x,s) = \sum_{i=0}^{k} P_i(s)\,\lambda_{h1i}\, e^{-sx-\int_0^x z_1(x)dx}$$

$$P_{h2}(x,s) = \sum_{i=0}^{k} P_i(s)\,\lambda_{h2i}\, e^{-sx-\int_0^x z_2(x)dx}$$

From equation (15) we have

$$s\,P_0(s)-1+(\lambda_{c10}+\lambda_{h10}+n\lambda+\lambda_{c20}+\lambda_{h20})P_0(s)$$

$$= \int_0^\infty \sum_{i=0}^{k} P_i(s)\lambda_{c1i}\, e^{-sx-\int_0^x r_1(x)dx}\, r_1(x)dx + \int_0^\infty \sum_{i=0}^{k} P_i(s)\lambda_{c2i}\, e^{-sx-\int_0^x r_2(x)dx}\, r_2(x)dx$$

$$+ \int_0^\infty \sum_{i=0}^{k} P_i(s)\lambda_{h1i}\, e^{-sx-\int_0^x z_1(x)dx}\, z_1(x)dx + \int_0^\infty \sum_{i=0}^{k} P_i(s)\lambda_{h2i}\, e^{-sx-\int_0^x z_2(x)dx}\, z_2(x)dx$$

$$+ \int_0^\infty (n-k)\lambda\, P_k(s)\, e^{-sx-\int_0^x \mu(x)dx}\, \mu(x)dx$$

$$\Rightarrow\; s\,P_0(s)-1+(\lambda_{c10}+\lambda_{h10}+n\lambda+\lambda_{c20}+\lambda_{h20})P_0(s) = \sum_{i=0}^{k} P_i(s)\,\lambda_{c1i}\int_0^\infty e^{-sx-\int_0^x r_1(x)dx}\, r_1(x)\; dx$$

$$+ \sum_{i=0}^{k} P_i(s)\,\lambda_{c2i}\int_0^\infty e^{-sx-\int_0^x r_2(x)dx}\, r_2(x)\; dx + \sum_{i=0}^{k} P_i(s)\,\lambda_{h1i}\int_0^\infty e^{-sx-\int_0^x z_1(x)dx}\, z_1(x)\; dx$$

$$+ \sum_{i=0}^{k} P_i(s)\,\lambda_{h2i}\int_0^\infty e^{-sx-\int_0^x z_2(x)dx}\, z_2(x) + (n-k)\,\lambda\, P_k(s)\int_0^\infty e^{-sx-\int_0^x \mu(x)dx}\, \mu(x)\; dx$$

$$\Rightarrow\; P_0(s)\left\{\begin{array}{l} s+\lambda_{c10}+\lambda_{h10}+n\lambda+\lambda_{c20}+\lambda_{h20}-\lambda_{c10}-\sum_{j=1}^{k} P_j(s)\,\lambda_{c1j}\; G_{c1}(s)-\lambda_{c20} \\ -\sum_{j=1}^{k} P_j(s)\,\lambda_{c2j}\; G_{c2}(s)-\lambda_{h10}-\sum_{j=1}^{k} P_j(s)\,\lambda_{h1j}\; G_{h1}(s)-\lambda_{h20} \\ -\sum_{j=1}^{k} P_j(s)\,\lambda_{h2j}\; G_{h2}(s)-(n-k)\,\lambda\; P_k(s)\,G_n(s) \end{array}\right\} = 1$$

$$\Rightarrow \quad P_0(s)\begin{Bmatrix} s+\lambda_{c10}+\lambda_{h10}+n\lambda+\lambda_{c20}+\lambda_{h20}-M_1G_{c1}(s) \\ -M_2G_{c2}(s)-M_3G_{h1}(s)-M_4G_{h2}(s)-M_5G_n(s) \end{Bmatrix} = 1$$

$$\Rightarrow \quad P_0(s) = \begin{Bmatrix} s+\lambda_{c10}+\lambda_{h10}+n\lambda+\lambda_{c20}+\lambda_{h20}-M_1G_{c1}(s) \\ -M_2G_{c2}(s)-M_3G_{h1}(s)-M_4G_{h2}(s)-M_5G_n(s) \end{Bmatrix}^{-1} \tag{32}$$

Where $M_1 = \lambda_{c10} + \sum_{j=1}^{k}\left\{\dfrac{\prod_{i=0}^{j-1}(n-i)\,\lambda^j\ \lambda_{c1j}}{\prod_{i=1}^{j}\{s+(n-i)\lambda+\lambda_{c1i}+\lambda_{c2i}+\lambda_{h1i}+\lambda_{h2i}\}}\right\}$

$$M_2 = \lambda_{c20} + \sum_{j=1}^{k}\left\{\frac{\prod_{i=0}^{j-1}(n-i)\,\lambda^j\ \lambda_{c2j}}{\prod_{i=1}^{j}\{s+(n-i)\lambda+\lambda_{c1i}+\lambda_{c2i}+\lambda_{h1i}+\lambda_{h2i}\}}\right\}$$

$$M_3 = \lambda_{h10} + \sum_{j=1}^{k}\left\{\frac{\prod_{i=0}^{j-1}(n-i)\,\lambda^j\ \lambda_{h1j}}{\prod_{i=1}^{j}\{s+(n-i)\lambda+\lambda_{c1i}+\lambda_{c2i}+\lambda_{h1i}+\lambda_{h2i}\}}\right\}$$

$$M_4 = \lambda_{h20} + \sum_{j=1}^{k}\left\{\frac{\prod_{i=0}^{j-1}(n-i)\,\lambda^j\ \lambda_{h2j}}{\prod_{i=1}^{j}\{s+(n-i)\lambda+\lambda_{c1i}+\lambda_{c2i}+\lambda_{h1i}+\lambda_{h2i}\}}\right\}$$

$$M_5 = (n-k)\lambda + \left\{\frac{\prod_{i=0}^{k-1}(n-i)\lambda^k}{\prod_{i=1}^{k} s+(n-i)\lambda+\lambda_{c1i}+\lambda_{c2i}+\lambda_{h1i}+\lambda_{h2i}}\right\}$$

$$P_n(s) = (n-k)\,\lambda\, P_k(s)\left[\frac{1-G_n(s)}{s}\right] \tag{33}$$

$$P_{c1}(s) = \sum_{i=0}^{k} P_i(s)\,\lambda_{c1i}\left[\frac{1-G_{c1}(s)}{s}\right] \tag{34}$$

$$P_{c2}(s) = \sum_{i=0}^{k} P_i(s)\,\lambda_{c2i}\left[\frac{1-G_{c2}(s)}{s}\right] \tag{35}$$

$$P_{h1}(s) = \sum_{i=0}^{k} P_i(s)\,\lambda_{h1i}\left[\frac{1-G_{h1}(s)}{s}\right] \tag{36}$$

$$P_{h2}(s) = \sum_{i=0}^{k} P_i(s)\,\lambda_{h2i}\left[\frac{1-G_{h2}(s)}{s}\right] \quad (37)$$

$$G_j(s) = \int_0^\infty e^{-sx}\, q_j(x)\,dx \quad \text{for} \quad j = n, c1, c2, h1, h2$$

$$q_n(x) = \mu(x)\, e^{-\int_0^x \mu(x)dx}$$

$$q_{c1}(x) = r_1(x)\, e^{-\int_0^x r_1(x)dx}$$

$$q_{c2}(x) = r_2(x)\, e^{-\int_0^x r_2(x)dx}$$

$$q_{h1}(x) = z_1(x)\, e^{-\int_0^x z_1(x)dx}$$

$$q_{h2}(x) = z_2(x)\, e^{-\int_0^x z_2(x)dx}$$

The Laplace transform of the system availability is

$$AV(s) = \sum_{i=0}^{k} P_i(s) \quad (38)$$

The Parallel system Steady – state availability is given by

$$AV(ss) = \lim_{s\to 0} s\, AV(s) \quad (39)$$

The Laplace Transform of the system unavailability is

$$UAV(s) = P_n(s) + P_{c1}(s) + P_{c2}(s) + P_{h1}(s) + P_{h2}(s) \quad (40)$$

SOME SPECIAL CASE MODELS:

This section presents five special case models.

SPECIAL CASE MODEL I:

For $n = 2$ and exponentially distributed repair times we developed equations for model I.

The Laplace Transforms of repair time density functions are given by

$$G_2(s) = \frac{\mu}{s+\mu} \quad (41)$$

$$G_{c1}(s) = \frac{r_1}{s+r_1} \tag{42}$$

$$G_{c2}(s) = \frac{r_2}{s+r_2} \tag{43}$$

$$G_{h1}(s) = \frac{z_1}{s+z_1} \tag{44}$$

$$G_{h2}(s) = \frac{z_2}{s+z_2} \tag{45}$$

Where μ, r_1, r_2, z_1, z_2 are the constant repair rates from failed system states $2, c1, c2, h1, h2$ respectively.

From equations (29) to (37) and (41) to (45) we get

$$P_0(s)\left[s+\lambda_{c10}+\lambda_{h10}+2\lambda+\lambda_{c20}+\lambda_{h20}\right]$$

$$= 1+\frac{\mu 2\lambda^2}{(s+\lambda+\lambda_{c11}+\lambda_{c21}+\lambda_{h11}+\lambda_{h12})(s+\mu)}P_0(s)$$

$$+\left[\lambda_{c10}+\frac{2\lambda\lambda_{c11}}{(s+\lambda+\lambda_{c11}+\lambda_{c21}+\lambda_{h11}+\lambda_{h21})}\right]\frac{r_1}{s+r_1}P_0(s)$$

$$+\left[\lambda_{c20}+\frac{2\lambda\lambda_{c21}}{(s+\lambda+\lambda_{c11}+\lambda_{c21}+\lambda_{h11}+\lambda_{h21})}\right]\frac{r_2}{s+r_2}P_0(s)$$

$$+\left[\lambda_{h10}+\frac{2\lambda\lambda_{c11}}{(s+\lambda+\lambda_{c11}+\lambda_{c21}+\lambda_{h11}+\lambda_{h21})}\right]\frac{z_1}{s+z_1}P_0(s)$$

$$+\left[\lambda_{h20}+\frac{2\lambda\lambda_{h21}}{(s+\lambda+\lambda_{c11}+\lambda_{c21}+\lambda_{h11}+\lambda_{h21})}\right]\frac{z_2}{s+z_2}P_0(s)$$

$$\Rightarrow P_0(s) = \frac{1+\mu P_2(s)+r_1 P_{c1}(s)+r_2 P_{c2}(s)+z_1 P_{h1}(s)+z_2 P_{h2}(s)}{\left[s+\lambda_{c10}+\lambda_{h10}+2\lambda+\lambda_{c20}+\lambda_{h20}\right]} \tag{46}$$

$$P_1(s) = \frac{2\lambda}{\left[s+\lambda+\lambda_{c11}+\lambda_{c21}+\lambda_{h11}+\lambda_{h21}\right]}P_0(s) \tag{47}$$

$$P_2(s) = \frac{2\lambda^2}{\left[s+\lambda+\lambda_{c11}+\lambda_{c21}+\lambda_{h11}+\lambda_{h21}\right]\left[s+\mu\right]}P_0(s) \tag{48}$$

$$P_{c1}(s) = \left[\frac{\lambda_{c10}}{s+r_1} + \frac{2\lambda\lambda_{c11}}{[s+\lambda+\lambda_{c11}+\lambda_{c21}+\lambda_{h11}+\lambda_{h21}][s+r_1]}\right] P_0(s) \quad (49)$$

$$P_{c2}(s) = \left[\frac{\lambda_{c20}}{s+r_2} + \frac{2\lambda\lambda_{c21}}{[s+\lambda+\lambda_{c11}+\lambda_{c21}+\lambda_{h11}+\lambda_{h21}][s+r_2]}\right] P_0(s) \quad (50)$$

$$P_{h1}(s) = \left[\frac{\lambda_{h10}}{s+z_1} + \frac{2\lambda\lambda_{h11}}{[s+\lambda+\lambda_{c11}+\lambda_{c21}+\lambda_{h11}+\lambda_{h21}][s+z_1]}\right] P_0(s) \quad (51)$$

$$P_{h2}(s) = \left[\frac{\lambda_{h20}}{s+z_2} + \frac{2\lambda\lambda_{h21}}{[s+\lambda+\lambda_{c11}+\lambda_{c21}+\lambda_{h11}+\lambda_{h21}][s+z_2]}\right] P_0(s) \quad (52)$$

Using equations (46) to (52), we get the Laplace transforms of the system availability and unavailability as follows:

$$AV(s) = \sum_{i=0}^{1} P_i(s)$$
$$= \left[\frac{1+\mu P_2(s)+r_1 P_{c1}(s)+r_2 P_{c2}(s)+z_1 P_{h1}(s)+z_2 P_{h2}(s)}{s+\lambda_{c10}+\lambda_{h10}+2\lambda+\lambda_{c20}+\lambda_{h20}}\right]$$
$$+ \left[\frac{2\lambda P_0(s)}{s+\lambda+\lambda_{c11}+\lambda_{c21}+\lambda_{h11}+\lambda_{h21}}\right] \quad (53)$$

and $UAV(s) = P_2(s)+P_{c1}(s)+P_{c2}(s)+P_{h1}(s)+P_{h2}(s)$ (54)

SPECIAL CASE MODEL II:

For $n = 2$ and exponentially distributed failed system repair times the following steady state probability expressions result:

$$P_0 = \left[1+\frac{2\lambda}{A}+\frac{2\lambda^2}{A\mu}+\frac{2\lambda\lambda_{c11}}{Ar_1}+\frac{2\lambda\lambda_{c21}}{Ar_2}+\frac{2\lambda\lambda_{h11}}{Az_1}+\frac{2\lambda\lambda_{h21}}{Az_2}+\frac{\lambda_{c10}}{r_1}+\frac{\lambda_{c20}}{r_2}+\frac{\lambda_{h10}}{z_1}+\frac{\lambda_{h20}}{z_2}\right]^{-1}$$

Where $A = \lambda+\lambda_{c11}+\lambda_{c12}+\lambda_{h11}+\lambda_{h21}$ (55)

$$P_1 = \frac{2\lambda}{A} P_0 \quad (56)$$

$$P_2 = \frac{2\lambda^2}{A\mu} P_0 \quad (57)$$

$$P_{c1} = \left[\frac{\lambda_{c10}}{r_1} + \frac{2\lambda\lambda_{c11}}{Ar_1}\right] P_0 \quad (58)$$

$$P_{c2} = \left[\frac{\lambda_{c20}}{r_2} + \frac{2\lambda\lambda_{c21}}{Ar_2}\right]P_0 \quad (59)$$

$$P_{h1} = \left[\frac{\lambda_{h10}}{z_1} + \frac{2\lambda\lambda_{h11}}{Az_1}\right]P_0 \quad (60)$$

$$P_{h2} = \left[\frac{\lambda_{h20}}{z_2} + \frac{2\lambda\lambda_{h21}}{Az_2}\right]P_0 \quad (61)$$

The system steady state availability $AV(ss)$ and unavailability $UAV(ss)$ are given by

$$AV(ss) = \sum_{i=0}^{1} P_i \quad (62)$$

$$UAV(ss) = P_2 + P_{c1} + P_{c2} + P_{h1} + P_{h2} \quad (63)$$

For $\mu = r = z = 15$, $\lambda = 10$, $\lambda_c = \lambda_{c10} = \lambda_{c11} = \lambda_{c20} = \lambda_{c21}$ and $\lambda_h = \lambda_{h10} = \lambda_{h11} = \lambda_{h20} = \lambda_{h21} = 2,3,4$.
The plots of Equation (62) are shown in fig 2.

S.No.	λ_h	λ_c	P_0	P_1	Availability
1	2	0	0.248	0.35	0.598
2	2	1	0.2510	0.31	0.5610
3	2	2	0.2513	0.279	0.5303
4	2	3	0.25	0.25	0.50
5	2	4	0.247	0.22	0.467

S.No.	λ_h	λ_c	P_0	P_1	Availability
1	3	0	0.2510	0.31	0.5610
2	3	1	0.2513	0.279	0.5303
3	3	2	0.25	0.25	0.50
4	3	3	0.247	0.22	0.467
5	3	4	0.243	0.20	0.443

S.No.	λ_h	λ_c	P_0	P_1	Availability
1	4	0	0.2513	0.279	0.5303
2	4	1	0.25	0.25	0.50
3	4	2	0.247	0.22	0.467
4	4	3	0.243	0.20	0.443
5	4	4	0.2398	0.18	0.4198

The parallel system steady-state availability decreases with the increasing values of system common-cause failure and critical human error rates.

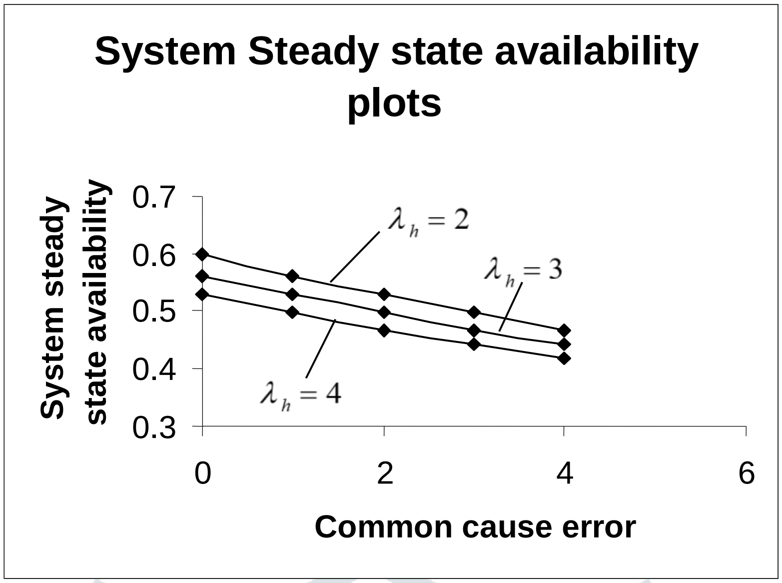

Fig.2

SPECIAL CASE MODEL III:

For $n = 3$ and exponentially distributed repair times, we developed equations for model I. The Laplace transforms of repair time density functions are given by

$$G_3(s) = \frac{\mu}{s+\mu} \tag{65}$$

$$G_{c1}(s) = \frac{r_1}{s+r_1} \tag{66}$$

$$G_{c2}(s) = \frac{r_2}{s+r_2} \tag{67}$$

$$G_{h1}(s) = \frac{z_1}{s+z_1} \tag{68}$$

$$G_{h2}(s) = \frac{z_2}{s+z_2} \tag{69}$$

Where μ, r_1, r_2, z_1, z_2 are the constant repair rates from failed system states $3, c1, c2, h1, h2$ respectively.

From equations (29) to (37) and (65) to (69) we get

$$P_0(s)\left[s+\lambda_{c10}+\lambda_{h10}+3\lambda+\lambda_{c20}+\lambda_{h20}\right]$$

$$= 1+\frac{\mu}{(s+\mu)}\frac{6\lambda^3}{A_1A_2}P_0(s) + \left[\lambda_{c10}+\frac{3\lambda\lambda_{c11}}{A_1}+\frac{6\lambda^2\lambda_{c12}}{A_1A_2}\right]\frac{r_1}{s+r_1}P_0(s)$$

$$+\left[\lambda_{c20}+\frac{3\lambda\lambda_{c21}}{A_1}+\frac{6\lambda^2\lambda_{c22}}{A_1A_2}\right]\frac{r_2}{s+r_2}P_0(s)$$

$$+\left[\lambda_{h10}+\frac{3\lambda\lambda_{h11}}{A_1}+\frac{6\lambda^2\lambda_{h12}}{A_1A_2}\right]\frac{z_1}{s+z_1}P_0(s)$$

$$+\left[\lambda_{h20}+\frac{3\lambda\lambda_{h21}}{A_1}+\frac{6\lambda^2\lambda_{h22}}{A_1A_2}\right]\frac{z_2}{s+z_2}P_0(s) \quad (70)$$

Where $A_1 = s+2\lambda+\lambda_{c11}+\lambda_{c21}+\lambda_{h11}+\lambda_{h21}$

$A_2 = s+\lambda+\lambda_{c21}+\lambda_{c22}+\lambda_{h12}+\lambda_{h22}$

$$\Rightarrow \quad P_0(s) = \frac{1+\mu\, P_3(s)+r_1\, P_{c1}(s)+r_2\, P_{c2}(s)+z_1\, P_{h1}(s)+z_2\, P_{h2}(s)}{\left[s+\lambda_{c10}+\lambda_{h10}+3\lambda+\lambda_{c20}+\lambda_{h20}\right]} \quad (71)$$

$$P_1(s) = \frac{3\lambda}{A_1}P_0(s) \quad (72)$$

$$P_2(s) = \frac{6\lambda^2}{A_1A_2}P_0(s) \quad (73)$$

$$P_{c1}(s) = \left[\frac{\lambda_{c10}}{s+r_1}+\frac{3\lambda\lambda_{c11}}{(s+r_1)A_1}+\frac{6\lambda^2\lambda_{c12}}{A_1A_2(s+r_1)}\right]P_0(s) \quad (74)$$

$$P_{c2}(s) = \left[\frac{\lambda_{c20}}{s+r_2}+\frac{3\lambda\lambda_{c21}}{(s+r_2)A_1}+\frac{6\lambda^2\lambda_{c22}}{A_1A_2(s+r_2)}\right]P_0(s) \quad (75)$$

$$P_{h1}(s) = \left[\frac{\lambda_{h10}}{s+z_1}+\frac{3\lambda\lambda_{h11}}{(s+z_1)A_1}+\frac{6\lambda^2\lambda_{h12}}{A_1A_2(s+z_1)}\right]P_0(s) \quad (76)$$

$$P_{h2}(s) = \left[\frac{\lambda_{h20}}{s+z_2}+\frac{3\lambda\lambda_{h21}}{(s+z_2)A_1}+\frac{6\lambda^2\lambda_{h22}}{A_1A_2(s+z_2)}\right]P_0(s) \quad (77)$$

Using equations (70) to (77), we get the Laplace transforms of the system availability and unavailability as follows:

$$AV(s) = \sum_{i=0}^{2}P_i(s)$$

$$= P_0(s)+P_1(s)+P_2(s) \quad (78)$$

and $$UAV(s) = P_3(s)+P_{c1}(s)+P_{c2}(s)+P_{h1}(s)+P_{h2}(s) \quad (79)$$

SPECIAL CASE MODEL IV:

For $n = 3$ and exponentially distributed failed system repair times the following steady-state probability expressions result:

$$P_0 = \left[1 + \frac{3\lambda}{A_3} + \frac{6\lambda^2}{A_3 A_4} + \frac{6\lambda^3}{\mu A_3 A_4} + \frac{\lambda_{c10}}{r_1} + \frac{3\lambda\lambda_{c11}}{A_3 r_1} + \frac{6\lambda^2\lambda_{c12}}{A_3 A_4 r_1} + \frac{\lambda_{c20}}{r_2} + \frac{3\lambda\lambda_{c21}}{A_3 r_2} + \frac{6\lambda^2\lambda_{c22}}{A_3 A_4 r_1} + \frac{\lambda_{h10}}{z_1} + \frac{3\lambda\lambda_{h11}}{A_3 z_1} + \frac{6\lambda^2\lambda_{h12}}{A_3 A_4 z_1} + \frac{\lambda_{h20}}{z_2} + \frac{3\lambda\lambda_{h21}}{A_3 z_2} + \frac{6\lambda^2\lambda_{h22}}{A_3 A_4 z_1} \right]^{-1} \quad (80)$$

Where $A_3 = 2\lambda + \lambda_{c11} + \lambda_{c21} + \lambda_{h11} + \lambda_{h21}$

$A_3 = \lambda + \lambda_{c12} + \lambda_{c22} + \lambda_{h12} + \lambda_{h22}$

$$P_1 = \frac{3\lambda}{A_3} P_0 \quad (81)$$

$$P_2 = \frac{6\lambda^2}{A_3 A_4} P_0 \quad (82)$$

$$P_3 = \frac{6\lambda^2}{\mu A_3 A_4} P_0 \quad (83)$$

$$P_{c1} = \left[\lambda_{c10} + \frac{3\lambda\lambda_{c11}}{A_3} + \frac{6\lambda^2\lambda_{c12}}{A_3 A_4} \right] \frac{P_0}{r_1} \quad (84)$$

$$P_{c2} = \left[\lambda_{c20} + \frac{3\lambda\lambda_{c21}}{A_3} + \frac{6\lambda^2\lambda_{c22}}{A_3 A_4} \right] \frac{P_0}{r_2} \quad (85)$$

$$P_{h1} = \left[\lambda_{h10} + \frac{3\lambda\lambda_{h11}}{A_3} + \frac{6\lambda^2\lambda_{h12}}{A_3 A_4} \right] \frac{P_0}{z_1} \quad (86)$$

$$P_{h2} = \left[\lambda_{h20} + \frac{3\lambda\lambda_{h21}}{A_3} + \frac{6\lambda^2\lambda_{h22}}{A_3 A_4} \right] \frac{P_0}{z_2} \quad (87)$$

The system steady state availability $AV(ss)$ and unavailability $UAV(ss)$ are given by

$$AV(ss) = \sum_{i=0}^{2} P_i \quad (88)$$

$$UAV(ss) = P_3 + P_{c1} + P_{c2} + P_{h1} + P_{h2} \quad (89)$$

SPECIAL CASE MODEL V:

This model is concerned with reliability analysis of the parallel system shown in Fig 1. Thus in the absence of repair, from fig 1, the following Laplace transforms of the state probabilities result.

$$P_0(s) = \frac{1}{s+N_1} \tag{90}$$

Where $N_1 = n\lambda + \lambda_{c10} + \lambda_{c20} + \lambda_{h10} + \lambda_{h20}$

$$P_1(s) = \frac{n\lambda}{(s+N_1)(s+N_2)} \tag{91}$$

Where $N_2 = (n-1)\lambda + \lambda_{c11} + \lambda_{c21} + \lambda_{h11} + \lambda_{h21}$

$$P_2(s) = \frac{n(n-1)\lambda^2}{(s+N_1)(s+N_2)(s+N_3)} \tag{92}$$

Where $N_3 = (n-2)\lambda + \lambda_{c12} + \lambda_{c22} + \lambda_{h12} + \lambda_{h22}$

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$$P_k(s) = \frac{\prod_{i=0}^{k-1}(n-i)\,\lambda^k}{\prod_{i=0}^{k} A_5} \tag{93}$$

for $k = 1,2,3,\ldots\ldots,(n-1)$,

Where $A_5 = s + (n-i)\,\lambda + \lambda_{c1i} + \lambda_{c2i} + \lambda_{h1i} + \lambda_{h2i}$

$$P_n(s) = \frac{\prod_{i=0}^{k}(n-i)\,\lambda^{k+1}}{s\prod_{i=0}^{k} A_5} \tag{94}$$

$$P_{c1}(s) = \frac{\lambda_{c10}}{s\,(s+N_1)} + \frac{1}{s}\sum_{j=1}^{k}\frac{\prod_{i=0}^{j-1}(n-i)\,\lambda^j\ \lambda_{c1j}}{\prod_{i=0}^{j} A_5} \tag{95}$$

$$P_{c2}(s) = \frac{\lambda_{c20}}{s\,(s+N_1)} + \frac{1}{s}\sum_{j=1}^{k}\frac{\prod_{i=0}^{j-1}(n-i)\,\lambda^j\ \lambda_{c2j}}{\prod_{i=0}^{j} A_5} \tag{96}$$

$$P_{h1}(s) = \frac{\lambda_{h10}}{s(s+N_1)} + \frac{1}{s}\sum_{j=1}^{k} \frac{\prod_{i=0}^{j-1}(n-i)\lambda^j \lambda_{h1j}}{\prod_{i=0}^{j} A_5} \tag{97}$$

$$P_{h2}(s) = \frac{\lambda_{h20}}{s(s+N_1)} + \frac{1}{s}\sum_{j=1}^{k} \frac{\prod_{i=0}^{j-1}(n-i)\lambda^j \lambda_{h2j}}{\prod_{i=0}^{j} A_5} \tag{98}$$

The Laplace Transform of the system reliability function is

$$R(s) = \sum_{i=0}^{k} P_i(s) = \frac{1}{s+N_1} + \sum_{j=1}^{k} \frac{\prod_{i=0}^{j-1}(n-i)\lambda^j}{\prod_{i=0}^{j} A_5} \tag{99}$$

The system mean time to failure

$$MTTF_S = \lim_{s\to 0} R(s) = \frac{1}{N_1} + \sum_{j=1}^{k} \frac{\prod_{i=0}^{j-1}(n-i)\lambda^j}{\prod_{i=0}^{j} A_5} \tag{100}$$

For $n=3$, $k=n-1=2$, $\lambda_c = \lambda_{c10} = \lambda_{c11} = \lambda_{c20} = \lambda_{c21} = \lambda_h = \lambda_{h10} = \lambda_{h11} = \lambda_{h20} = \lambda_{h21}$.

The equation (100) simplifies to

$$MTTF_S = \frac{1}{3\lambda + \lambda_{c10} + \lambda_{c20} + \lambda_{h10} + \lambda_{h20}} + \frac{6\lambda}{(2\lambda + 2\lambda_c + 2\lambda_h)(\lambda + 2\lambda_c + 2\lambda_h)} \tag{101}$$

For $\lambda = 10$ and $\lambda_c = \lambda_h = 2,3$ the plots of equation (101) are shown in Fig. 3.

These plots indicate that system mean time to failure decreases with the increasing values of λ_c and λ_h .

S.No	λ_c	λ_h	M.T.T.F
1	0	2	0.2079
2	1	2	01720
3	2	2	0.1453
4	3	2	0.125
5	4	2	0.1090
6	5	2	0.0962

S.No	λ_c	λ_h	M.T.T.F
1	0	3	01720
2	1	3	0.1453
3	2	3	0.125
4	3	3	0.1090
5	4	3	0.0962
6	5	3	0.0858

Similarly,

for $n = 2$, $k = n - 1 = 1$, $\lambda_c = \lambda_{c10} = \lambda_{c11} = \lambda_{c20} = \lambda_{c21} = \lambda_h = \lambda_{h10} = \lambda_{h11} = \lambda_{h20} = \lambda_{h21}$.

from equation (100) we get

$$MTTF_S = \frac{1}{2\lambda + \lambda_{c10} + \lambda_{c20} + \lambda_{h10} + \lambda_{h20}} + \frac{4\lambda}{(2\lambda + 2\lambda_c + 2\lambda_h)(\lambda + 2\lambda_c + 2\lambda_h)} \quad (102)$$

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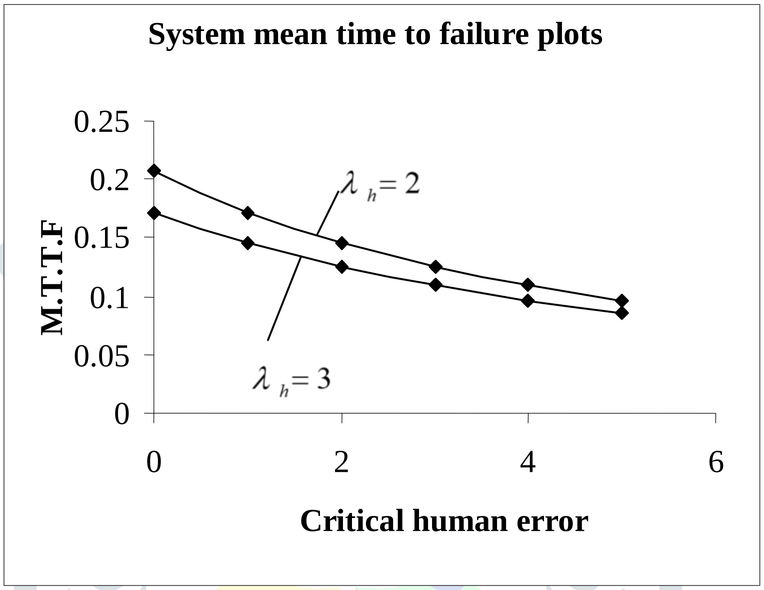

Fig.3

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