

Partition function and symmetrical applications of 1D,2D,3D and DD monoatomic ideal gas

Lalit Kumar¹, Jyoti Chaudhary², Sudarshan Shiromani¹

1. Meerut College Meerut
2. Shaheed Mangal Pandey Govt.Girls PG College, Meerut

Abstract:

Partition function has highly significant role in statistical physics. Once it is known in any thermodynamic system then various physical properties of the system can be easily and quickly evaluated. symmetry always play a marvellous role with respect to law of nature, as nature love symmetry so to make things beautiful. Now a days we are realizing that symmetry principals are most powerful in this nature. Success of symmetrical approach in context of general relativity and particle physics inspire us to apply this in statistical physics too. In this research paper we have applied symmetrical approach to evaluate partition function and its various application in various dimension for monoatomic ideal gas.

Introduction:

Monoatomic gas is such gas in which atoms are not bound to each other. noble gases such as Argon[Ar],Krypton[Kr] and Xenon [Xe] etc all are example of such gases. Such gases behave as ideal monoatomic gases at high temperature. monoatomic gases possesses only translatory motion and independent upon rotational and vibrational motion. Ideal monoatomic gas molecules does not exert any force upon each other,so such gases does not possesses any potential energy. Internal energy of monoatomic ideal gas is in the form of kinetic energy form only.

According to Maxwell Boltzmann statistics, no of particles in r^{th} state depends upon density of states g_r and energy ϵ_r and temperature T of any thermodynamic system

$$n_r = A g_r(\epsilon_r) e^{-\beta \epsilon_r}$$

Or

$$\frac{n_r}{g_r} = A e^{-\beta \epsilon_r}$$

$$f_{MB} = A e^{-\beta \epsilon_r}$$

$$\sum_r f_{MB} = 1$$

$$\sum_r A e^{-\beta \epsilon_r} = 1$$

$$A = \frac{1}{\sum_r e^{-\beta \epsilon_r}} = \frac{1}{Z}$$

Z is the partition function or sum over state of the system.

Suppose the level of energy ϵ_r be degenerate having degeneracy $g_r(\epsilon_r)$, then partition function

$$Z = \sum_r g_r(\epsilon_r) e^{-\beta \epsilon_r}$$

Helmholtz function $F = -NkT \log Z$

$$\text{Pressure } P = - \left[\frac{\partial F}{\partial V} \right]_{N,T}$$

$$\text{Energy } E = NkT^2 \frac{\partial}{\partial T} \log z$$

$$\text{Specific heat at constant volume } C_V = \left(\frac{\partial E}{\partial T} \right)_V$$

$$\text{Specific heat at constant pressure } C_p = \left(\frac{\partial E}{\partial T} \right)_p$$

$$\text{Entropy } S = - \left[\frac{\partial F}{\partial T} \right]_{N,V} = k \log Z + \frac{E}{T}$$

Symmetrical evaluation of partition function:

From the expression $Z = \sum_r g_r(\epsilon_r) e^{-\beta \epsilon_r}$

$$Z = \iint \frac{dx dp_x}{h} e^{-\beta \frac{p_x^2}{2m}}$$

Now using limit of x and p_x from $-\infty$ to $+\infty$ solving we get

$$Z_x = \frac{L_x}{h} (2\pi m k T)^{\frac{1}{2}} \quad \text{Partition function in 1D box}$$

For Rectangular box (2D) $Z = Z_x Z_y$

$$= \frac{L_x}{h} (2\pi m k T)^{\frac{1}{2}} \frac{L_y}{h} (2\pi m k T)^{\frac{1}{2}} = \frac{A}{h^2} (2\pi m k T)$$

Here $A = L_x L_y =$ area of rectangle box

$$\text{For Square box (2D) } Z = \frac{L}{h} (2\pi m k T)^{\frac{1}{2}} \frac{L}{h} (2\pi m k T)^{\frac{1}{2}} = \frac{A}{h^2} (2\pi m k T)$$

Here $A = L^2 =$ Area of square box

For Cuboid box (3D) $Z = Z_x Z_y Z_z$

$$= \frac{L_x}{h} (2\pi m k T)^{\frac{1}{2}} \frac{L_y}{h} (2\pi m k T)^{\frac{1}{2}} \frac{L_z}{h} (2\pi m k T)^{\frac{1}{2}}$$

$$= \frac{V}{h^3} (2\pi m k T)^{\frac{3}{2}} \quad \text{here } V = L_x L_y L_z = \text{volume of cuboid box}$$

$$\text{For Cubic box (3D) } Z = \frac{V}{h^3} (2\pi m k T)^{\frac{3}{2}} \quad \text{here } V = L^3 = \text{volume of cubic box}$$

From symmetry we have

$$\text{For D dimensional box } Z = \frac{L^D}{h^D} (2\pi m k T)^{\frac{D}{2}}$$

Result and Discussion:

Table 1.1:

Dimension	Partition Function Z
1D	$\frac{L_x}{h} (2\pi mkT)^{\frac{1}{2}}$
2D	$\frac{A}{h^2} (2\pi mkT)^{\frac{2}{2}}$
3D	$\frac{V}{h^3} (2\pi mkT)^{\frac{3}{2}}$
DD	$\frac{L^D}{h^D} (2\pi mkT)^{\frac{D}{2}}$

$$F^D = -NkT \log Z^D$$

$$P^D = - \left[\frac{\partial F^D}{\partial L^D} \right] = NkT \left[\frac{\partial}{\partial L^D} \log Z^D \right]$$

Table 1.2:

Dimension	Pressure like properties	SI unit
1D	$P^1 = F = \frac{NkT}{L}$	Force
2D	$P^2 = \tau = \frac{NkT}{A}$	Torque or Work
3D	$P^3 = p = \frac{NkT}{V}$	Pressure
DD	$P^D = NkT \left[\frac{\partial}{\partial L^D} \log Z^D \right]$	Depends upon D

In this table $P^1 = F =$ force exerted at the two end points of a segment of length L

$P^2 = \tau =$ surface tension exerted at the boundary of area A

$P^3 = p =$ pressure of a gas exerted at the two dimensional boundary

$$E^D = NkT^2 \frac{\partial}{\partial T} \log Z^D = NkT^2 \frac{\partial}{\partial T} \log \frac{L^D}{h^D} (2\pi mkT)^{\frac{D}{2}} = \frac{D}{2} Nk_B T$$

Table 1.3:

Dimension	Energy	Specific Heat $C_V = \left(\frac{\partial E}{\partial T} \right)_V$	Specific Heat $C_p = \left(\frac{\partial E}{\partial T} \right)_p$
1D	$\frac{1}{2} Nk_B T$	$\frac{1}{2} Nk_B$	$\frac{3}{2} Nk_B$
2D	$\frac{2}{2} Nk_B T$	$\frac{2}{2} Nk_B$	$\frac{4}{2} Nk_B$
3D	$\frac{3}{2} Nk_B T$	$\frac{3}{2} Nk_B$	$\frac{5}{2} Nk_B$
DD	$\frac{D}{2} Nk_B T$	$\frac{D}{2} Nk_B$	$\frac{D+2}{2} Nk_B$

$$\text{Entropy } S = k \log Z + \frac{E}{T}$$

Table 1.4:

Dimension	Entropy S
1D	$S^1 = Nk_B \left[\log \frac{L^1}{h^1} (2\pi mkT)^{\frac{1}{2}} + \frac{1}{2} \right]$
2D	$S^2 = Nk_B \left[\log \frac{L^2}{h^2} (2\pi mkT)^{\frac{2}{2}} + \frac{2}{2} \right]$
3D	$S^3 = Nk_B \left[\log \frac{L^3}{h^3} (2\pi mkT)^{\frac{3}{2}} + \frac{3}{2} \right]$
DD	$S^D = Nk_B \left[\log \frac{L^D}{h^D} (2\pi mkT)^{\frac{D}{2}} + \frac{D}{2} \right]$

$$\text{Enthalpy } H = E + PV$$

$$\text{For 1D } P^1 = F \text{ and } V^1 = L$$

$$\text{For 2D } P^2 = \tau \text{ and } V^2 = L^2 = A$$

$$\text{For 3D } P^3 = p \text{ and } V^3 = L^3 = V$$

Table 1.5:

Dimension	Enthalpy H(E,P,V)	Enthalpy H(D,T)
1D	$H^1 = E^1 + P^1V^1$	$H^1 = \frac{N}{2} k_B T + Nk_B T$
2D	$H^2 = E^2 + P^2V^2$	$H^2 = \frac{2N}{2} k_B T + Nk_B T$
3D	$H^3 = E^3 + P^3V^3$	$H^3 = \frac{3N}{2} k_B T + Nk_B T$
DD	$H^D = E^D + P^DV^D$	$H^D = \frac{DN}{2} k_B T + Nk_B T$

From the table 1.1 it can be concluded that partition function depends upon length L and mass of the particle m and temperature T and these parameters also depends upon power of the dimension D. Partition function dependency upon the length L^D , mass $m^{\frac{D}{2}}$ and temperature $T^{\frac{D}{2}}$ of the thermodynamic system.

From the table 1.2 it can be concluded that pressure like properties of ideal monoatomic gas depends upon total number of molecules and volume in different dimensions and it is directly proportional to total number of molecules and temperature T and inversely proportional to D dimensional volume of thermodynamic system.

From the table 1.3 it can be concluded that energy of ideal monoatomic ideal gas in different dimension is directly proportional to total number of molecules, temperature T and dimension D. Specific heat at constant volume and constant pressure is directly proportional to total number of the molecules and it also depends upon dimension D and remains independent upon temperature T.

From the table 1.4 it can be concluded that entropy of ideal monoatomic gas is directly proportional to total number of the molecules and it also depends upon logarithm of L^D , mass $m^{\frac{D}{2}}$, temperature $T^{\frac{D}{2}}$.

From the table 1.5 it can be concluded that enthalpy H of ideal monoatomic gas is directly proportional to total number of molecules and temperature T and it also depends upon dimension D of thermodynamic system.

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