A STATISTICAL ANALYSIS OF MACHINE REPAIR MODEL IN QUEUEING SYSTEM AND IT'S PERFORMANCE CRITERIA WITH MIXED STAND BY COMPONENTS AND SERVICE EFFICIENCY

Ankit Kaparwan*

* Deptt. of STATISTICS, H.N.B. Garhwal University, Srinagar Garhwal, Uttarakhand
E. Mail: ankitkaparwan@rediffmail.com

ABSTRACT

The present paper is aims to understand queueing model in order to analyze performance issues in machining system. We investigate queue model using variety of performance in different frame due to the range of the applicability and potential growth of controlling their system. The breakdown machine or component will lead to decrease in desire demand of production in any manufacturing system. The concept of mixed standby components is to replace the breakdown spare by cold or warm standby and breakdowncomponents go to the repair facility. Both the arrival patterns of breakdown parts of machine in service station and service pattern follows exponential distribution with queue pattern FCFS basis for C permanent repairperson and C subsidiary removal repairperson. If all the fixed repairpersons are engaged, the subsidiary removal repairperson will be available for repair work. With this connection we try to find of the issued performance of machining system consists of mixed spares and operating components maintained by a multi repairperson facility and queue size distribution in the form product.

KEY WORDS: Machining system, warm and cold spares, repairing method, subsidiary removalrepairperson, queueing model etc.

INTRODUCTION

We are involved or dependent upon the machining system for every activity of our lives. A machine become prone failure with regular uses or as the time passes. Due to the failure in machine or any components causes in decrease in output, money goodwilletc. when any machining systems failed, it is sent to the repair facilities. The plant or company always maintains additional standby machine to avoid any loss or failure so that a standby machines or spares can instantly replace with the substitutes when a spare of machine get failed. Any standby whichhas zero failure rates is said to be cold standby whereas any standby having non-zero failure rate and not more than the online unit failure as per warm standby. The machining systems presented in modern time are highly complex in nature. These are few complicated number of parts. Breakdown of any parts, machine directly affects the service system being shouted and sometimes the machine can have different reasons of being out service and its modes of failures. In this modern era, the failure and repair of spares are common events in complicated machining systems. Several frameworks are developed by many researchers to investigate the issues of machine repair system. Shawky [17] considered the system interference model M/M/c/K/N with balking, reneging and spares. Armstrong[1] discussed age repair concept for the machine repair issues while Arulmozhi [2] disclosed the reliability of an M-out-of-N warm standby with R repair functions. Jain and Singh [11] investigated M/M/m queue with balking, reneging and subsidiary servers. Probability analysis of a repairable system with warm standbys plus balking and reneging was discussed by Ke and Wang [14]. The M/M/c interdependent machining system with mixed spares and controlled rate of failure and repair was studied by Jain and Sharma [10]. Wang and Chiu [19] deals with cost benefits analysis of availability systems with warm standby units and imperfect coverage. Wang et al [22] discussed profit analysis ofthe M/M/R machine repair problem with balking, reneging and standby switching failures. Hassan etal [8] discussed a discrete time Geo/G/1 retrial queue with general retrial times and balking customers. Jain, Sharma and Sharma [12] considered the activities modelling of state dependent systems including mixed standby and breakdown modes. Mehrgani et al [16] studied the lockout/ tagout and operationalrisks in the manufacturing control of production system with additional redundancy. Reliability analysis of a warm standby repairable system with priority in use was discussed by Yuan and Meng [23]. Wang et al [21] defined optimization of cost and sensitivity analysis of the machine repair issues while Wang et al [20] defined comparative analysis of the machine repair issues without perfect coverage and service presume situation. Boudali and Economou [3] investigated optimal and equilibrium balking strategies with the single server M-queue with catastrophes. Ke and Wu [15] discussed multi-server machine repair model with standby and synchronous multiple vacation. Supriya Maheshwari and Shazia Ali [18] defined repair issues of machinery with mixed spares, balking and reneging. Gross and Harris [6] was studied machine repair model with standby. Gopalan [5] investigate probability analysis of a single server n unit system with (n-1) warm standby. Hilliard [9] defines a maintenance of float system with cost analysis approach. Ke and Wang [14] use balking, reneging and server breakdowns to analyzed the cost analysis of M/M/R machine repair problem with. Daduna [4] deals with exchangeable items in repair systems: delay times.

MODEL DESCRIPTIONS AND NOTATIONS

We prefer machine repair model with Balking process, reneging and mixed standby elements. To studythe issues we designed the model mathematically using following assumptions-

- ✓ The breakdown spares arrive at service station as FCFS discipline and service patterns follow exponential distribution life time.

- ✓ The system requires N normal functioning unit while work at least n operating units.
- ✓ In the system C fixed repairperson and c subsidiary removal repairperson to marginally maintain the units of manufaccturing. When the number of breakdown spares are more than C then subsidiary removal repairperson plays role depending upon work load.
- ✓ On finishing the repairing job, the repaired spare join the standby group. The cold standby is used in place of the operating unit in case of any operating failure if it is available otherwise replaced by warm standby.
- ✓ β is balking probability of the machine when all fixed repairperson are engaged and few standby units are ready to use, β_0 is balking probability if each standby units are exhausted andno subsidiary repairperson turn on.
- ✓ α is reneging rate when all the fixed repairperson are engaged and there is an availability of the standby units,α_0 is reneging rate when all the available standby units are exhausted and number of breakdown units is less and equal to threshold level T.

To develop the mathematical model, we use the following notations-

N : The number of operating unit in the system

n : The number of breakdown spares in the process

C : The number of fixed repairperson in the process

c : The number of subsidiary removal repairperson in the system

S : The number of warm standby units

Y : The number of cold standby units

λ : Operating unit failure rate

μ : The service rate of permanent repairperson

μ_i : The service rate of subsidiary removal repairperson

α : Reneging rate of spares

β : Joining probability of breakdown unit in the queue if few standby are ready to use

β_0 : Joining probability of breakdown unit in the queue when all standby units are exhausted andsubsidiary removal repairperson are not turns on

P_0 : Probability that there are not any breakdown spares in the system

P_n : Probability that there are n breakdown spares in the system at steady-state

a :The rate to which degree of repair is affected by the process state

K : The system’s capacity

PERFORMANCE MEASURE

Study of failure and service rates of the units are

$$\lambda(n) = \begin{cases} N\lambda + S\alpha & 0 \le n < C \\ (N\lambda + S\alpha\,) & C \le n < Y \\ [N\lambda + (S+Y-n)\alpha\,]\beta & Y \le n < Y+S \\ [N\lambda\beta_0 + (S+Y-n)\alpha\,]\beta & Y+S \le n < T \end{cases} \qquad (1)$$

$$\mu(n) = \begin{cases} n\mu & 0 \le n \le R \\ C\mu + (n-C)\alpha & C < n \le S+Y \\ C\mu_i + [n-(C-c)]\mu_i & rT < n < N+Y+S-m \end{cases} \qquad (2)$$

The steady-state equations, with suitable dependent rates given in (1) and (2)

$$-(N\lambda + S\alpha)P_0 + \mu P_1 = 0 \qquad (3)$$

$$-(N\lambda + S\alpha + n\mu)P_n + (N\lambda + S\alpha)P_{n-1} + (n+1)\mu P_{n+1} = 0 \qquad 0< n < C \qquad (4)$$

$$-[(N\lambda + S\alpha)\beta + C\mu]\ P_C + (N\lambda + S\alpha)P_{C-1} + \left(\frac{C+1}{C}\right)^a C\mu P_{C+1} = 0 \qquad (5)$$

$$-\left[(N\lambda + S\alpha)\beta + \left(\frac{n}{c}\right)^a C\mu\right]P_n + (N\lambda + S\alpha)\beta\ P_{n-1} + \left(\frac{n+1}{c}\right)^a C\mu\ P_{n+1} = 0;\quad C \le n \le Y$$

(6)

$$-\left|[N\lambda + (S+Y-n)\alpha]\,\beta + \left(\frac{n}{c}\right)^a C\mu\right|P_n + [N\lambda + (S+Y-n+1)\alpha]\beta P_{n-1} + \left(\frac{n+1}{c}\right)^a C\mu P_{n+1} = 0$$

$$Y < n < Y+S \qquad (7)$$

$$-\left|[N\lambda\beta_0 + (S+Y-n)\beta\lambda] + \left(\frac{n}{c}\right)^a C\mu\right|P_n + [N+S+Y-n+1]\beta\lambda\, P_{n-1} + \left[C\mu_i + \left(\frac{n+1}{c}\right)^a\right]C\mu P_{n+1} = 0$$

$$S+Y < n \le S+Y+K-1 \qquad (8)$$

$$-\left[(N+Y+S-T)\beta\lambda + \left(\frac{n}{c}\right)^a + (T-C)\right]P_T + [N\lambda\beta_0 + (Y+S-T)\lambda]P_{T-1} + \left[\left(\frac{n+1}{c}\right)^a + (T-C)\alpha_j\right]$$

$$C\mu P_{T+1} = 0 \qquad (9)$$

$$-\left(\frac{Y+S+K}{c}\right)^a C\mu P_{Y+S+K} + (N-K+1)\beta\lambda P_{Y+S+K-1} = 0 \qquad (10)$$

Solution of the equations (3) to (10) may be obtained by product type solution at steady-state, as

$$P_n = \begin{cases} \left(\dfrac{N\lambda+S\alpha}{\mu}\right)^n \cdot \dfrac{1}{n!} P_0 \quad ; & 0 \le n \le C \\ \dfrac{(N\lambda+S\alpha)^n.\beta^{n-C}}{\prod_{k=C+1}^{n}(C\mu+(k-C)\alpha)}.A_1P_0 \quad ; & C < n \le Y \\ \dfrac{(N\lambda+S\alpha)^{Y-c}.\prod_{i=Y+1}^{n}[N\lambda+(S+Y-i+1)\alpha].\beta^{n-C}}{\prod_{k=C+1}^{n}(C\mu+(k-C)\alpha)}.A_1P_0 \;; & Y+1 \le n \le Y+S \\ (N\lambda+S\alpha)^{Y-c}.\prod_{i=Y+1}^{Y+S}[N\lambda+(S+Y-i+1)\alpha]\lambda^{n-Y-S}.\beta^{n-C} & \\ \quad \times \dfrac{\prod_{i=Y+S+1}^{n}[N+Y+S-i+1]}{\prod_{k=C+1}^{n}(C\mu+(k-C)\alpha)}.A_2P_0 \;; & Y+S < n \le Y+S+K \end{cases} \tag{11}$$

where $A_1 = \left(\frac{N\lambda+S\alpha}{\mu}\right)^C \cdot \left(\frac{n!}{C!}\right)^a$ and $A_2 = (N\lambda\beta_0 + S\alpha)^{Y-C}.A_1$

Using normalizing condition, $\sum_{n=0}^{Y+S+K} P_n = 1$, we find P_0 as

$$P_0^{-1} = \left[\sum_{n=0}^{C}\frac{(N\lambda+S\alpha)^n}{n!.\mu^n} + \frac{1}{C!}\sum_{n=C+1}^{Y}\frac{(N\lambda+S\alpha)^n.\beta^{n-C}}{\prod_{k=C+1}^{n}(C\mu+(k-c)\alpha)} + \frac{(N\lambda+S\alpha)^{Y-C}.\sum_{n=Y+S+1}^{Y+S+K}\prod_{i=Y+1}^{Y+S}[N\lambda+(Y+S-i+1)\alpha].\beta^{n-C}}{\prod_{k=C+1}^{n}[C\mu+(k-C)\alpha]}.A_1 + \frac{\prod_{i=Y+S+1}^{n}[N+Y+S-i].\beta_j^{n-C}\lambda^{n-Y-S}}{\prod_{k=C+1}^{n}[C\mu+(k-C)\alpha].\beta_j}A_2\right] \tag{12}$$

PERFORMANCE PROCESS OF THE MEASURE OF THE MODEL

Some fundamental measure, we may obtain using queue size distributions as follows:

(i) Anticipated number of breakdown components in the process

$$E(n) = \sum_{n=0}^{Y+S+K} n.P_n \tag{13}$$

(ii) Anticipated number of breakdown components in the system

$$E(m) = \sum_{n=C}^{Y+S+K} (n-C).P_n \tag{14}$$

(iii) Anticipated number of handling units

$$E(o) = \sum_{n=Y+S+1}^{Y+S+K} (n-Y-S).P_n \tag{15}$$

(iv) Anticipated number of unused cold spares wait in the system

$$E(UCS) = \sum_{n=0}^{Y} (Y-n).P_n \tag{16}$$

(v) Anticipated number of unused warm spares wait in the system

$$E(UWS) = \sum_{n=0}^{S} P_n + \sum_{n=Y+1}^{Y+S} (Y + S - n).P_n \quad (17)$$

(vi) Anticipated idle fixed repairperson

$$E(IPR) = \sum_{n=0}^{C-1} (C - n).P_n \quad (18)$$

(vii) Anticipated number of busy permanent repairperson

$$E(BPR) = R - E(IPR) \quad (19)$$

(viii) Anticipated number of busy subsidiary removal repairperson

$$E(BAR) = \sum_{j=1}^{r-1} J. \sum_{n=jT+1}^{(j+1)T} P_n + \sum_{n=rT+1}^{Y+S+K} P_n \quad (20)$$

SPECIAL CASES

(a) When $a = 0$ & $\beta = \beta_j = 1$; Then the model is reduced to model without balking and reneging.

(b) When $Y = S = 0$; The the model is reduced to model with no spares.

(c) When $S = 0$; then the model is reduced to model with cold spares.

COST FUNCTION

To determine the optimal number of repairperson and spares, the cost function can be keep to a minimum.

$$E(C) = C_1 \sum_{n=0}^{Y+S} NP_n + C_2E(IPR) + C_3E(UCS) + C_4E(UWS) + C_5E(BPR) + C_6E(BAR) \quad (21)$$

where,

C_1 = Per unit cost time of a handling unit in normal working mode of system

C_2 = Per unit cost time for every ideal fixed repairman

C_3 = Per unit time cost for allowing a cold spare unit

C_4 = Per unit time cost for giving a warm spare unit.

C_5 = Per unit time cost of permanent repairperson when the person is engaged in repaire activities.

C_6 = Per unit time cost of subsidiary removal repairperson.

CONCLUSION

In the present paper, we discussed the machine repair model with balking, reneging and spare with subsidiary removal repairperson. In such type of machining system, we find the product type solutions for queue size distribution at steady- state. The concept of multiple standby and subsidiary features helps the officiating system in arranging consistence manufacturing to a inclination to words the demand in specific where there increase breakdown components in the procedure. Some measures anticipated number of breakdown component, anticipated number of handling unit, anticipated number of permanent repairperson and anticipate number of engaged and additional repairperson are discussed. The special cases have been deduced subject to earlier work done in this field also the costfunction to this model is suggested.

REFERENCES

1. Armstrong, M.J.: Age repair policies for the machine repair problem, Euro. J. Oper. Res., 138(1), 2002, 127-141.
2. Arulmozhi, G.: Reliability of an M-out-of-N warm standby with R repairs facilities, Opsearch, 39, 2002, 77-87.
3. Boudali, O. and Economou, A.: Optimal and equilibrium balking strategies in the single server M-queue with catastrophes, Euro. J. Oper. Res., 218(3), 2012, 708-715.
4. Daduna, H.: Exchangeable items in repair systems: delay times, Oper. Res., 38, 1990, 349-355.
5. Gopalan, M.N.: Probability analysis of a single server n unit system with (n-1) warm standby, Oper. Res., 23, 1975, 591-595,
6. Gross, D. and Harris, C.M.: Fundamental of queueing theory, IInd Edi., John Willey & Sons, New York, 1985.
7. Gupta, S.M.: N policy queueing system with finite sources and warm spares, OPSEARCH, 36(3),1999, 189-217.
8. Hassan, A.A., Rabia, S.I. and Taboly, F.A.: A discrete time Ge0/G/1 retrial queue with general retail times and balking customers, J. Korean Stati. Soci., 34(4), 2008, 335-348.
9. Hilliard, J.E.: An approach to cost analysis of maintenance float systems, AIIE Trans., 8, 1976, 128-133.
10. Jain, M and Sharma, G.C.: The M/M/c interdependent machining system with mixed spares and controlled rate of failure and repair, J. Raj. Acad. Phy. Sci., 3(2), 2004, 45-54.
11. Jain, M. and Singh, P.: M/M/m queue with balking, reneging and subsidiary servers, Int. J. Engg., 3, 2002, 107-114
12. Jain, M., Sharma, G.C. and Sharma, R.: Performance modelling of state dependent system with mixed standby and two modes of failure, Appl. Math. Model., 32(5), 2008, 712-724.
13. Ke, J.C. and Wang, K.H.: Cost analysis of the M/M/R machine repair problem with balking, reneging and server breakdowns, J. Oper. Res. Soci., 50, 1999, 275-282.
14. Ke, J.C. and Wang, K.H.: Probability analysis of a repairable system with warm standby plus balking and reneging, Appl. Math. Model., 24(4), 2003, 327-336.
15. Ke, J.C. and Wu, C.H.: Multi server machine repair model with standby and synchronous multiple vacation, Comp. Ind. Engg., 62, 2012, 296-212.
16. Mehrgani, B.E., Nadeau, S. and Kenne, J.P.: Lockout/ tagout and operational risks in the

production control of manufacturing system with passive redundancy, Int. J. Prod. Eco., 132(2), 2011, 296-305.

17. Shawky, A.I.: The machine interference model M/M/c/K/N with balking, reneging and spares, Opsearch, 37(1), 2000, 25-35.
18. Supriya Maheswari and Sazia Ali: Machine repair problem with mixed spares, balking and reneging, Int. J. Theo. And Appli. Sci., 5(1), 2013, 75-83.
19. Wang, K.H. and Chiu, L.W.: Cost benefits analysis of availability systems with warm standby units and imperfect coverage, Appl. Math. Comp., 172(2), 2006, 1239-1256.
20. Wang, K.H. Liu, Y.C. and Yang, D.Y.: Cost optimization and sensitivity analysis of the machine repair problem with variable server and balking, Proc. Soc. Beha. Sci., 25, 2011, 178-188.
21. Wang, K.H., Ke, J.B. and Ke, J.C.: Profit analysis of the M/M/R machine repair problem with balking, reneging and standby switching failures, Comp. and Oper. Res., 34(3), 2007, 835-847.
22. Wang, K.H., Liou, C.D. and Liu, Y.H.: Comparative analysis of the machine repair problem with imperfect coverage and service pressure condition, Appl. Math. Model.,
23. Yuan, L. and Meng, X.Y.: Reliability analysis of a warm standby repairable system with priority in use, Appl. Math. Model., 35(9), 2011, 4295-4303.