Rate 3/4 TCM Code having 16 State with 16 QAM for fading Channel

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Abstract- In designing of any digital communication system, coding and modulation are generally treated as two independent functions. The role of modulator and demodulator is restricted primarily to convert an analog waveform channel to a discrete channel and the role of encoder and decoder is to correct the errors those occurred in this discrete channel. To achieve the higher performance, code rate is lowered and the redundancy in the code increased at the cost of bandwidth expansion. This code can be integrated with the expanded bandwidth efficient signal set so as to utilize the redundancy resulted from such an expansion in order to avoid bandwidth expansion. The convolutional code when integrated with a bandwidth efficient modulation scheme is termed as Trellis Coded Modulation (TCM) [1]. TCM is a bandwidth efficient transmission scheme that can achieve high coding gain by integrating coding and modulation. It was discovered in the late 1970s and is presently used in many systems for modern information transmission. In this paper, a new scheme has been proposed for designing 16 State Rate 3/4 TCM code for fading Channel. The result has been quite encouraging and has indicated the coding gain of about 2dB over uncoded 8-Ary QAM. This helps in increasing the date rate especially in the case of Bandwidth Limited Channel.

I. INTRODUCTION

The idea of using an error correction coding scheme followed by a suitable modulation scheme has been in existence for long, in fact, multilevel modulation of convolutionally encoded symbols was a known concept before the introduction of TCM. Although the expansion of a signal set provides the redundancy required for coding, it shrinks the distance between the signal points if the average energy is kept constant. This has been shown in the Fig 1.

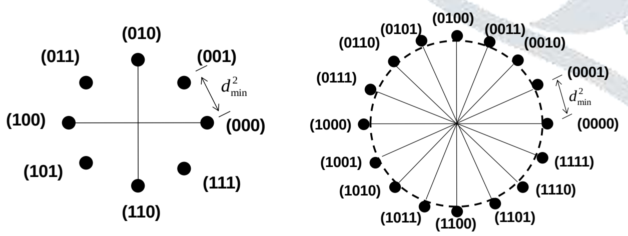

Fig. 1 Shrinking of distance after expansion of signal set

This reduction in the distance between the signal points increases the error rate, which needs to be compensated with the coding. Trellis Coded Modulation is an efficient coding technique, which achieves the coding gain at no cost of bandwidth. It allows reliable high data rate communication over channels with limited bandwidth. This makes the use of TCM very attractive for land mobile radio communications where the spectrum and the power are limited resources. The innovative aspect of TCM is the concept in which the convolutional encoding and the modulation are not treated as separate entities, but, as a unique combined operation. It combines the choice of a modulation scheme with that of a convolutional code together for the purpose of gaining noise immunity over uncoded transmission without expanding the signal bandwidth or increasing the transmitted power. Therefore the performance of TCM scheme in general is measured by coding gain over an uncoded signal.

The design of TCM scheme requires increasing the minimum free distance by considering the way modulation is related to the trellis. The symbol sent out from the modulator is not simply mapped from the encoder output. Instead, the overall signal constellation must be partitioned into signal sets. The encoder output chooses the signal set, while an additional systematic input to the modulator chooses the symbol within the signal set. Therefore, the different methods of symbol selection have a great impact on the performance of TCM.

The most important type of error in convolutional codes is pair-wise error, which occurs when the decoder selects a sequence other than the sequence the transmitter sent. The pair-wise error comes from incorrect state transitions. Therefore we need to assign constellation points whose Euclidean distance is as large as possible to the divergent branches and emergent branches. In this way, modulation is combined with the convolutional encoder.

The detection process is based on soft decision, rather than on the hard decision. The use of hard-decision demodulation prior to the decoding in a coded scheme causes an irreversible loss of information, which translates into a loss of SNR. If the maximum likelihood criterion is applied in soft-decision decoding on the fading channel, the decision rule of the optimum sequence decoder will depend on the Euclidean distance. In other words, the optimum decoder will choose the code sequence that is nearest to the received sequence in terms of Euclidean distance. TCM combined with interleaver of sufficient depth, can provide significant coding gains, compared to uncoded ones in case of fading channels, provided appropriate design criteria [2] is utilized in designing the code.

The rest of this paper is organized as follows. In Section II, the System Model in brief has been described. The proposed design along with the rules for systematically designing the optimum code for rate 3/4, 16-state, 16-QAM TCM scheme for the fading channel has been presented in section III. The construction of the code has been explained in Section IV. Performance analysis is presented in Section V. Section VI presents the result and also concludes the finding of the results achieved through simulation.

II. SYSTEM MODEL

A general block diagram of a TCM scheme [3] on a fading channel is shown in Fig. 2. Input bits are encoded by a trellis encoder to produce a sequence of signals $\boldsymbol{s}_l = (s_1, s_2 \dots s_l)$, where each signal S_i is a k-dimensional vector chosen from an M-QAM signal set and i denotes the current time index. Using complex notation, each of the signals S_i can also be represented by a point in a complex plane. The coded signals can also be interleaved to spread the bursts of errors. Also

before modulating the signal for transmitting over the channel pulse shaping be undertaken for eliminating the ISI. Finally the transmitted signal gets corrupted by an additive white Gaussian noise term while passing through channel.
At the receiver, the received signal is demodulated and quantized for decoding. The channel estimator can also be used to estimate the channel gain so that the same can be used in the decoding process to improve the performance of the coded system. The sequence $\boldsymbol{r}_l = (r_1, r_2, \ldots r_l)$ is the input to TCM decoder which performs maximum likelihood (ML) decoding.

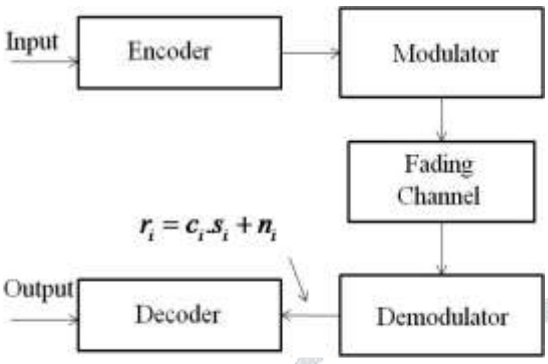

Fig. 2 System block diagram.

A discrete time model for the system of Fig. 2 is shown in Fig. 3. Using this model the received signal at time i can be written as

$$r_i = c_i . s_i + n_i \qquad (1)$$

where n_i is a sample of a zero-mean complex Gaussian noise process with variance $\sigma^2_n = N_0/2$ and the complex channel gain c_i is a sample of a complex Gaussian process with variance σ^2_c.
Using phasor notation, the complex channel gain c_i is expressed as

$$c_i = a_i \cdot e^{j\phi_i} \qquad (2)$$

where a_i and ϕ_i are the amplitude and the phase processes, respectively.

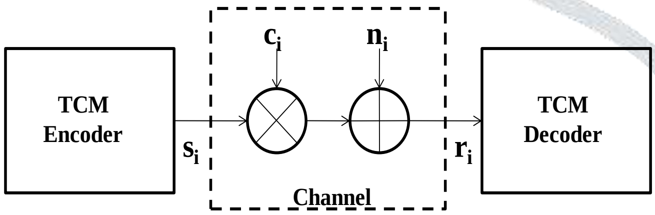

Fig. 3 Baseband system model of Fig.2

It is assumed that the receiver performs coherent detection, and hence the channel phase shift is compensated by the receiver and therefore (1) can be further simplified as

$$r_i = a_i s_i + n_i \qquad (3)$$

where a_i is the amplitude of the noise process.

III. Proposed Design

In this section a set of rules have been formulated which are used in designing and constructing rate 3/4, 16-state TCM codes, optimum for fading channels. These rules have been build up on similar rules that have been proposed earlier for 4-state, rate 2/3 TCM codes for fading channels [3].

Using heuristics, Ungerboeck has designed a rate 2/3, 8-state, 8PSK TCM that provides an effective length, L, of 2 and $d^2_{free} = 4.586E_s$, which was optimized for the fading channel [3].

In this paper, following guidelines have been proposed for designing the similar 16-state, 16-QAM TCM schemes optimized for the fading channel.

The signals associated with transitions between states of consecutive stages are represented by a matrix of dimension 16 x 16, whose ij^{th} element represents the signal associated with the path from state i, at stage k, to state j, at stage $k+1$, of the trellis. Also, the elements of the i^{th} row indicate signals associated with path diverging from state i and the elements of the j^{th} column show signals associated with paths reemerging at state j.

Using set partitioning as shown in Fig. 4, the 16-QAM signal set can be partitioned into two subsets, viz., $A_0 = \{s_0, s_2, s_4, s_6, s_8, s_{10}, s_{12}, s_{14}\}$ and $A_1 = \{s_1, s_3, s_5, s_7, s_9, s_{11}, s_{13}, s_{15}\}$ with minimum intra-set distance of δ_1.

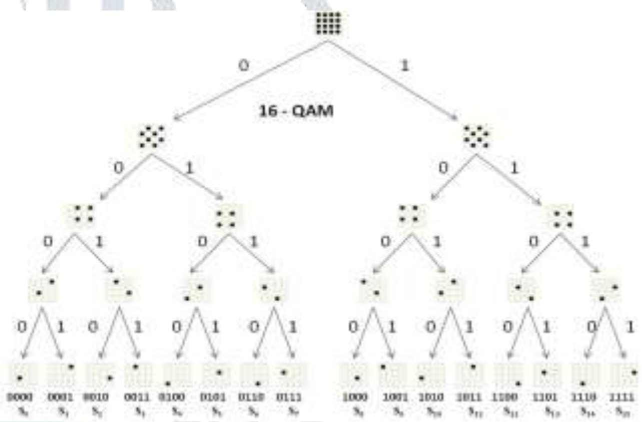

Fig. 4 Set partitioning method.

The signal constellation diagram is shown in Fig. 5.

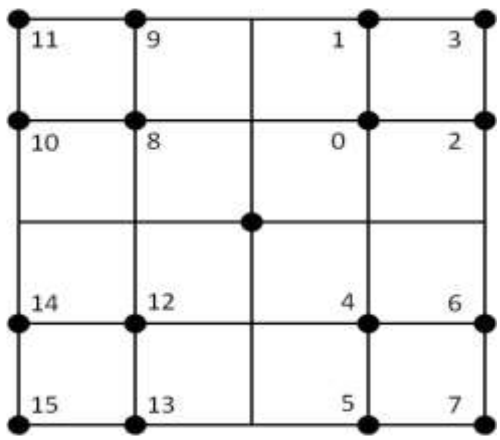

Fig. 5 16-QAM signal constellation.

To assign signal points to the elements of matrix, the following rules have been proposed:-

a. A signal can occur only once in a given row or column.

b. As the number of paths emerging from a state (for a rate 3/4 code) is only 8, and there are 16 states, all transitions are not possible. A signal can be associated with a transition path between two states, only if the LSB of the label of the initial state is the same bit, $y \in \{0,1\}$, as the MSB of the destination state. For a given value of y, all the signals associated should strictly be from one of the two sets A_0 or A_1. The output matrix for the proposed design scheme is shown in the Fig. 6.

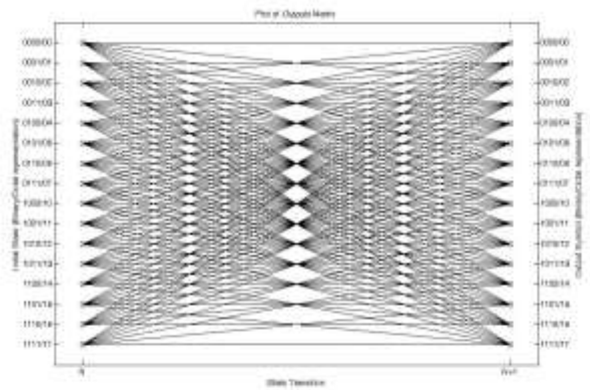

Fig. 6 Output matrix

c. The State Transition matrix for the proposed design is shown in Table 1.

State	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010	1011	1100	1101	1110	1111
0000	**S_{00}**	**S_{01}**	**S_{02}**	**S_{03}**	**S_{04}**	**S_{05}**	**S_{06}**	**S_{07}**	S_{08}	S_{09}	S_{0A}	S_{0B}	S_{0C}	S_{0D}	S_{0E}	S_{0F}
0001	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}	S_{16}	S_{17}	**S_{18}**	**S_{19}**	**S_{1A}**	**S_{1B}**	**S_{1C}**	**S_{1D}**	**S_{1E}**	**S_{1F}**
0010	**S_{20}**	**S_{21}**	**S_{22}**	**S_{23}**	**S_{24}**	**S_{25}**	**S_{26}**	**S_{27}**	S_{28}	S_{29}	S_{2A}	S_{2B}	S_{2C}	S_{2D}	S_{2E}	S_{2F}
0011	S_{30}	S_{31}	S_{32}	S_{33}	S_{34}	S_{35}	S_{36}	S_{37}	**S_{38}**	**S_{39}**	**S_{3A}**	**S_{3B}**	**S_{3C}**	**S_{3D}**	**S_{3E}**	**S_{3F}**
0100	**S_{40}**	**S_{41}**	**S_{42}**	**S_{43}**	**S_{44}**	**S_{45}**	**S_{46}**	**S_{47}**	S_{48}	S_{49}	S_{4A}	S_{4B}	S_{4C}	S_{4D}	S_{4E}	S_{4F}
0101	S_{50}	S_{51}	S_{52}	S_{53}	S_{54}	S_{55}	S_{56}	S_{57}	**S_{58}**	**S_{59}**	**S_{5A}**	**S_{5B}**	**S_{5C}**	**S_{5D}**	**S_{5E}**	**S_{5F}**
0110	**S_{60}**	**S_{61}**	**S_{62}**	**S_{63}**	**S_{64}**	**S_{65}**	**S_{66}**	**S_{67}**	S_{68}	S_{69}	S_{6A}	S_{6B}	S_{6C}	S_{6D}	S_{6E}	S_{6F}
0111	S_{70}	S_{71}	S_{72}	S_{73}	S_{74}	S_{75}	S_{76}	S_{77}	**S_{78}**	**S_{79}**	**S_{7A}**	**S_{7B}**	**S_{7C}**	**S_{7D}**	**S_{7E}**	**S_{7F}**
0000	**S_{80}**	**S_{81}**	**S_{82}**	**S_{83}**	**S_{84}**	**S_{85}**	**S_{86}**	**S_{87}**	S_{88}	S_{89}	S_{8A}	S_{8B}	S_{8C}	S_{8D}	S_{8E}	S_{8F}
1001	S_{90}	S_{91}	S_{92}	S_{93}	S_{94}	S_{95}	S_{96}	S_{97}	**S_{98}**	**S_{99}**	**S_{9A}**	**S_{9B}**	**S_{9C}**	**S_{9D}**	**S_{9E}**	**S_{9F}**
1010	**S_{A0}**	**S_{A1}**	**S_{A2}**	**S_{A3}**	**S_{A4}**	**S_{A5}**	**S_{A6}**	**S_{A7}**	S_{A8}	S_{A9}	S_{AA}	S_{AB}	S_{AC}	S_{AD}	S_{AE}	S_{AF}
1011	S_{B0}	S_{B1}	S_{B2}	S_{B3}	S_{B4}	S_{B5}	S_{B6}	S_{B7}	**S_{B8}**	**S_{B9}**	**S_{BA}**	**S_{BB}**	**S_{BC}**	**S_{BD}**	**S_{BE}**	**S_{BF}**
1100	**S_{C0}**	**S_{C1}**	**S_{C2}**	**S_{C3}**	**S_{C4}**	**S_{C5}**	**S_{C6}**	**S_{C7}**	S_{C8}	S_{C9}	S_{CA}	S_{CB}	S_{CC}	S_{CD}	S_{CE}	S_{CF}
1101	S_{D0}	S_{D1}	S_{D2}	S_{D3}	S_{D4}	S_{D5}	S_{D6}	S_{D7}	**S_{D8}**	**S_{D9}**	**S_{DA}**	**S_{DB}**	**S_{DC}**	**S_{DD}**	**S_{DE}**	**S_{DF}**
1110	**S_{E0}**	**S_{E1}**	**S_{E2}**	**S_{E3}**	**S_{E4}**	**S_{E5}**	**S_{E6}**	**S_{E7}**	S_{E8}	S_{E9}	S_{EA}	S_{EB}	S_{EC}	S_{ED}	S_{EF}	S_{6F}
1111	S_{F0}	S_{F1}	S_{F2}	S_{F3}	S_{F4}	S_{F5}	S_{F6}	S_{F7}	**S_{F8}**	**S_{F9}**	**S_{FA}**	**S_{FB}**	**S_{FC}**	**S_{FD}**	**S_{FE}**	**S_{FF}**

Table 1. State transitions

IV. CODE CONSTRUCTION

In this section TCM code for rate 3/4, 16-state, 16-QAM will be designed as per the rules outlined in the previous section. According to the second rule, as a first step, the transitions which are not permissible are eliminated and associated the subset A_0 with the LSB $z = 0$ and A_1 with LSB $z = 1$. Therefore, for signals of even numbered rows, the subset A_0 will be used and for signals of odd numbered rows the subset A_1 will be used. At this stage, any of the signal points of subset A_0 can be used as the first element of the first row and similarly any of the signal points of subset A_1 as the first valid element (i.e. one associated with the first possible transition) of the second row of transition matrix. Let us choose them to be s_0 and s_1 respectively. In the second step, the signals from subset A_0 and A_1 are assigned to the first, second, third and fourth row, correspondingly, as per the second and the third rule. Now, in the next step, either s_2 or s_6 can be chosen as the first element of the fifth row and s_3 or s_7 as the first valid element of the sixth row. Choosing s_6 and s_7 respectively, the remaining signals in remaining rows are assigned as per the second and the third rule. This completes the code design. The trellis diagram for this code construction is depicted in Fig. 7.

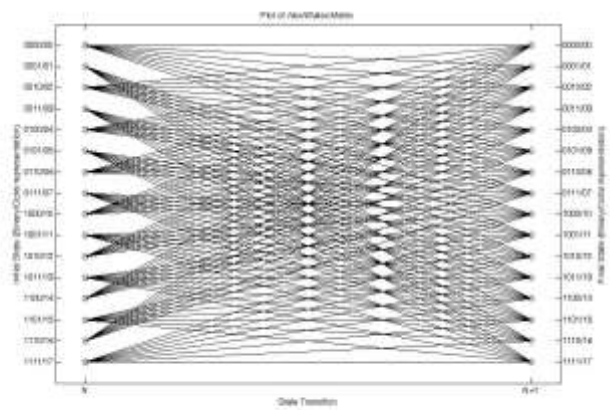

Fig. 7 Trellis diagram of the rate 3/4, 16-state, 16 QAM TCM

V. PERFORMANCE ANALYSIS

Using the union bound, an upper bound on the bit error probability can be obtained by taking into consideration the effect of all possible error events of all lengths. Unfortunately, in enumerating such error events the generating function approach [3] cannot be employed due to the intractable form of the exact expression for $P_2(\boldsymbol{s}_l, \hat{\boldsymbol{s}}_l)$. However, the bit error probability can be estimated reasonably well by considering a small number of dominant error events rather than by accounting for the error event paths of all lengths. Hence the bit error probability can be approximated as

$$P_b \approx \frac{1}{n}\sum_{l=1}^{\lambda}\sum_{s_l \neq \hat{s}_l} \bar{n}_l \, P_2(s_l, \hat{s}_l) \qquad (4)$$

where n_l is the average number of bit errors associated with the error event $(\boldsymbol{s}_l, \hat{\boldsymbol{s}}_l)$, n is the number of information bits per symbol and λ is a limit imposed on the actual length of the error events to be considered. The performance bound was evaluated for the fading channel and was used for the evaluation of the pairwise error event probability through the MATLAB simulation. The Bit Error Rate Vs SNR for the Rate 3/4, 16 State 16-QAM TCM code designed using the rules proposed has been shown in Fig. 8. Bit Error Rate Vs SNR for the uncoded 8-QAM and in respect of BPSK has also been plotted in the same figure.

VI. RESULT AND CONCLUSION

It is concluded from the result that approximately two dB gain has been achieved by using the proposed scheme over the uncoded 8-QAM scheme, however the gain achieved is significant over BPSK. This implies that higher data rate can be achieved by implementing the proposed TCM coding scheme for communicating over the fading channel.

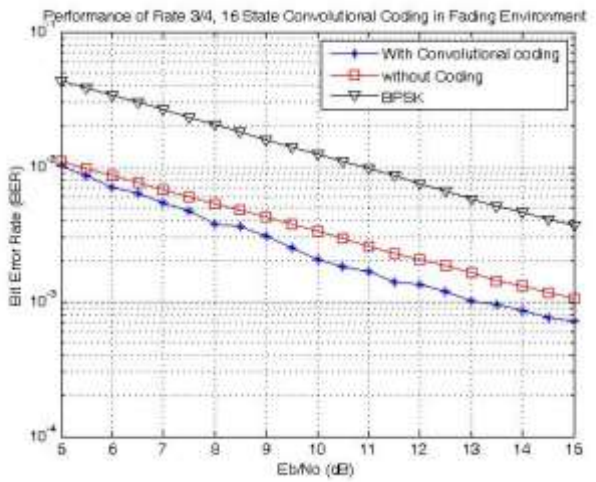

Fig. 8 Bit Error Rate Vs SNR

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