A Single Server Batch Arrival Queueing System with Constant Batch Size and State Dependent Service

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Abstract:

Single server batch arrival queueing model with constant batch size and state dependent service is considered. Customers arrive in fixed batch size b according to Poisson stream with the rate λ. State dependent service is provided with exponential service rates μ and $\mu_1 (\mu < \mu_1)$. The performance measures of the system such as mean number of consumers in the system and in the queue, mean waiting time in the queue, mean time spent in the system, by using generating function method are found.

Keywords: Bulk queue, Poisson, exponential, generating function.

1. Introduction

In queueing theory, customers arrive randomly according to a Poisson process and form a queue. In a typical computer and communication system the situation that the messages arrive at a center from source in batches of constant size 'b' and they transmitted to the device as soon as possible according to its requirement. Such a situation can be modelled as queues with bulk arrivals and constant batch size. For comprehensive study one can refer Gross [7] et al., and Choudhry and Templeton [5] . There have been a number of researches on the bulk queueing system. Some of them are briefly discussed here. Abolnikov et al. [1] considered a class of bulk queueing systems with a compound Poisson input modulated by semi-Markov process, multilevel control service time and queue length dependent, service delay discipline. Bagyam et al. [2] analysed bulk arrival general service retrial queueing system where server provides two phases of service-essential and optimal. After each service completion, the server The repair process does not start immediately after a breakdown and there is a delay time waiting for repairs to start. Baruah et al. [3] explored a queueing model with two stage heterogeneous service where customer arrival in batches and has a single server providing service in two stages, one after the other in succession. Baruah et al. [4] again studied the behaviour of a batch arrival queuing system equipped with a single server providing general arbitrary service to customers with different service rates in two fluctuating modes of service. In addition, the server is subject to random breakdown. As soon as the server faces breakdown, the customer whose service is interrupted comes back to the head of the queue.

Ghimire et al.,[6] studied M/M/1 queue with heterogeneous arrival and departure with server vacations and breakdowns. Haridass et al., [8] derived some operating characteristics of a queueing system with unreliable server and single vacation in which, if the server is on and subjected to fail, the arrival rate depends on the up and down states of the server. Jain and Bhargava [9] analysed the unreliable server bulk arrival retrial queue with two types of subscribers in which type I is the priority subscribers and type II is the non-pre-emptive priority subscribers. Khalaf et al. [10] studied a batch arrival queueing system in which the server may face occasional random breakdowns. The repair process does not start immediately after a breakdown and there is a delay time waiting for repairs to start. Khalaf et al. [11] dealt a queueing system with four different main server interruptions and a stand-by server replaces the main server during any potential stop. The main server has five statues in this system where it is either services the customers or it does not work.

Sikdar et al.,[12] investigated a batch arrival single-server queue with renewal input and multiple exponential vacations. Shinde and Patankar [13] studied the state dependent bulk service queueing system with balking, reneging and multiple vacations where (a-1) arrivals waiting in the queue, server will wait for some time (called the change over time), in spite of going for a vacation. Singh et al. [14] investigated a single-server bulk arrival queueing model, wherein Poisson arrival rate of the units is state dependent, and service time is arbitrary distributed. It is also assumed that the system is subject to breakdown, and the failed server immediately joins the repair facility which takes constant duration to repair the server. The rest of the paper is organized as follows: Section 2 presents the mathematical model of the queueing system, Section 3 describes the probability distributions, the performance measure of the system is derived in Section 4, Section 4 illustrates the numerical study for a particular situation of the model and finally Section 5 concludes the paper.

2. The System

In modern computer and communication system, transmitting messages/calls are modelled as bulk arrival queueing system with constant batch size and state dependent service. Messages/calls arrive in batches of constant size 'b' according to Poisson distribution with rate λ. Service time of each message / call follows exponential distribution with rates μ and μ_1 respectively, where $\mu < \mu_1$. If there are less than b messages/ calls in the system, server provides service with rate μ whereas more than or equal b messages / calls in the system, each receive service with rate μ_1 so that the system is more realistic and avoid losing subscribers from the system network. The system is stable if

$$\rho = \frac{\lambda b}{\mu}\text{, when batch size} < b$$

and

$$\rho_1 = \frac{\lambda b}{\mu_1}\text{, when batch size} \geq b$$

The Notations:

Define the random variables for the system in steady state.

W_s - waiting time of messages / calls in the system.

W_q - waiting time messages / calls in the queue.

L_s - mean number of messages / calls in the system.

L_q - mean number of messages / calls in the queue.

Let π_k (k = 0, 1, ... , n, …..) be steady-state probability that there are k messages / calls in the queueing system.

For $|z| \leq 1$, define $G(z) = \sum_{k=0}^{\infty} \pi_k z^k$

G(z) – Probability generating function of the number of messages / calls in the system in steady state.

3. The Probability Distributions

We derive the probability distribution of number of messages / calls in the system in steady state at arbitrary time the usual arguments lead to the following steady state balance equation.

$\lambda\pi_0 = \mu\pi_1$ $\quad k = 0$ (1)

$(\lambda+\mu)\pi_k = \mu\pi_{k+1}$ $\quad 1 \le k \le b-1$ (2)

$(\lambda+\mu_1)\pi_k = \lambda\pi_{k-b} + \mu_1\pi_{k+1}$ $\quad k \ge b$ (3) and the normalization condition $\sum_{k=0}^{\infty}\pi_k = 1$.

Multiplying z^k both sides of (2) and (3) and then summing from 1 to ∞. Using (1), we get

$$\lambda\pi_0 + \sum_{k=1}^{b-1}(\lambda + \mu)\pi_k z^k + \sum_{k=b}^{\infty}(\lambda + \mu_1)\pi_k z^k$$

$$= \mu\pi_1 + \mu \sum_{k=1}^{b-1} \pi_{k+1} z^k + \lambda \sum_{k=b}^{\infty} \pi_{k-b} z^k + \mu_1 \sum_{k=b}^{\infty} \pi_{k+1} z^k$$

On simplifying above equation, we get

$$[\lambda z(1-z^b) + \mu_1(1-z)]G(z) = (1-z)\left[(\mu-\mu_1)\sum_{k=1}^{b-1}\pi_k z^k - \mu_1\pi_0\right]$$

$$G(z) = \frac{(\mu_1-\mu)\sum_{k=1}^{b-1}\pi_k z^k + \mu_1\pi_0}{\mu_1 - \lambda\sum_{k=1}^{b} z^k}$$

Since G(1) = 1,

$$\pi_0 = 1 - \frac{\lambda b}{\mu_1} - \left(1 - \frac{\mu}{\mu_1}\right)\sum_{k=1}^{b-1}\pi_k$$

$$G(z) = \frac{\mu_1 - \lambda b - (\mu_1 - \mu)\sum_{k=1}^{b-1}\pi_k(1-z^k)}{\mu_1 - \lambda\sum_{k=1}^{b} z^k}$$

4. Performance Measures of the System

By straightforward calculations, the mathematical expressions for the operating characteristics are obtained for this queueing system:

(i) Mean number of consumers in the system, L_s:

$$L_s = \frac{\left(1-\frac{\mu}{\mu_1}\right)\frac{\lambda}{\mu}\,\pi_0\sum_{k=0}^{b-1} k\left(\frac{\lambda}{\mu}+1\right)^{k-1} + \lambda b\,\frac{(b+1)}{2}}{(1-\rho_1)}$$

Where $\pi_0 = \left(1-\frac{\lambda b}{\mu_1}\right)\left[1+\left(1-\frac{\mu}{\mu_1}\right)\left(\frac{\lambda}{\mu}\right)\sum_{k=0}^{b-1}\left(\frac{\lambda}{\mu}+1\right)^{k-1}\right]^{-1}$

(ii) Mean waiting time of consumer in the system W_s:

By Little's formula,

$$W_s = \frac{L_s}{\lambda b} = \frac{\left(1-\frac{\mu}{\mu_1}\right)\frac{\lambda}{\mu}\,\pi_0\sum_{k=0}^{b-1} k\left(\frac{\lambda}{\mu}+1\right)^{k-1} + \lambda b\,\frac{(b+1)}{2}}{\lambda b(1-\rho_1)}$$

(iii) Average number consumers in the queue, L_q:

$$L_q = L_s - \frac{\lambda b}{\mu_1} = \frac{\left(1-\frac{\mu}{\mu_1}\right)\frac{\lambda}{\mu}\,\pi_0 \sum_{k=0}^{b-1} k\left(\frac{\lambda}{\mu}+1\right)^{k-1} + \lambda b\,\frac{(b+1)}{2}}{(1-\rho_1)} - \rho_1$$

(iv) Average waiting time of consumers in the queue is W_q:

$$W_q = \frac{L_q}{\lambda b} = \frac{\left(1-\frac{\mu}{\mu_1}\right)\frac{\lambda}{\mu}\,\pi_0 \sum_{k=0}^{b-1} k\left(\frac{\lambda}{\mu}+1\right)^{k-1} + \lambda b\,\frac{(b+1)}{2}}{\lambda b\,(1-\rho_1)} - \frac{\rho_1}{\lambda b}$$

$$W_q = \frac{\left(1-\frac{\mu}{\mu_1}\right)\frac{\lambda}{\mu}\,\pi_0 \sum_{k=0}^{b-1} k\left(\frac{\lambda}{\mu}+1\right)^{k-1}}{\lambda b\,(1-\rho_1)} + \frac{(b+1)}{2\,(1-\rho_1)} - \frac{1}{\mu_1}$$

5. Numerical Study

Numerical result is studied for a particular situation. The parameter values and the batch size is as follows: Table – 5.1

λ	μ	μ_1	b	Ls	Ws	Lq	Wq
2	20	35	3	0.53961	0.08994	0.36818	0.06136
4	20	35	3	1.28867	0.10739	0.94581	0.07882
6	20	35	3	2.47487	0.13749	1.96058	0.10892
8	20	35	3	4.82518	0.20105	4.13946	0.17248
10	20	35	3	12.55814	0.41861	11.701	0.39003

Table – 5.2

λ	μ	μ_1	b	ls	ws	lq	wq
10	100	200	3	0.497738	0.016591	0.347738	0.011591
15	100	200	3	0.793778	0.01764	0.568778	0.01264
20	100	200	3	1.135831	0.018931	0.835831	0.013931
25	100	200	3	1.541463	0.020553	1.166463	0.015553
30	100	200	3	2.037851	0.022643	1.587851	0.017643

From the table, as the arrival rate increases gradually with different service rates, the corresponding increases is observed in the performance measures Ls , Lq, Ws and in the Wq where as there is a marginal increase in the performance measures between $\lambda = 8$ and $\lambda = 10$, which is observed from the table 5.1.

Particular case:

If $\mu_1 = \mu$, then the above system is coincided with the system considered by D. Gross et. al., [7]

6. Conclusion

The explicit formulae derived in this paper have their real life applications in computer and communication system. Some performance measures explicitly by using the probability generating function method have been obtained. This model can be studied under the provision of state dependent service with different service rates, which takes the system to more realistic environment and advantage over the model with constant batch size, which was discussed by Gross et.al.[7].

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